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# Definition System and Criteria Related to the Static Stability of Airplanes (First Report) —Definition System, Static Flight Speed Stability, and Static Angle of Attack Stability—

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This paper discusses the static stability concepts of airplanes. The definition of static stability has not been systematically discussed. For example, although "$C_{m\alpha} < 0$," "$C_{\ell\beta} < 0$," and "$C_{n\beta} > 0$" are merely the aerodynamic criteria corresponding to each static stability, they are used just like the definition of static stability. Moreover, "flight path stability" is well known as the static stability concept on one translational motion, but the stability concepts in regard to the heaving and sliding motions of an airplane are indefinite. And though "tuck under" and "flight path divergence" are also static stability problems, they are treated not systematically but individually as special problems. So, this paper proposes a definition system consisting of six static stability concepts. Moreover, in the first report of this paper, "static flight speed stability" and "static angle of attack stability" are discussed in detail, and their stability criteria in terms of the aerodynamic characteristics of an airframe are obtained.

**Key Words:** Aircraft, Stability and Control, Static Stability

## Nomenclature

| | |
|---|---|
| $b$ : | wing span |
| $\bar{c}$ : | mean aerodynamic chord of wing |
| $C_D, C_L, C_m$ : | coefficients of drag, lift, and pitching moment |
| $C_{D\alpha}, C_{L\alpha}, C_{m\alpha}$ : | $\partial C_D/\partial\alpha$, $\partial C_L/\partial\alpha$, $\partial C_m/\partial\alpha$ |
| $C_{Du}, C_{Lu}, C_{mu}$ : | $\partial C_D/\partial(\Delta U/U)$, $\partial C_L/\partial(\Delta U/U)$, $\partial C_m/\partial(\Delta U/U)$ |
| $C_{D\delta_e}, C_{L\delta_e}, C_{m\delta_e}$ : | $\partial C_D/\partial\delta_e$, $\partial C_L/\partial\delta_e$, $\partial C_m/\partial\delta_e$, |
| $C_{L\delta_{DLC}}, C_{Y\delta_{DSFC}}, C_{\ell\delta_a}, C_{n\delta_r}$ : | $\partial C_L/\partial\delta_{DLC}$, $\partial C_Y/\partial\delta_{DSFC}$, $\partial C_\ell/\partial\delta_a$, $\partial C_n/\partial\delta_r$ |
| $C_\ell, C_n, C_Y$ : | coefficients of rolling moment, yawing moment, and side force |
| $C_{\ell\beta}, C_{n\beta}, C_{Y\beta}$ : | $\partial C_\ell/\partial\beta$, $\partial C_n/\partial\beta$, $\partial C_Y/\partial\beta$ |
| $C_T$ : | thrust coefficient |
| $D, D_{net}$ : | aerodynamic drag and net drag $(= D - T)$ |
| DLC, DSFC : | direct lift control and direct side force control |
| FC.0 : | initial trimmed flight condition |
| FC.1 : | steady flight condition after a flight variable of interest is changed from FC.0 |
| FC.2 : | nonequilibrium flight condition just after the change in primary control in FC.1 is quickly returned to zero |
| $L$ : | aerodynamic lift |
| $M, N$ : | aerodynamic pitching moment and yawing moment |
| $p, q, r$ : | rolling, pitching, and yawing angular velocities |
| $R$ : | aerodynamic rolling moment |
| $S$ : | wing area |
| $T$ : | thrust produced by propulsion system |
| $U$ : | flight speed |
| $W$ : | weight of airplane |
| $Y$ : | aerodynamic side force |
| $\alpha, \beta, \gamma$ : | angle of attack, sideslip angle, and flight path angle |
| $\delta_a$ : | aileron deflection (positive: right aileron down and left aileron up) |
| $\delta_e$ : | elevator deflection (positive: elevator trailing edge down) |
| $\delta_r$ : | rudder deflection (positive: rudder trailing edge left) |
| $\delta_{DLC}$ : | DLC deflection (positive: lift $L$ increases) |
| $\delta_{DSFC}$ : | DSFC deflection (positive: side force $Y$ increases) |
| $\rho$ : | air density |
| $\Theta, \Phi, \Psi$ : | pitch, roll, and yaw attitudes |
| $\Delta$ : | small increment |
| Subscript 0, 1, 2 : | FC.0, FC.1, FC.2 |



## 1. Introduction

"$C_{m\alpha} < 0$" (or "$\partial C_m/\partial C_L < 0$"), "$C_{\ell\beta} < 0$," and "$C_{n\beta} > 0$" are usually used as the criteria of static stability on pitching, rolling, and yawing motions (we call them, respectively, "*static pitch stability,*" "*static roll stability,*" and "*static yaw stability*") and appear to be considered just the same as the definition of each static stability.[1–6] However, although "$C_{m\alpha} < 0$," "$C_{\ell\beta} < 0$," and "$C_{n\beta} > 0$" are the effective aerodynamic characteristics for an airplane to be statically stable, the problem is that they can be confused with the essential definitions of static stability. For example, the flight variable $\alpha$ in $C_{m\alpha}$ does not denote the rotational motion, but the heaving translational motion, of an airplane, though static pitch stability is a stability concept on pitching motion. This inconsistency can be pointed out similarly in "$C_{\ell\beta} < 0$" and "$C_{n\beta} > 0$."

To know these fundamental definitions, we should remember how flight tests for judging static stability are carried out. For example, in the flight test on "static pitch stability," at first the new steady flight condition in which the pitch attitude is different from the initial trimmed attitude is established by use of the elevator. Next, the elevator is soon returned quickly to the initial deflection. At this flight condition, if the resulting pitching moment restores pitch attitude, the static pitch stability is judged to be stable. This means that the essence of static pitch stability is whether the pitching moment $\Delta M$ restoring the change in pitch attitude $\Delta\Theta$ arises or not. Therefore "$\mathrm{d}M/\mathrm{d}\Theta < 0$" should be used as the fundamental definition of static pitch stability. Similarly, "$\mathrm{d}R/\mathrm{d}\Phi < 0$" and "$\mathrm{d}N/\mathrm{d}\Psi < 0$" should be respectively used for static roll stability and static yaw stability.

