

Decoupling Implementation Inheritance From Interface Inheritance as a *clean* and *general* solution to multiple inheritance

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The most difficult challenge in multiple inheritance (MI) is exemplified by the well-known "diamond problem", leading to MI's avoidance in most contemporary OOP languages, such as Java, C#, and Scala etc., which primarily advocate for single inheritance. Nevertheless, to address the absence of MI while maximizing code reuse, alternative techniques such as object composition, mixins / traits have been employed. However, these existing approaches have fallen short in effectively resolving naming conflicts, especially for inherited *fields* conflicts. In particular, they have not provided a satisfactory mechanism for programmers to specify, on an *individual* basis, how each inherited field should be joined or separated. In this paper we introduce a novel method that can achieve this, hence providing a *clean* and *general* solution to multiple inheritance. Our method is applicable across a wide spectrum of industry-strength OOP languages. We will compare the advantages of our method with those existing methods, including (C++'s) virtual inheritance, object composition, and mixins / traits.

Traditionally in class based OOP languages, *both the fields and methods* from the super-classes are inherited by the sub-classes. However we believe this is the root cause of many problems in multiple inheritance, e.g. most notably the diamond problem. In this paper, we propose to *stop inheriting data fields* and Decouple Implementation Inheritance From Interface Inheritance (DIIFII), which is a derived principle from the more general principle: Decoupling Data Interface From data Implementation (DDIFI), as a clean and general solution to such problems. We first present a design pattern called DDIFI which cleanly solves the diamond problem C++. It can handle the class fields of the multiple inheritance exactly according to the programmer's intended application semantics. It gives programmers flexibility when dealing with the diamond problem for instance variables: each instance variable can be configured *individually* either as one joined copy or as multiple independent copies in the implementation class. The key ideas are:

- (1) decouple implementation inheritance from interface inheritance by stopping inheriting data fields;
- (2) in the regular methods implementation use *virtual* property methods instead of direct raw fields; and
- (3) after each semantic *branching* add (and override) the new semantic assigning property.

Then we show our method is general enough, and can also achieve clean multiple inheritance in any OOP languages:

- that natively support multiple inheritance (e.g. C++, Python, OCaml, Lisp, Eiffel, etc.)
- single inheritance languages that support default interface methods (e.g. Java, C# etc.)
- single inheritance languages that support mixins (e.g. D), or traits (e.g. Scala)

(The supplementary DDIFI.tgz contains the implementation source code of this design pattern DDIFI in these 9 languages.)

Moreover, in this paper we introduced a **<Person, Student, Faculty, ResearchAssistant>** inheritance Problem 1 as a working example of our method. We believe if one can solve this problem,

¹The work reported in this paper is patent pending.

then one can solve *all* the problems that can arise in multiple inheritance. Finally, we would like to propose the following challenges to the whole programming language research community:

- (1) Design another alternative cleaner solution (than ours) to the diamond problem of ResearchAssistant Problem.
- (2) Devise another multiple inheritance problem (be it diamond problem or not), which *cannot* be solved by using the method of this paper.

We hope the success of DDIFI will guide the future OOP languages design and implementations.

CCS Concepts: • **Software and its engineering** → **General programming languages**.

Additional Key Words and Phrases: multiple inheritance, diamond problem, program to interfaces, virtual property, data interface, data implementation, semantic branching site, reusability, modularity

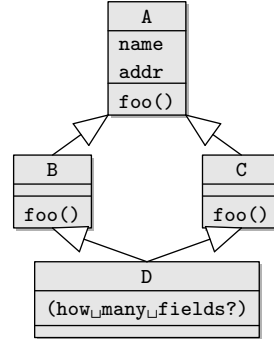
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1 MOTIVATION: THE DIAMOND PROBLEM

The most well known problem in multiple inheritance (MI) is the diamond problem [Snyder 1987] [Knudsen 1988] [Sakkinen 1989], for example on the popular website wikipedia¹ (for everyday working programmers) it is described as:

The "diamond problem" is an ambiguity that arises when two classes B and C inherit from A, and class D inherits from both B and C. If there is a method in A that B and C have overridden, and D does not override it, then which version of the method does D inherit: that of B, or that of C?



Actually in the real world engineering practice, for any *method's* ambiguity e.g. `foo()`, it is *relatively easy* to resolve *by the programmers*:

- just *override* it in `D.foo()`, or
- explicitly use fully quantified method names, e.g. `A.foo()`, `B.foo()`, or `C.foo()`.

And there is no *auto-magical* way for the compiler to *guess* which method has the correct *application semantics* that the programmer has in mind.