These three stability concepts are concerned with the rotational motions of an airplane. Since an airplane can move in six degrees of freedom directions, three static stability concepts on translational motions must exist similarly. One is well known as "flight path stability."[7,8] This stability depends on whether the net drag $D_{\mathrm{net}}$ $(= D - T)$ becomes the restoring force for flight speed $U$. Thus this static stability is stable when $\mathrm{d}D_{\mathrm{net}}/\mathrm{d}U > 0$, called "*static flight speed stability*." However, the stability concepts on heaving velocity (or angle of attack) and the sliding velocity (or sideslip angle) of an airplane have not necessarily been clearly discussed. The former is called "*static angle of attack stability*" and the latter "*static sideslip stability*." These stability concepts should also be incorporated systematically in the definition system on static stability.

Furthermore, "tuck under"[5,9] and "flight path divergence"[10] are peculiar static problems. These phenomena should be also explained on the basis of the definition system mentioned above.

The first report of this paper proposes a definition system of static stability on the six degrees of freedom motions, and "static flight speed stability" and "static angle of attack stability" are discussed in detail to deduce their stability criteria in terms of the aerodynamic characteristics of an airplane. The other static stability concepts are then discussed in the second and third reports.

## 2. Definition System of Static Stability

Static stability concepts can be established for each of the six degrees of an airplane's flight motions. The combination of flight variable and restoring force are (1) flight speed $U$ and net drag $D_{\mathrm{net}}$; (2) sideslip angle $\beta$ (or sliding velocity) and side force $Y$; (3) angle of attack $\alpha$ (i.e., heaving velocity) and lift $L$; (4) roll attitude $\Phi$ and rolling moment $R$; (5) pitch attitude $\Theta$ and pitching moment $M$; (6) yaw attitude $\Psi$ and yawing moment $N$.

### 2.1. Procedure for judging static stability

The procedures in flight tests for judging the static stability of an airplane are as follows. At first, an airplane flies in an initial trimmed flight condition (a flight condition called "*FC.0*"). Next, to give a small change to a flight variable of interest in FC.0, an external force is applied to the airplane by use of its control surfaces, propulsion system, for example, and a new steady flight condition is established (a condition called "*FC.1*"). The relation between an applied force and a flight variable of interest is the same as the above relation between a flight variable and a restoring force. For example, $L$ is applied to change $\alpha$, and $M$ is applied to change $\Theta$. (We call the control to generate these applied forces "*primary control*.")

Then by returning the primary control quickly to the initial position, the applied force in FC.1 is quickly removed (a condition called "*FC.2*"). If the nonequilibrium aerodynamic force resulting at this FC.2 acts to restore the flight variable of interest, the static stability is stable, and vice versa.

Incidentally, the applied force in FC.1 balances against the restoring force mentioned above. This means that static stability can be judged by checking the sense of the primary control in FC.1.

Besides aerodynamic control surfaces and propulsion systems, it is then also possible to utilize the components of gravity force as the primary control giving the external thrust or side force. They are "$-W\sin\gamma$" and "$W\cos\gamma\sin\Phi$," respectively, and depend on flight path angle $\gamma$ or roll attitude $\Phi$. Thus $\gamma$ or $\Phi$ plays the role of the parameter of primary control.

To detect the restoring force as purely as possible at FC.2, we must satisfy the following requirements.

(1) The aerodynamic damping forces caused by angular velocities $(p, q, r)$ of an airframe must not be mixed into the aerodynamic restoring forces. Therefore in this paper, FC.0 and FC.1 are assumed to be steady straight flights (i.e., $p = q = r = 0$) for simplicity.

(2) Gravity force components must not work as restoring forces. Thus if the restoring force of interest is drag, lift, or side force, the flight path angle $\gamma$ and the roll attitude $\Phi$ in FC.1 must be the same as those in FC.0.

(In this paper, the subsidiary use of control surfaces for satisfying the requirement (1) and (2) is called "*secondary control*.")

(3) It is desirable that the applied force produced by the primary or secondary control acts as the corresponding flight variable as directly as possible. Therefore a thrust force of the propulsive system is used for $U$. Similarly, an elevator, an aileron, and a rudder are respectively used for $\Theta$, $\Phi$, and $\Psi$. And a direct lift control (DLC) system and a direct side

force control (DSFC) system are necessary to obtain the lift changing $\alpha$ and the side force changing $\beta$, respectively. Conventional airplanes are equipped without DLC and DSFC systems. This is apparently one reason why the static stability concepts on $\alpha$ and $\beta$ have been weakly recognized.

### 2.2. Systematic definition of static stability

Based on the discussion in the previous section, a total of six static stability concepts can be set systematically as follows. It is assumed that the thrust force produced by a propulsion system acts on the line of aerodynamic drag and is independent for $U$ and $\alpha$ (i.e., if the throttle position is constant, $T_0 = T_1 = T_2$). And as a general rule in this paper, the aerodynamic coefficients and derivatives without subscript 0, 1, or 2 show the values in FC.0.

(a) ***Static flight speed stability***: Although the flight path angle in FC.0 is maintained with the elevator as a secondary control, FC.1 is established by changing the thrust force of the propulsion system as a primary control. When the change in thrust $\Delta T$ is quickly returned to zero (i.e., at FC.2), if the resulting nonequilibrium net drag $\Delta D_{\mathrm{net}}$ works to restore the change in flight speed $\Delta U$, the static flight speed stability is stable. Therefore the fundamental definition for static flight speed stability to be stable is given by

$$\mathrm{d}D_{\mathrm{net}}/\mathrm{d}U > 0 \tag{1}$$

The additive thrust in FC.1 is equal to $\Delta D_{\mathrm{net}}$ (i.e., $\Delta T = \Delta D_{\mathrm{net}}$). Thus the derived form of definition can be shown by

$$\mathrm{d}T/\mathrm{d}U > 0 \tag{2}$$

Furthermore, the thrust component of gravity force can be used as the primary control and is shown by

$$\begin{aligned}&\Delta(\text{thrust componet of gravity force})\\ &\quad = -W\sin(\gamma_0+\Delta\gamma) - (-W\sin\gamma_0) \cong -W\cos\gamma_0\cdot\Delta\gamma\end{aligned} \tag{3}$$