Using any kind of fixed precedence order (of one class over the another class) also does not work: for example, suppose in the application semantics the ResearchAssistant simultaneously wants

- `Faculty.library_privilege` precedence over `Student.library_privilege`, AND
- `Student.cafe_discount_benefit` precedence over `Faculty.cafe_discount_benefit`

The *more difficult* problem is how to handle the *data members* (i.e. *fields*) inherited from A:

- (1) Shall D have one joined copy of A's fields? or
- (2) Shall D have two separate copies of A's fields? or
- (3) Shall D have mixed fields from A, with some fields being joined, and others separated?

¹https://en.wikipedia.org/wiki/Multiple_inheritance#The_diamond_problem

For example, in C++ [Stroustrup 1989c] [Stroustrup 1994], [Stroustrup 2003] (1) is called virtual inheritance, and (2) is default (regular) inheritance. But C++ does not completely solve this problem, it is difficult to achieve (3). This is the main problem that we will solve in this paper.

Suppose we want to build an object model for Person, Student, Faculty, and ResearchAssistant in a university:

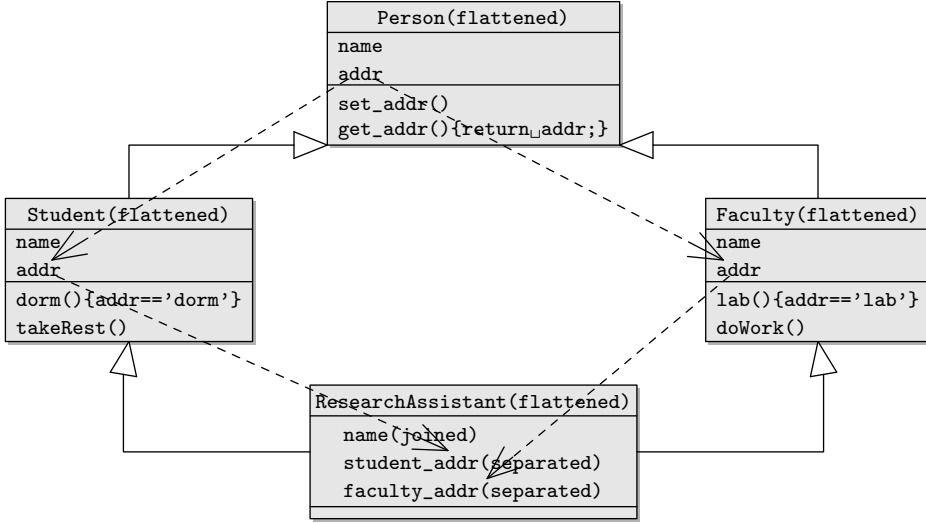


Fig. 1. the diamond problem: the ideal semantics of *fields* name & addr, which is not achievable in C++'s plain MI mechanism: with name joined into one field, and addr separated into two fields

We would like to formulate this ResearchAssistant inheritance problem as:

Problem 1 (The intended application semantics). *In the diamond inheritance of <Person, Student, Faculty, ResearchAssistant>, a ResearchAssistant should have*

- only 1 name field,
- but 2 different address fields:
 - (1) one "dorm" as Student to takeRest(), and
 - (2) one "lab" as Faculty to doBenchwork()

so in total 3 fields.

In the following we will use this problem as the working example to illustrate our method to simultaneously achieve both the field joining and separation in multiple inheritance, just as the dining philosophers problem as the working example in concurrency algorithm design.

2 LITERATURE SURVEY AND CURRENT ENGINEERING PRACTICES

2.1 C++'s virtual inheritance

In C++'s plain MI we can do either:

- (1) virtual inheritance: ResearchAssistant will have 1 name, and 1 addr; in total 2 fields, or
- (2) default inheritance: ResearchAssistant will have 2 names, and 2 addrs; in total 4 fields

It is difficult to achieve the intended application semantics. As is shown in the following C++ code:

Listing 1. plain_mi.cpp

```

1  #include <iostream>
2  #include <string>
3  typedef std::string String;
4
5  #define VIRTUAL // virtual // no matter we use virtual inheritance or not, it's problematic
6
7  class Person {
8  protected:
9      String _name; // need to be joined into one single field in ResearchAssistant
10     String _addr; // need to be separated into two addresses in ResearchAssistant
11 public:
12     virtual String name() {return _name;}
13     virtual String addr() {return _addr;}
14 };
15
16 class Student : public VIRTUAL Person {
17 public:
18     virtual String dorm() {return _addr;} // assign dorm semantics to _addr
19     void takeRest() {
20         std::cout << name() << " takeRest in the " << dorm() << std::endl;
21     }
22 };
23
24 class Faculty : public VIRTUAL Person {
25 public:
26     virtual String lab() {return _addr;} // assign lab semantics to _addr
27     void doBenchwork() {
28         std::cout << name() << " doBenchwork in the " << lab() << std::endl;
29     }
30 };
31
32 class ResearchAssistant : public VIRTUAL Student, public VIRTUAL Faculty {
33 };
34
35 int main() {
36     std::cout << "sizeof(Person) = " << sizeof(Person) << std::endl;
37     std::cout << "sizeof(Student) = " << sizeof(Student) << std::endl;
38     std::cout << "sizeof(Faculty) = " << sizeof(Faculty) << std::endl;
39     std::cout << "sizeof(ResearchAssistant) = " << sizeof(ResearchAssistant) << std::endl;
40 }
41