The small increment in flight path angle $\Delta\gamma$ is obtained by using the elevator. Because this thrust component equals $\Delta D_{\mathrm{net}}$ in FC.1 (i.e., $-W\cos\gamma_0\Delta\gamma = \Delta D_{\mathrm{net}}$) and "$W\cos\gamma_0 = \text{const.} > 0$," the other derived form of definition can be obtained as follows:

$$(-W\cos\gamma_0)\mathrm{d}\gamma/\mathrm{d}U > 0, \quad \text{or} \quad \mathrm{d}\gamma/\mathrm{d}U < 0 \tag{4}$$

This is well known as the definition of "flight path stability."[10)]

(b) ***Static angle of attack stability***: Although the flight path angle in FC.0 is held with the elevator as a secondary control, FC.1 is established by changing the lift force produced by the DLC deflection as a primary control. At FC.2, when the change in DLC deflection is quickly returned to zero, if the nonequilibrium lift $\Delta L$ acts to restore the increment in angle of attack $\Delta\alpha$, the static angle of attack stability is stable. Thus the fundamental definition on static angle of attack stability is given by

$$\mathrm{d}L/\mathrm{d}\alpha > 0 \tag{5}$$

The additive lift caused by the DLC deflection in FC.1 balances against $\Delta L$ (i.e., $[\rho U_1{}^2/2]SC_{L\delta_{\mathrm{DLC}}}\Delta\delta_{\mathrm{DLC}} = -\Delta L$). Since $C_{L\delta_{\mathrm{DLC}}}$ is defined to be positive in this paper, the derived form of the above-mentioned definition is given by

$$(\rho U_1{}^2/2)SC_{L\delta_{\mathrm{DLC}}}(\mathrm{d}\delta_{\mathrm{DLC}}/\mathrm{d}\alpha) < 0, \quad \text{or} \quad \mathrm{d}\delta_{\mathrm{DLC}}/\mathrm{d}\alpha < 0 \tag{6}$$

(c) ***Static pitch stability***: The pitch attitude is changed from FC.0 by the elevator control as a primary control, and FC.1 is then established. At FC.2, when the change in the elevator is quickly returned to zero, if the nonequilibrium pitching moment $\Delta M$ acts to restore the increment in pitch attitude $\Delta\Theta$, the static pitch stability is stable, and its fundamental definition is given by

$$\mathrm{d}M/\mathrm{d}\Theta < 0 \tag{7}$$

Since the additive pitching moment produced by the elevator in FC.1 balances against $\Delta M$ (i.e., $(\rho U_1{}^2/2)S\bar{c}C_{m\delta_e}\Delta\delta_e = -\Delta M$) and $C_{m\delta_e} < 0$, the derived form of the above fundamental definition is given by

$$(\rho U_1{}^2/2)S\bar{c}C_{m\delta_e}\cdot(\mathrm{d}\delta_e/\mathrm{d}\Theta) > 0 \quad \text{or} \quad \mathrm{d}\delta_e/\mathrm{d}\Theta < 0 \tag{8}$$

(d) ***Static sideslip stability***: Although the roll attitude and the straight flight in FC.0 is maintained with the aileron and the rudder as a secondary control, FC.1 is established by controlling the side force produced by the DSFC system as a primary control. At FC.2, when the additive DSFC deflection is quickly returned to zero, if the resulting nonequilibrium side force $\Delta Y$ works to restore the increment in sideslip angle $\Delta\beta$, the static sideslip stability is stable, and thus its fundamental definition is given by

$$\mathrm{d}Y/\mathrm{d}\beta < 0 \tag{9}$$

Since the additive side force of the DSFC deflection in FC.1 balances against $\Delta Y$ (i.e., $[\rho U_1{}^2/2]SC_{Y\delta_{\mathrm{DSFC}}}\Delta\delta_{\mathrm{DSFC}} = -\Delta Y$), the derived form of the above definition is given by

$$(\rho U_1{}^2/2)SC_{Y\delta_{\mathrm{DSFC}}}(\mathrm{d}\delta_{\mathrm{DSFC}}/\mathrm{d}\beta) > 0 \quad \text{or} \quad \mathrm{d}\delta_{\mathrm{DSFC}}/\mathrm{d}\beta > 0 \tag{10}$$

where $C_{Y\delta_{\mathrm{DSFC}}}$ is defined to be positive in this paper.

Moreover, the side force component of gravity force caused by the change in roll attitude can be used as the primary control and is shown by

$$\begin{aligned}&\Delta(\text{side force componet of gravity force})\\ &\quad = W\cos\gamma_0\sin(\Phi_0+\Delta\Phi) - W\cos\gamma_0\sin\Phi_0\\ &\quad \cong W\cos\gamma_0\cos\Phi_0\cdot\Delta\Phi\end{aligned} \tag{11}$$

The increment in roll attitude $\Delta\Phi$ is obtained by using the aileron, and the steady straight flight is maintained by using the rudder. Since this side force component balances against $\Delta Y$ in FC.1 (i.e., $W\cos\gamma_0\cos\Phi_0\Delta\Phi = -\Delta Y$) and $W\cos\gamma_0\cos\Phi_0 = \text{const.} > 0$, the other derived form of definition can be obtained as follows:

$$(W\cos\gamma_0\cos\Phi_0)\mathrm{d}\Phi/\mathrm{d}\beta > 0, \quad \text{or} \quad \mathrm{d}\Phi/\mathrm{d}\beta > 0 \tag{12}$$

This definition is the same as the requirement in MIL-F-8785B 3.3.6.2 "Side forces in steady sideslips."[7)]