```

Hence if the programmers use C++'s multiple inheritance mechanism plainly as it is, **ResearchAssistant** will have either one whole copy, or two whole copies of **Person**'s all data members. This leaves something better to be desired. E.g this is why the Google C++ Style Guide [Google 2022] (last updated: Jul 5, 2022) gives the following negative advice about the diamond problem in MI:

Multiple inheritance is especially problematic, ... because it risks leading to "diamond" inheritance patterns, which are prone to ambiguity, confusion, and outright bugs.

Because the C++ inheritance mechanism (virtual or not) always treat all the fields from the super-class as a *whole*, no matter how to combine virtual and non-virtual inheritance in any possible way, it will not achieve the goal what we want: i.e. support both field join and separation flexibly according to any application semantics the programmers needed. Moreover, [Wasserrab et al. 2006] presented an operational semantics and type safety proof for multiple inheritance in C++, and they concluded the combination of virtual and non-virtual inheritance caused additional complexity at the semantics level.

2.2 Other industry strength OOP languages

Other OOP languages have designed different mechanisms, among the most popular OOP languages (besides C++) used in the industry:

- In Python [Van Rossum and Drake Jr 2014] all the inherited fields are joined by name (a Python object's fields are keys of an internal dictionary), hence there is no direct language inheritance mechanism to achieve field separation.
- Java [Gosling et al. 2000] and C# [Hejlsberg et al. 2003] get rid of multiple inheritance in favor of the simple single inheritance and multiple interfaces, and advise programmers to use composition to simulate multiple inheritance when needed.

2.3 Method resolution order

The Eiffel [Meyer 1993] `select` clause allows the programmer to *explicitly* resolve name clash on each inherited feature *individually*, which we believe is a better solution than imposing the *same* method resolution order (MRO) [Barrett et al. 1996] to *all* features as in many other OOP languages, e.g. Python [van Rossum 2010]: the base classes' order in the inheritance clause should *not* matter.

2.4 Multiple inheritance via composition

For OOP languages which do not directly support multiple inheritance, it is usually suggested to simulate multiple inheritance via composition. As illustrated in the following:

Listing 2. MI via composition in Java

```
1 // multiple inheritance via composition
2 class ResearchAssistant implements StudentI, FacultyI { // suffix 'I' means Interface
3     Student _theStudentSubObject; // composition
4     Faculty _theFacultySubObject; // composition
5
6     // Problem 1, code duplication: manual forwarding for *every* methods is very tedious
7     void doBenchWork() { _theFacultySubObject.doBenchWork(); }
8     void takeRest() { _theStudentSubObject.takeRest(); }
9     String lab() { return _theFacultySubObject._addr; }
10    String dorm() { return _theStudentSubObject._addr; }
11
12    // Problem 2, data duplication: need mutex, and keep *multiple duplicate* fields in sync
13    Object set_name_mtx; // need extra mutex var
14
15    String name() {
16        synchronized (set_name_mtx) {
17            String r = _theStudentSubObject._name;
18            return r;
19        }
20    }
21
22    String name(String name) {
23        synchronized (set_name_mtx) {
24            _theStudentSubObject._name = name; // dup fields
25            _theFacultySubObject._name = name;
26        }
27    }
28 };
```

First, *logically* speaking we think this method is abusing "Has-A" relationship as "Is-A" relationship. (i.e. a ResearchAssistant "Is-A" both Student and Faculty object, not "Has-A" both Student and Faculty objects).

Furthermore, with *manual* method forwarding, which is not only very tedious, but also incur data duplication, e.g.

- Each ResearchAssistant object will contain both a Student and Faculty sub-object, so the fields `name` are duplicated in both the sub-objects.
- In multi-threaded environment, assign new value to the two `name` fields in these two sub-objects need to be protected by a synchronization lock, which increases the complexity of the software.