(e) ***Static roll stability***: Although the straight flight in FC.0 is held with the rudder as a secondary control, FC.1 is established by the additive rolling moment produced by the aileron as a primary control. This means that FC.1 is a steady sideslip flight. At FC.2, when the additive aileron deflection is quickly returned to zero, if the resulting nonequilibrium rolling moment $\Delta R$ acts to restore the increment in roll attitude $\Delta\Phi$, the static roll stability is stable and its fundamental definition is given by

$$\mathrm{d}R/\mathrm{d}\Phi < 0 \tag{13}$$

The additive rolling moment of the aileron in FC.1 balances against $\Delta R$ (i.e., $[\rho U_1{}^2/2]SbC_{\ell\delta_a}\Delta\delta_a = -\Delta R$). Thus since $C_{\ell\delta_a} < 0$, the derived form of the above fundamental definition is given by

$$(\rho U_1{}^2/2)SbC_{\ell\delta_a}(\mathrm{d}\delta_a/\mathrm{d}\Phi) > 0, \quad \text{or} \quad \mathrm{d}\delta_a/\mathrm{d}\Phi < 0 \tag{14}$$

(f) ***Static yaw stability***: Although the straight flight in FC.0 is held with the aileron as a secondary control, FC.1 is established by the change in yawing moment produced by the rudder as a primary control. That is, FC.1 is a steady sideslip flight. At FC.2, when the additive rudder deflection is quickly returned to zero, if the resulting nonequilibrium yawing moment $\Delta N$ works to restore the increment in yaw attitude $\Delta\Psi$, the static yaw stability is stable and its fundamental definition is given by

$$\mathrm{d}N/\mathrm{d}\Psi < 0 \tag{15}$$

The additive yawing moment of the rudder in FC.1 balances against $\Delta N$ (i.e., $[\rho U_1{}^2/2]SbC_{n\delta_r}\Delta\delta_r = -\Delta N$). Since $C_{n\delta_r} < 0$, the derived form of the above definition is given by

$$(\rho U_1{}^2/2)SbC_{n\delta_r}(\mathrm{d}\delta_r/\mathrm{d}\Psi) > 0, \quad \text{or} \quad \mathrm{d}\delta_r/\mathrm{d}\Psi < 0 \tag{16}$$

A summary of the matter mentioned in (a)∼(f), that is, the definition system of static stability, is shown in Table 1.

## 3. Static Flight Speed Stability

In this chapter, the detail of "static flight speed stability" is discussed in two instances; (1) a propulsion system is used as a primary control, and (2) a gravity force component is used as a primary control. The stability criteria in terms of the aerodynamic characteristics of airframe are then deduced.

### 3.1. In a situation in which a propulsion system is used as a primary control

FC.1, in which the flight speed is changed from that in FC.0, that is $U_1 = U_0 + \Delta U$, is established by use of the thrust change $\Delta T$ of the propulsion system, while maintaining the same flight path angle as in FC.0 by use of the elevator. At FC.2, when the throttle position is quickly returned to the initial position, if Eq. (1) is satisfied the static flight speed stability is stable. Moreover, because $\Delta T$ equals the restoring force $\Delta D_{\mathrm{net}}$ in FC.1, the derived form of definition can be described by Eq. (2).

#### 3.1.1. Equilibrium equation and restoring force

The equilibrium equations on the drag, lift, and pitching moment in FC.0 and FC.1 are respectively described as follows.

"In FC.0":

$$\text{Net drag:} \quad D_{\mathrm{net}0}(= D_0 - T_0) = (\rho U_0{}^2/2)SC_D - T_0 = -W\sin\gamma_0 \tag{17a}$$

$$\text{Lift:} \quad L_0 = (\rho U_0{}^2/2)SC_L = W\cos\gamma_0 \tag{17b}$$

$$\text{Pitching moment:} \quad M_0 = (\rho U_0{}^2/2)S\bar{c}C_m = 0 \tag{17c}$$

Table 1. Definition system of static stability.

| | Foundamental definition | Derived form of definition | Control equipment | |
|---|---|---|---|---|
| | | | Primary control | Secondary control |
| Static flight speed stability | $\frac{\mathrm{d}D_{\mathrm{net}}}{\mathrm{d}U} > 0$ | $\frac{\mathrm{d}T}{\mathrm{d}U} > 0$ | Propulsion system | Elevator (Holding flight path angle) |
| | | $\frac{\mathrm{d}\gamma}{\mathrm{d}U} < 0$ | Gravity force component, or flight path angle | — |
| Static angle of attack stability | $\frac{\mathrm{d}L}{\mathrm{d}\alpha} > 0$ | $\frac{\mathrm{d}\delta_{\mathrm{DLC}}}{\mathrm{d}\alpha} < 0$ | DLC system | Elevator (Holding flight path angle) |
| Static pitch stability | $\frac{\mathrm{d}M}{\mathrm{d}\Theta} < 0$ | $\frac{\mathrm{d}\delta_e}{\mathrm{d}\Theta} < 0$ | Elevator | — |
| Static sideslip stability | $\frac{\mathrm{d}Y}{\mathrm{d}\beta} < 0$ | $\frac{\mathrm{d}\delta_{\mathrm{DSFC}}}{\mathrm{d}\beta} > 0$ | DSFC system | Aileron/rudder (Holding roll attitude and straight flight) |
| | | $\frac{\mathrm{d}\Phi}{\mathrm{d}\beta} > 0$ | Gravity force component or roll attitude | Rudder (Holding straight flight) |
| Static roll stability | $\frac{\mathrm{d}R}{\mathrm{d}\Phi} < 0$ | $\frac{\mathrm{d}\delta_a}{\mathrm{d}\Phi} < 0$ | Aileron | Rudder (Holding straight flight) |
| Static yaw stability | $\frac{\mathrm{d}N}{\mathrm{d}\Psi} < 0$ | $\frac{\mathrm{d}\delta_r}{\mathrm{d}\Psi} < 0$ | Rudder | Aileron (Holding straight flight) |

"In FC.1":

Net drag: $D_{\mathrm{net1}}(= D_1 - T_1) = -W\sin\gamma_0$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D + C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e\right) - (T_0+\Delta T) = -W\sin\gamma_0 \tag{18a}$$

Lift: $L_1 = W\cos\gamma_0$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_L + C_{Lu}\frac{\Delta U}{U_0} + C_{L\alpha}\Delta\alpha + C_{L\delta_e}\Delta\delta_e\right) = W\cos\gamma_0 \tag{18b}$$

Pitching moment: $M_1 = 0$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\bar{c}\left(C_{mu}\frac{\Delta U}{U_0} + C_{m\alpha}\Delta\alpha + C_{m\delta_e}\Delta\delta_e\right) = 0 \tag{18c}$$

The nonequilibrium net drag $\Delta D_{\mathrm{net}}$ at FC.2, when the additive thrust $\Delta T$ in FC.1 is returned to zero, is the aerodynamic net drag resulting when $\Delta T$ is eliminated from the equilibrium condition "$D_{\mathrm{net1}} + W\sin\gamma_0 = 0$" in FC.1 and is shown by

$$\begin{aligned}\Delta D_{\mathrm{net}} &= (D_{\mathrm{net1}} + W\sin\gamma_0) - (-\Delta T)\\ &= \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D + C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{L\delta_e}\Delta\delta_e\right) - T_0 + W\sin\gamma_0\\ &= \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D + C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{L\delta_e}\Delta\delta_e\right) - \frac{1}{2}\rho {U_0}^2 S C_D\end{aligned} \tag{19a}$$

Once the change in thrust $\Delta T$ from Eq. (17a) and Eq. (18a) is obtained, this $\Delta T$ is reasonably shown to be also equal to $\Delta D_{\mathrm{net}}$, and thus the definitions Eq. (1) and Eq. (2) are equivalent.

By expanding Eq. (19a) and neglecting the insignificantly small products and squares of the changes in variables, we can obtain the following linear equation.

$$\Delta D_{\mathrm{net}} = \frac{1}{2}\rho {U_0}^2 S\left\{(2C_D + C_{Du})\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e\right\} \tag{19b}$$

### 3.1.2. Criteria for static flight speed stability

Total differential d$z$ in the vicinity of point $(x, y, \cdots)$ of "$z = f(x, y, \cdots)$" has the following relation.

$$\mathrm{d}z = (\partial z/\partial x)\mathrm{d}x + (\partial z/\partial y)\mathrm{d}y + \cdots \tag{20}$$

Therefore, since $\Delta D_{\mathrm{net}}$ is a function of $\Delta U$, $\Delta\alpha$ and $\Delta\delta_e$ as shown in Eq. (19b), Eq. (1) can be described by

$$\frac{\mathrm{d}D_{\mathrm{net}}}{\mathrm{d}U} = \frac{\partial D_{\mathrm{net}}}{\partial U} + \frac{\partial D_{\mathrm{net}}}{\partial\alpha}\cdot\frac{\mathrm{d}\alpha}{\mathrm{d}U} + \frac{\partial D_{\mathrm{net}}}{\partial\delta_e}\cdot\frac{\mathrm{d}\delta_e}{\mathrm{d}U} > 0 \tag{21}$$

The partial differentials in Eq. (21) are given by differentiating Eq. (19b) as follows.

$$\partial D_{\mathrm{net}}/\partial U = \rho U_0 S(2C_D + C_{Du})/2 \tag{22a}$$

$$\partial D_{\mathrm{net}}/\partial\alpha = \rho {U_0}^2 S\cdot C_{D\alpha}/2 \tag{22b}$$

$$\partial D_{\mathrm{net}}/\partial\delta_e = \rho {U_0}^2 S\cdot C_{D\delta_e}/2 \tag{22c}$$

The differentials d$\alpha$/d$U$ and d$\delta_e$/d$U$ in Eq. (21) are deduced as follows. By subtracting Eq. (17a) from Eq. (18a) and ignoring the insignificantly small products and squares of the changes in variables, the following equation is obtained.

$$\frac{1}{2}\rho {U_0}^2 S\left(C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e\right) + \rho U_0 S C_D\Delta U - \Delta T = 0$$

or

$$(2C_D + C_{Du})(\Delta U/U_0) + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e = \Delta C_T \tag{23a}$$

where

$$\Delta C_T = \Delta T/(\rho {U_0}^2 S/2) \tag{23b}$$

In the same way, the following equations are obtained from Eq. (17b) and (18b) and Eq. (17c) and (18c).

$$(2C_L + C_{Lu})(\Delta U/U_0) + C_{L\alpha}\Delta\alpha + C_{L\delta_e}\Delta\delta_e = 0 \tag{24}$$

$$C_{mu}(\Delta U/U_0) + C_{m\alpha}\Delta\alpha + C_{m\delta_e}\cdot\Delta\delta_e = 0 \tag{25}$$

Next, once the simultaneous equations of Eq. (23a), (24), and (25) are solved, the ratio of $\Delta U$, $\Delta\alpha$, and $\Delta\delta_e$ to $\Delta C_T$ is obtained as follows:

$$(\Delta U/U_0)/\Delta C_T = (C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha})/A \tag{26a}$$

$$\Delta\alpha/\Delta C_T = \{C_{L\delta_e}C_{mu} - C_{m\delta_e}(2C_L + C_{Lu})\}/A \tag{26b}$$

$$\Delta\delta_e/\Delta C_T = \{C_{m\alpha}(2C_L + C_{Lu}) - C_{mu}C_{L\alpha}\}/A \tag{26c}$$

where

$$\begin{aligned}A = {} & C_{m\delta_e}\{(2C_D + C_{Du})C_{L\alpha} - (2C_L + C_{Lu})C_{D\alpha}\}\\ &+ C_{D\delta_e}\{(2C_L + C_{Lu})C_{m\alpha} - C_{mu}C_{L\alpha}\}\\ &- C_{L\delta_e}\{(2C_D + C_{Du})C_{m\alpha} - C_{mu}C_{D\alpha}\}\end{aligned} \tag{27}$$