2.5 Mixins / traits

In some other single inheritance OOP languages, various forms of mixins are introduced to remedy the lack of MI. Informally a mixin / trait is a named compilation unit which contains fields and methods to be "*inlined (copy/pasted)*"² rather than *inherited* by the client class to avoid the inheritance relationship, e.g.:

- Mixins [Bracha and Cook 1990] in Dart, D [Bright et al. 2020], Ruby [Flanagan and Matsumoto 2008], etc.
- Traits [Schärli et al. 2003] [Ducasse et al. 2006] in Scala [Odersky 2010], PHP [Lockhart 2015], Pharo Language [Tesone et al. 2020], Hack [Ottoni 2018] etc.

However, the problems with mixins and traits are:

- (1) There is no clean and flexible way to resolve field (of the same name) conflicts included from multiple different mixins, this is what our method will achieve.
- (2) Furthermore, an object of the type of the including class cannot be cast to, and be used as the named mixin type, which means it paid the price of the inheritance ambiguity of (e.g. as C++'s plain) MI, but does not enjoy the benefit of it.

2.6 Other experimental languages / constructs

LAVA programming language [Kniesel 1999] is an extension of Java with a new language construct called wrappers [Büchi and Weck 2000], [Truyen et al. 2004] presented a method in LAVA to solve the diamond problem using a delegation-based object system with wraps clauses. However, object delegation increases the complexity of the overall system with many extra participating objects at runtime. It also has the same issues as composition we discussed above.

[Malayeri and Aldrich 2009] proposed a new language CZ, which supports multiple inheritance but forbids diamond inheritance, and showed how to convert a diamond inheritance scheme to one without diamonds. While this works for their new language, we think it is counter-intuitive since diamonds arise naturally in the languages that support multiple inheritance. Instead of restricting what the programmers cannot do, we want a solution that can work with most current mainstream OOP languages.

In short, while these experimental languages are of interest to the academic researchers, We want to find a practical solution that can be directly applied in the current industry-strength languages. In this paper we have designed a new design pattern which can cleanly achieve multiple inheritance according to the programmers intended semantics. It gives programmers flexibility when dealing with the diamond problem for instance variables: each instance variable can be configured *individually* either as one joined copy or as multiple independent copies in the implementation class.

In Section 3 we will demo our method in C++ using the previous example step by step; in Section 4 we will formalize our design pattern and present new programming rules that make our method also work in some other OOP languages; in Section 5 we will demo our method in Java with the same example using these rules. In Section 6 we will compare our method with MI via composition, and other approaches like mixins / traits. Finally, in the Appendix, we will show our method in Python and C#.

²E.g. from Hack Documentation: <https://docs.hhvm.com/hack/traits-and-interfaces/using-a-trait>

3 DECOUPLING DATA INTERFACE FROM DATA IMPLEMENTATION

One of the most important OOP concepts is encapsulation, which means bundling *data* and *methods* that work on that data within one unit (i.e. class). As noted, inherited method conflicts are relatively easy to solve by the programmers by either overriding or using fully quantified names in the derived class.

3.1 Troublemaker: the inherited fields

But for fields, traditionally in almost all OOP languages, if a base class has field **f**, then the derived class will also have this field **f**. The reason that the inherited data members (fields) from the base classes causing so much troubles in MI is because fields are the actual memory implementations, which are hard to be adapted to the new derived class, e.g.:

- Should the memory layouts of all the different base classes' fields be kept intact in the derived class? and in which (linear memory) order?
- How to handle if the programmers want *some* of the inherited fields from different base classes to be merged into one field (e.g. **name** in the above example), and *others* separated (e.g. **addr** in the above example) according to the application semantics?
- What are the proper rules to handle all the combinations of these scenarios?

The key inspiring question: since class fields are the troublemakers for MI, can we just remove them from the inheritance relation? or delay their implementation to the last point?

3.2 The key idea: reduce the data dependency on fields to methods dependency on properties

Let us step back, and check what is the minimal dependency of the class methods on the class data? Normally there are two ways for a method to read / write class fields:

- (1) directly read / write the raw fields
- (2) read / write through the getter / setter methods

Definition 1 (getter and setter method). In OOP we have

- The getter method returns the value of a class field.
- The setter method takes a parameter and assigns it to a class field.

In the following, we call getter and setter as *property method* or just *property*; and we call the collection of properties of a class as the *data interface* of the class; In contrast we call the other non-property class methods as *regular methods* or just *methods*.

In Fig.1 and `plain_mi.cpp`, we can see the field `Person._addr` has been assigned two different meanings in the two different inheritance branches: in class `Student` it's assigned "dorm" semantics, while in class `Faculty` it's assigned "lab" semantics.