Thus

$$\frac{\mathrm{d}\alpha}{\mathrm{d}U} = \frac{1}{U_0}\left(\frac{\Delta\alpha}{\Delta C_T}\right)\Big/\left(\frac{\Delta U/U_0}{\Delta C_T}\right) = \frac{C_{L\delta_e}C_{mu} - C_{m\delta_e}(2C_L + C_{Lu})}{U_0(C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha})} \tag{28a}$$

$$\frac{\mathrm{d}\delta_e}{\mathrm{d}U} = \frac{1}{U_0}\left(\frac{\Delta\delta_e}{\Delta C_T}\right)\Big/\left(\frac{\Delta U/U_0}{\Delta C_T}\right) = \frac{C_{m\alpha}(2C_L + C_{Lu}) - C_{mu}C_{L\alpha}}{U_0(C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha})} \tag{28b}$$

When Eq. (22a)~(22c), (28a), and (28b) are applied to Eq. (21), the stability criterion in terms of the aerodynamic characteristics of the airplane can be obtained as follows:

$$2C_D + C_{Du} + \frac{C_{L\delta_e}C_{mu}C_{D\alpha}}{C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha}} + \frac{C_{D\delta_e}\{C_{m\alpha}(2C_L + C_{Lu}) - C_{mu}C_{L\alpha}\}}{C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha}} - \frac{C_{m\delta_e}C_{D\alpha}(2C_L + C_{Lu})}{C_{m\delta_e}C_{L\alpha} - C_{L\delta_e}C_{m\alpha}} > 0 \tag{29}$$

When $C_{L\delta_e}$ and $C_{D\delta_e}$ are small and may be ignored as they are in regard to conventional airplanes, Eq. (29) is given by

$$\{C_{L\alpha}(2C_D + C_{Du}) - C_{D\alpha}(2C_L + C_{Lu})\}/C_{L\alpha} > 0 \tag{30}$$

This Eq. (30) is the general form of criterion on static flight speed stability for conventional airplanes.

Moreover, it can be considered that $C_{Lu} = C_{Du} = 0$ in low subsonic flights and that $C_L > 0$ in normal flights. Then Eq. (30) is further simplified to be the same as the familiar form of criterion on "flight path stability," as follows:

$$C_D/C_L - C_{D\alpha}/C_{L\alpha} > 0 \tag{31}$$

It is well known that in the frontside flight region, Eq. (31) is satisfied and the static flight speed stability is stable; this does not occur in the backside region. However, the static flight speed stability in the stall region is stable because $C_D/C_L > 0$, $C_{L\alpha} < 0$ and $C_{D\alpha} > 0$.

### 3.2. In a situation in which a gravity force component as a primary control

It is possible to use the thrust (or drag) component of gravity force instead of the propulsion system. This component can be obtained by $\gamma$ change as a primary control, and $\gamma$ is controlled by use of the elevator. When $\Delta U$ arises with $\Delta\gamma$ from FC.0 to FC.1, the definition of static flight speed stability is shown by Eq. (4). In this section it is shown that Eq. (4) is equivalent to the fundamental definition Eq. (1).

The equilibrium equations in "FC.0" are given by Eq. (17a)~(17c). The equations in "FC.1" are given by

Net drag: $D_{\mathrm{net1}} (= D_1 - T_1) = -W\sin(\gamma_0 + \Delta\gamma)$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D + C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e\right) - T_0 = -W\sin(\gamma_0+\Delta\gamma) \tag{32a}$$

Lift: $L_1 = W\cos(\gamma_0 + \Delta\gamma)$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_L + C_{Lu}\frac{\Delta U}{U_0} + C_{L\alpha}\Delta\alpha + C_{L\delta_e}\Delta\delta_e\right) = W\cos(\gamma_0+\Delta\gamma) \tag{32b}$$

Pitching moment: $M_1 = 0$,

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\bar{c}\left(C_{mu}\frac{\Delta U}{U_0} + C_{m\alpha}\Delta\alpha + C_{m\delta_e}\Delta\delta_e\right) = 0 \tag{32c}$$

When Eq. (17a) is subtracted from Eq. (32a), the change in the thrust component of gravity force is obtained as the following:

$$\text{the thrust component change of gravity force} = -W\cos\gamma_0\Delta\gamma = \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D + C_{Du}\frac{\Delta U}{U_0} + C_{D\alpha}\Delta\alpha + C_{D\delta_e}\Delta\delta_e\right) - \frac{1}{2}\rho {U_0}^2 S C_D \tag{33}$$

When Eq. (33) is compared with Eq. (19a), it is obvious that this gravity force component is equal to $\Delta D_{\mathrm{net}}$. Consequently, since $W\cos\gamma_0 = \text{const.} > 0$, the definition Eq. (1) can be rewritten to Eq. (4), that is, "$\mathrm{d}\gamma/\mathrm{d}U < 0$."

## 4. Static Angle of Attack Stability

In this chapter, the detail of "static angle of attack stability" is discussed, assuming that the airplane of interest has a DLC system.

FC.1, in which the angle of attack is changed from that in FC.0, that is $\alpha_1 = \alpha_0 + \Delta\alpha$, is obtained with the lift change $\Delta L_{\mathrm{DLC}}$ by use of the DLC system, maintaining the same flight path angle as in FC.0 by use of the elevator. At FC.2, when the additive DLC deflection in FC.1 is quickly returned to zero, if the resulting nonequilibrium lift $\Delta L$ acts to restore $\Delta\alpha$ to zero, the static angle of attack stability is stable and is defined by Eq. (5). Moreover, because the additive lift of the DLC system is balanced against $\Delta L$ in FC.1, the derived form of definition is given by Eq. (6).

In the following discussion, it is assumed for simplicity that the DLC system is an aerodynamic control surface and gives only the change in aerodynamic lift $\Delta L_{\mathrm{DLC}}$, that is, "$\Delta L_{\mathrm{DLC}} = (1/2)\rho U^2 S C_{L\delta_{\mathrm{DLC}}}\Delta\delta_{\mathrm{DLC}}$."