Definition 2 (semantic branching site of property). If a class *C*'s property *p* has more than one semantic meanings in its immediate sub-classes, we call *C* the *semantic branching site* of *p*; If class *A* inherits from class *B*, we call *A* is *below* *B*.

In our previous example, class `Person` is the semantic branching site of property `addr`; and class `Student` is below `Person`.

Since properties are methods which are more manipulatable than the raw fields, we can reduce the data dependency on fields to methods dependency on properties, by only using fields' getter and setter in the regular methods.

Traditionally, the getter and setter methods are defined in the *same* scope as the field is in, i.e. in the same class body (as we can see from the class `Person` in `plain_mi.cpp` of the previous example). But due to the troubles the class fields caused us in MI, we would like to isolate them into another scope (as data implementation). Then to make other regular methods in the original class continue to work, we will add abstract property definitions to the original class (as data interface). For example, as shown in the class UML diagram on the right:

To make the distinction more clear, we intentionally colored the data interface class `Person` with lighter background color, and the data implementation class `PersonImpl` with darker background.

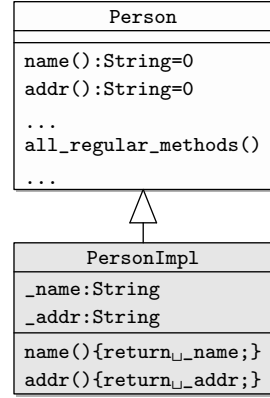


Fig. 2. decouple data interface (class `Person` with abstract property methods) from data implementation (class `PersonImpl` where the fields and property methods are actually defined)

The key point here is that: the programmers have the freedom to either add new or override existing property *methods* in the derived class' data interface to achieve any application semantics, without worrying about the *data implementation*, which will be eventually defined in the implementation class. Thus remove the data dependency of the derived class' implementation on the base classes' implementation.

The UML of our DDIFI classes:

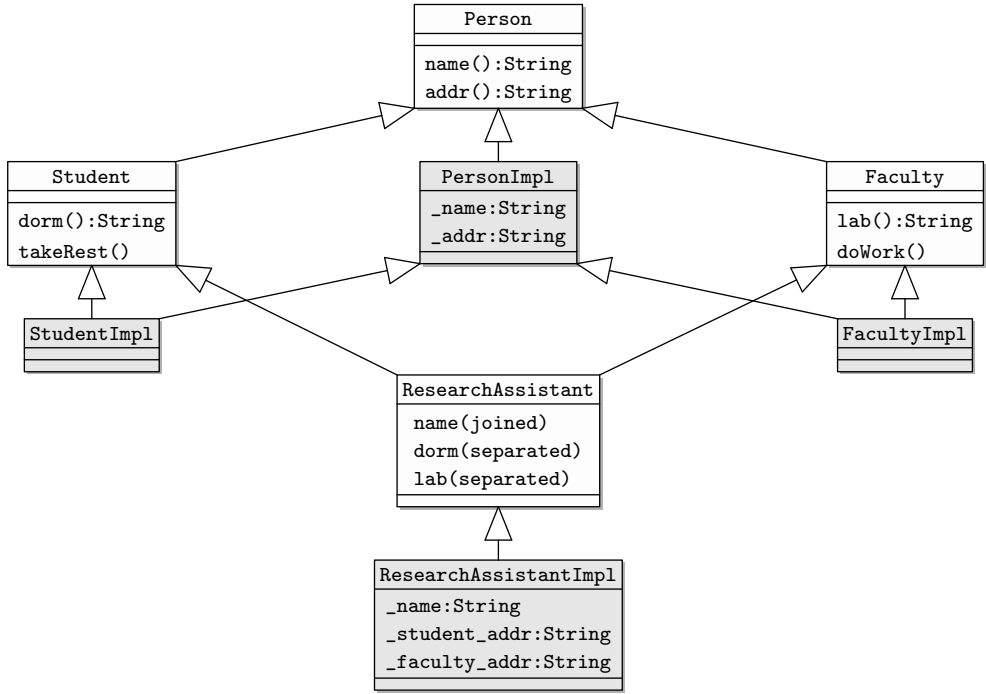


Fig. 3. DDIFI achieved the ideal semantics of *fields* name & addr: with name joined into one field, and addr separated into two fields

Please note: implementation inheritance is still an *option*, e.g. `StudentImpl` inherits `PersonImpl`, and `FacultyImpl` inherits `PersonImpl` for maximal code reuse; but it is not mandatory, e.g. `ResearchAssistantImpl` is totally *independent* of `StudentImpl`, `FacultyImpl`, and `PersonImpl`.

In fact, we can draw the above DDIFI UML classes in a 3D fashion: the interface classes are at the top, and the implementation classes are at the bottom

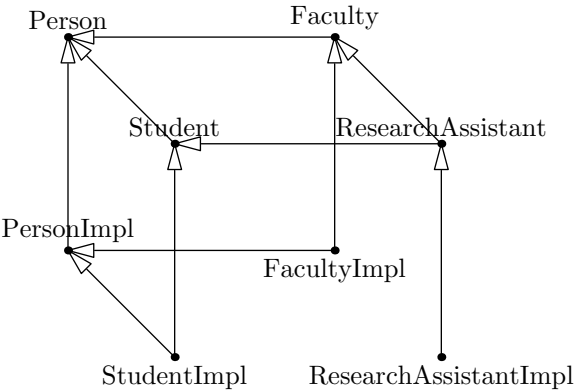


Fig. 4. DDIFI 3D UML

In the following we will demo how this data interface and implementation decoupling can solve the diamond problem in a clean way with concrete C++ code.

Listing 3. person.h

```

1 // define abstract virtual property, in Person's data-interface
2 class Person {
3 public:
4     virtual String name() = 0; // C++ abstract virtual method
5     virtual String addr() = 0; // C++ abstract virtual method
6
7     // all_public_or_protected_regular_methods() are defined in the data-interface
8     // to be inherited and code-reused
9 };
10
11 // define fields and property method, in Person's data-implementation
12 class PersonImpl : Person {
13 protected:
14     String _name;
15     String _addr;
16 public:
17     virtual String addr() override { return _addr; }
18     virtual String name() override { return _name; }
19 };

```

First, split `person.h` into two classes: `Person` as data interface (with regular methods), and move fields definition into `PersonImpl` as data implementation.

Listing 4. student.h

```

1 class Student : public Person {
2 public:
3     // add new semantic assigning virtual property
4     virtual String dorm() { // give it a new exact name matching its new semantics
5         return addr(); // but the implementation here can be just super's addr()
6     }
7
8     // regular methods' implementation
9     void takeRest() {
10         cout << name() << " takeRest in the "
11             << dorm() // MUST use the new property, not the inherited addr() whose semantics has branched!
12             << endl;
13     }
14 };
15
16
17 class StudentImpl : public Student, PersonImpl {
18     // no new field: be memory-wise efficient, while function-wise flexible
19 };

```

We do the same for `student.h`, please also note:

- (1) We added a *new* semantic assigning virtual property `dorm()`, which currently just return `addr()`; but can be overridden in the derived classes.
- (2) We implemented all other regular methods in the data-interface class `Student`, which when needed can read / write (but not direct access) any class field via the corresponding (abstract) property method.
- (3) Please also take notice here `Student.takeRest()` calls `dorm()` (which in turn calls `addr()`), instead of calling `addr()` directly. We will discuss this treatment of semantic branching property in the next section.
- (4) `StudentImpl` inherits all the data fields from `PersonImpl`, this is just for convenience; alternatively, the programmer can choose to let `StudentImpl` define its own data implementation totally *independent* of `PersonImpl`, as we will show in the following `ResearchAssistantImpl`. This is the key to solve the inherited field conflicts of the diamond problem.

Listing 5. faculty.h

```

1 class Faculty : public Person {

```

```
2 public:
3 // add new semantic assigning virtual property
4 virtual String lab() { // give it a new exact name matching its new semantics
5     return addr(); // but the implementation here can be just super's addr()
6 }
7
8 // regular methods' implementation
9 void doBenchwork() {
10     cout << name() << " doBenchwork in the "
11         << lab() // MUST use the new property, not the inherited addr() whose semantics has branched!
12         << endl;
13 }
14 };
15
16 class FacultyImpl : public Faculty, PersonImpl {
17     // no new field: be memory-wise efficient, while function-wise flexible
18 };
```

We do the same also for `faculty.h`, and added a new semantic assigning property `lab()`.

Listing 6. `ra.h`

```
1 class ResearchAssistant : public Student, public Faculty { // MI with regular-methods code reuse!
2 };
3
4 class ResearchAssistantImpl : public ResearchAssistant { // only inherit from ResearchAssistant interface
5 protected:
6 // define three fields: NOTE: totally independent to those fields in PersonImpl, StudentImpl, and FacultyImpl
7     String _name;
8     String _faculty_addr;
9     String _student_addr;
10 public:
11     ResearchAssistantImpl() { // the constructor
12         _name = NAME;
13         _faculty_addr = LAB;
14         _student_addr = DORM;
15     }
16
17 // override the property methods
18     virtual String name() override { return _name; }
19     virtual String addr() override { return dorm(); } // use dorm as ResearchAssistant's main addr
20     virtual String dorm() override { return _student_addr; }
21     virtual String lab() override { return _faculty_addr; }
22 };
23
24 ResearchAssistant* makeResearchAssistant() { // the factory method
25     ResearchAssistant* ra = new ResearchAssistantImpl();
26     return ra;
27 };
```