### 4.1. Equilibrium equation and restoring force

The equilibrium equations in "FC.0" are given by Eq. (17a)~(17c), and the equations in "FC.1" are given by

$$\text{Net drag:}\quad D_{\mathrm{net1}}(=D_1-T_1)=-W\sin\gamma_0,$$

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_D+C_{Du}\frac{\Delta U}{U_0}+C_{D\alpha}\Delta\alpha+C_{D\delta_e}\Delta\delta_e\right)-T_0=-W\sin\gamma_0 \tag{34a}$$

$$\text{Lift:}\quad L_1=W\cos\gamma_0,$$

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_L+C_{Lu}\frac{\Delta U}{U_0}+C_{L\alpha}\Delta\alpha+C_{L\delta_e}\Delta\delta_e\right)+\Delta L_{\mathrm{DLC}}=W\cos\gamma_0 \tag{34b}$$

$$\text{Pitching moment:}\quad M_1=0,$$

$$\text{or}\quad \frac{1}{2}\rho(U_0+\Delta U)^2 S\bar{c}\left(C_{mu}\frac{\Delta U}{U_0}+C_{m\alpha}\Delta\alpha+C_{m\delta_e}\Delta\delta_e\right)=0 \tag{34c}$$

The nonequilibrium lift $\Delta L$ at FC.2 when the additive lift $\Delta L_{\mathrm{DLC}}$ of the DLC system in FC.1 is returned to zero is the aerodynamic lift resulting when $\Delta L_{\mathrm{DLC}}$ is subtracted from the equilibrium condition "$L_1-W\cos\gamma_0=0$" in FC.1 and is shown by

$$\begin{aligned}\Delta L&=(L_1-W\cos\gamma_0)-\Delta L_{\mathrm{DLC}}\\&=\frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_L+C_{Lu}\frac{\Delta U}{U_0}+C_{L\alpha}\Delta\alpha+C_{L\delta_e}\Delta\delta_e\right)-W\cos\gamma_0\\&=\frac{1}{2}\rho(U_0+\Delta U)^2 S\left(C_L+C_{Lu}\frac{\Delta U}{U_0}+C_{L\alpha}\Delta\alpha+C_{L\delta_e}\Delta\delta_e\right)-\frac{1}{2}\rho {U_0}^2 S C_L\end{aligned} \tag{35a}$$

When $\Delta L_{\mathrm{DLC}}$ produced by the DLC system from Eq. (17b) and Eq. (34b) is obtained, $\Delta L_{\mathrm{DLC}}$ is reasonably shown to balance against $\Delta L$ (i.e., $\Delta L_{\mathrm{DLC}}=-\Delta L$), and thus the definitions Eq. (5) and Eq. (6) are equivalent.

Linearizing Eq. (35a) gives the following equation.

$$\Delta L=\frac{1}{2}\rho {U_0}^2 S\left\{(2C_L+C_{Lu})\frac{\Delta U}{U_0}+C_{L\alpha}\Delta\alpha+C_{L\delta_e}\Delta\delta_e\right\} \tag{35b}$$

### 4.2. Criteria for static angle of attack stability

Since $\Delta L$ in Eq. (35b) is a function of $\Delta U$, $\Delta\alpha$, and $\Delta\delta_e$, Eq. (5) is represented by

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}=\frac{\partial L}{\partial U}\cdot\frac{\mathrm{d}U}{\mathrm{d}\alpha}+\frac{\partial L}{\partial\alpha}+\frac{\partial L}{\partial\delta_e}\cdot\frac{\mathrm{d}\delta_e}{\mathrm{d}\alpha}>0 \tag{36}$$

Each partial differential in Eq. (36) is then given by

$$\partial L/\partial U=\rho U_0 S(2C_L+C_{Lu})/2 \tag{37a}$$

$$\partial L/\partial\alpha=\rho {U_0}^2 S C_{L\alpha}/2 \tag{37b}$$

$$\partial L/\partial\delta_e=\rho {U_0}^2 S C_{L\delta_e}/2 \tag{37c}$$

The total differential $\mathrm{d}U/\mathrm{d}\alpha$ and $\mathrm{d}\delta_e/\mathrm{d}\alpha$ in Eq. (36) are deduced by the same procedures as in Paragraph 3.1.2. From Eq. (2a) and (34a), Eq. (2b) and (34b), and Eq. (2c) and (34c), the following equations are obtained, respectively:

$$(2C_D+C_{Du})(\Delta U/U_0)+C_{D\alpha}\Delta\alpha+C_{D\delta_e}\Delta\delta_e=0 \tag{38}$$

$$(2C_L+C_{Lu})(\Delta U/U_0)+C_{L\alpha}\Delta\alpha+C_{L\delta_e}\Delta\delta_e=-\Delta C_{L_{\mathrm{DLC}}} \tag{39}$$

$$C_{mu}(\Delta U/U_0)+C_{m\alpha}\Delta\alpha+C_{m\delta_e}\Delta\delta_e=0 \tag{40}$$

where

$$\Delta C_{L_{\mathrm{DLC}}}=\Delta L_{\mathrm{DLC}}/(\rho {U_0}^2 S/2) \tag{41}$$

By solving the simultaneous equations of Eq. (38)~(40), we can obtain each ratio of $\Delta U$, $\Delta\alpha$, and $\Delta\delta_e$ to $\Delta C_{L_{\mathrm{DLC}}}$ as follows:

$$(\Delta U/U_0)/\Delta C_{L_{\mathrm{DLC}}}=(C_{m\delta_e}C_{D\alpha}-C_{D\delta_e}C_{m\alpha})/A \tag{42a}$$

$$\Delta\alpha/\Delta C_{L_{\mathrm{DLC}}}=\{C_{D\delta_e}C_{mu}-C_{m\delta_e}(2C_D+C_{Du})\}/A \tag{42b}$$

$$\Delta\delta_e/\Delta C_{L_{\mathrm{DLC}}}=\{C_{m\alpha}(2C_D+C_{Du})-C_{D\alpha}C_{mu}\}/A \tag{42c}$$

where $A$ is given by Eq. (27).