Finally, we define research assistant, please note:

- (1) The fields of `ResearchAssistantImpl`: `_name`, `_faculty_addr`, and `_student_addr` are totally *independent* of the fields in `PersonImpl`, `StudentImpl`, and `FacultyImpl`. This is what we mean: removing the data dependency of the derived class' data implementation on the base classes' data implementations
- (2) Now indeed each `ResearchAssistant` object has exactly 3 fields: 1 name, 2 addrs!
- (3) We added a factory method to create new `ResearchAssistant` objects.

Let's create a `ResearchAssistant` object, also assign it to `Faculty*`, `Student*` variables, and make some calls of the corresponding methods on them:

```
1 #include <iostream>
2 #include <string>
3 typedef std::string String;
4 using namespace std;
5
6 String NAME = "ResAssis";
7 String HOME = "home";
8 String DORM = "dorm";
9 String LAB = "lab";
10
11 #include "person.h"
12 #include "student.h"
13 #include "faculty.h"
```

```

14 #include "ra.h"
15 #include "biora.h"
16
17 int main() {
18     ResearchAssistant* ra = makeResearchAssistant();
19     Faculty* f = ra;
20     Student* s = ra;
21
22     ra->doBenchwork(); // ResAssis doBenchwork in the lab
23     ra->takeRest();    // ResAssis takeRest in the dorm
24
25     f->doBenchwork();  // ResAssis doBenchwork in the lab
26     s->takeRest();     // ResAssis takeRest in the dorm
27
28     return 0;
29 }

```

As we can see, all the methods generate expected correct outputs.

To the best of the authors' knowledge, this design pattern that we introduced in this section to achieve multiple inheritance so cleanly has never been reported in any previous OOP literature. It is the first design pattern that cleanly solves the diamond problem in a number of mainstream industry-strength OOP programming languages³, e.g. C++ [Stroustrup 1991], Java, C#, Python, Ocaml [Leroy et al. 2021], D, etc, which we will show in the following sections.

3.3 Virtual property

It is very important to define the property method as *virtual*, this gives the programmers the freedom to choose the appropriate implementation of the concrete representation in the derived class. Properties can be:

- fields (data members) with memory allocation, or
- methods via computation if needed.

For example, a biology research assistant may alternate between two labs (labA, labB) every other weekday to give the micro-organism enough time to develop. We can implement `BioResearchAssistantImpl` as the following (please pay special attention to the new `lab()` property):

Listing 7. biora.h

```

1 #include "util.h"
2
3 String LAB_A = "labA";
4 String LAB_B = "labB";
5
6 // only inherit from ResearchAssistant, but not from any other xxxImpl class
7 class BioResearchAssistantImpl : public ResearchAssistant {
8     protected:
9         // define two fields: NOTE: totally independent to those fields in PersonImpl, StudentImpl, and FacultyImpl
10        String _name;
11        String _student_addr;
12    public:
13        BioResearchAssistantImpl() { // the constructor
14            _name = NAME;
15            _student_addr = DORM;
16        }
17
18        // override the property methods
19        virtual String name() override { return _name; }
20        virtual String addr() override { return dorm(); } // use dorm as ResearchAssistant's main addr
21        virtual String dorm() override { return _student_addr; }
22        virtual String lab() override {
23            int weekday = get_week_day();
24            return (weekday % 2) ? LAB_A : LAB_B; // alternate between two labs
25        }
26 };
27

```

³By the TIOBE Programming Community index <https://www.tiobe.com/tiobe-index/>

```
28 ResearchAssistant* makeBioResearchAssistant() { // the factory method
29     ResearchAssistant* ra = new BioResearchAssistantImpl();
30     return ra;
31 }
```

Note: both `ResearchAssistantImpl` and `BioResearchAssistantImpl` are at the bottom point of the diamond inheritance, but their actual fields are quite different. In our approach the derived class data implementation does *not* inherit the actual fields from the base classes' data implementation, but only inherits the data interface of the base classes (i.e. the property methods, and will override them). This is the key difference from C++'s plain MI mechanism. That's why our approach is so flexible that it can achieve the intended semantics the programmers needed.

In the next section we will summarize the new programming rules to formalize our approach to achieve general MI.