Thus

$$\begin{aligned}\frac{\mathrm{d}U}{\mathrm{d}\alpha}&=U_0\left(\frac{\Delta U/U_0}{\Delta C_{L_{\mathrm{DLC}}}}\right)\Big/\left(\frac{\Delta\alpha}{\Delta C_{L_{\mathrm{DLC}}}}\right)\\&=\frac{U_0(C_{m\delta_e}C_{D\alpha}-C_{D\delta_e}C_{m\alpha})}{C_{D\delta_e}C_{m\alpha}-C_{m\delta_e}(2C_D+C_{Du})}\end{aligned} \tag{43a}$$

$$\begin{aligned}\frac{\mathrm{d}\delta_e}{\mathrm{d}\alpha}&=\left(\frac{\Delta\delta_e}{\Delta C_{L_{\mathrm{DLC}}}}\right)\Big/\left(\frac{\Delta\alpha}{\Delta C_{L_{\mathrm{DLC}}}}\right)\\&=\frac{C_{m\alpha}(2C_D+C_{Du})-C_{D\alpha}C_{mu}}{C_{D\delta_e}C_{mu}-C_{m\delta_e}(2C_D+C_{Du})}\end{aligned} \tag{43b}$$

We obtain the following criterion in terms of the aerodynamic characteristics by applying Eq. (37a)~(37c), (43a), and (43b) to Eq. (36).

$$C_{L\alpha}+\frac{C_{m\delta_e}C_{D\alpha}(2C_L+C_{Lu})}{C_{D\delta_e}C_{mu}-C_{m\delta_e}(2C_D+C_{Du})}$$

Table 2. Criteria on static flight speed stability and static angle of attack stability.

| Subject | Definition | Criteria for conventional airplanes | Note |
|---|---|---|---|
| Static flight speed stability | $\dfrac{\mathrm{d}D_{\mathrm{net}}}{\mathrm{d}U} > 0$<br>$\dfrac{\mathrm{d}T}{\mathrm{d}U} > 0$<br>$\dfrac{\mathrm{d}\gamma}{\mathrm{d}U} < 0$ | General form of stability criterion:<br>$\dfrac{C_{L\alpha}(2C_D + C_{Du}) - C_{D\alpha}(2C_L + C_{Lu})}{C_{L\alpha}} > 0$<br>Stability criterion in low subsonic region:<br>$C_D/C_L - C_{D\alpha}/C_{L\alpha} > 0$ | Unstable in backside region |
| Static angle of attack stability | $\dfrac{\mathrm{d}L}{\mathrm{d}\alpha} > 0$<br>$\dfrac{\mathrm{d}\delta_{\mathrm{DLC}}}{\mathrm{d}\alpha} < 0$ | General form of stability criterion:<br>$\dfrac{C_{L\alpha}(2C_D + C_{Du}) - C_{D\alpha}(2C_L + C_{Lu})}{2C_D + C_{Du}} > 0$<br>Stability criterion in low subsonic region:<br>$C_{L\alpha}(C_D/C_L - C_{D\alpha}/C_{L\alpha}) > 0$ | Unstable in backside region and stall region |

$$- \frac{C_{D\delta_e} C_{m\alpha}(2C_L + C_{Lu})}{C_{D\delta_e} C_{mu} - C_{m\delta_e}(2C_D + C_{Du})} + \frac{C_{L\delta_e}\{C_{m\alpha}(2C_D + C_{Du}) - C_{D\alpha} C_{mu}\}}{C_{D\delta_e} C_{mu} - C_{m\delta_e}(2C_D + C_{Du})} > 0 \quad (44)$$

Since $C_{L\delta_e}$ and $C_{D\delta_e}$ are small and may be ignored as they are in conventional airplanes, Eq. (44) is given by

$$\{C_{L\alpha}(2C_D + C_{Du}) - C_{D\alpha}(2C_L + C_{Lu})\}/(2C_D + C_{Du}) > 0 \quad (45)$$

This is the general form of criterion on the static angle of attack stability for conventional airplanes.

Moreover, $C_{Lu}$ and $C_{Du}$ are small and may be ignored in low subsonic flights, and $C_L$ and $C_D$ are positive in normal flights. Thus Eq. (45) in such a situation is simplified as follows:

$$C_{L\alpha}(C_D/C_L - C_{D\alpha}/C_{L\alpha}) > 0 \quad (46)$$

The expression in the parentheses on the left side of Eq. (46) is equal to the expression in the left side of Eq. (31). Therefore the static angle of attack stability in the low subsonic region depends on both the static flight speed stability (or flight path stability) and the lift curve slope $C_{L\alpha}$.

In the frontside flight region in which the flight speed stability is stable and "$C_{L\alpha} > 0$," the static angle of attack stability is stable. In the backside region before the stall (i.e., $C_{L\alpha} > 0$), both the static flight speed stability and the static angle of attack stability are unstable. Moreover, in the stall region (i.e., $C_{L\alpha} < 0$) the static flight speed stability is stable, as mentioned in Paragraph 3.1.2, but Eq. (46) is negative; thus the static angle of attack stability is unstable. As a result, the conditions for static angle of attack stability to be stable are given by

$$C_{L\alpha} > 0 \quad \text{and} \quad C_D/C_L - C_{D\alpha}/C_{L\alpha} > 0 \quad (47)$$

## 5. Summary of This Report

This paper proposes the definitions of static stability for six degrees of freedom motion as summarized in Table 1. "Static flight speed stability" and "static angle of attack stability" are discussed in detail, and their criteria in terms of the aerodynamic characteristics of an airplane are obtained as shown in Table 2.

The fundamental definition for static flight speed stability is "$\mathrm{d}D_{\mathrm{net}}/\mathrm{d}U > 0$." Furthermore, the derived forms are "$\mathrm{d}T/\mathrm{d}U > 0$" and "$\mathrm{d}\gamma/\mathrm{d}U < 0$." The definition "$\mathrm{d}\gamma/\mathrm{d}U < 0$" is well known as the definition of so-called flight path stability. The stability criterion in a low subsonic region shows that static flight speed stability is stable in the frontside region, unstable in the backside region before stalling, and stable in the stall region.

To judge a static angle of attack stability, we must use a DLC system. The definitions on static angle of attack stability are "$\mathrm{d}L/\mathrm{d}\alpha > 0$" and "$\mathrm{d}\delta_{\mathrm{DLC}}/\mathrm{d}\alpha < 0$." The criteria deduced from these definitions require not only "positive lift curve slope," but also "flying in the frontside region" in a low subsonic region.

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