4 NEW PROGRAMMING RULES

Rule 1 (split data interface class and data implementation class). *To model an object foo, define two classes:*

- (1) *class Foo as data interface, which does not contain any field; and Foo can inherit multiple-ly from any other data-interfaces.*
- (2) *class FooImpl inherit from Foo, as data implementation, which contains fields (if any) and implement property methods.*

For example, we can see from `person.h` and Fig. 2: class `Person` and `PersonImpl` in the previous section.

Rule 2 (data interface class). *In the data-interface class Foo:*

- (1) *define or override all the (abstract) properties, and always make them virtual (to facilitate future unplanned MI).*
- (2) *implement all the (especially public and protected) regular methods, using the property methods when needed, as the default regular methods implementation.*
- (3) *add a static (or global) Foo factory method to create FooImpl object, which the client of Foo can call without exposing the FooImpl's implementation detail.*

Note: although Foo is called *data* interface, the regular methods are also *implemented* here, because:

- it's good engineering practice to program to (the data) interfaces, instead of using the raw fields directly
- other derived classes will inherit from Foo, (instead of FooImpl which is data implementation specific), so these regular methods can be reused to achieve the other OOP goal: maximal code reuse.

Of course, for the *private* regular methods, the programmer may choose to put them in FooImpl to hide their implementation.

Rule 3 (data implementation class). *In the data-implementation class FooImpl:*

- (1) *implement all the properties in the class FooImpl: a property can be either*
 - (a) *via memory, define the field and implement the getter and setter, or*
 - (b) *via computation, define property method*
- (2) *implement at most the private regular methods (or just leave them in class Foo by the program to (the data) interfaces principle, instead of directly accessing the raw fields).*

So, because of Rule 2 all the data-interface classes (which also contains regular method implementations) can be multiple-ly inherited by the derived interface class without causing fields conflict. And because of Rule 3 each data-implementation class can provide the property implementations exactly as the intended application semantics required.

Rule 4 (sub-classing). *To model class bar as the subclass of foo:*

- (1) *make Bar inherit from Foo, and override any virtual properties according to the application semantics.*
- (2) *make BarImpl inherit from Bar, but BarImpl can be implemented independently from FooImpl (hence no data dependency of BarImpl on FooImpl).*

Rule 5 (add and use new semantic assigning property after branching). *If class C is the semantic branching site of property p, in every data-interface class D that is immediate below C:*

- (1) *add a new semantic assigning virtual property p' (of course, p' and p are different names),*
- (2) *all other regular methods of D should choose to use p' instead of p according to the corresponding application semantics when applicable.*

The following is an example of applying this Rule 5:

- Class **Person** is the semantic branching site of property **addr**.
- In class **Student**, we added a new semantic assigning property **dorm()**; and **Student.takeRest()** uses property **dorm()** instead of **addr()**.
- In class **Faculty**, we added a new semantic assigning property **lab()**; and **Faculty.doBenchwork()** uses property **lab()** instead of **addr()**.

The reason to add new semantic assigning virtual property after branching is to facilitate fields separation, as we have shown in **ra.h**, the derived class **ResearchAssistant** implementation can override the new properties (i.e. **dorm()**, and **lab()**) differently; otherwise without adding such new properties, **ResearchAssistant** can only override the single property **addr()**, then at least one of the inherited method **takeRest()** or **doBenchwork()** will be wrong (since they can only both call **addr()** in that case).

In summary: the goal is to make fields joining or separation as flexible as possible, to allow programmers to achieve any intended semantics (in the derived data implementation class) that the application needed:

- field joining can be achieved by overriding the corresponding virtual property method of the same name from multiple base classes
- field separation can be achieved by implementing / overriding the new semantic assigning property introduced in Rule 5.

5 JAVA WITH INTERFACE DEFAULT METHODS

Despite many modern programming languages (Java, C#) tried to avoid multiple inheritance (MI) by only using single inheritance + multiple interfaces in their initial design and releases, to remedy the restrictions due to the lack of MI. they introduced various other mechanisms in their later releases, e.g.

- (1) Java v8.0 added default interface methods in 2014 [Oracle 2014]
- (2) C# v8.0 added default interface methods in 2019 [Microsoft 2022]

Actually, the programming rules we introduced in the previous section works perfectly well with Java's (\geq v8.0) interface default methods, which now allows methods be implemented in Java interfaces. In the following, we show how the previous example can be coded in Java.

Listing 8. MI.java

```
1 interface Person {
2     public String name(); // abstract property method, to be implemented
3     public String addr(); // abstract property method, to be implemented
4     // no actual field
5 }
6
7 class PersonImpl implements Person {
8     // only define fields and property methods in data implementation class
9     String _name;
10    String _addr;
11    @Override public String name() { return _name; }
12    @Override public String addr() { return _addr; }
13 }
14
15 interface Faculty extends Person {
16     default String lab() {return addr();} // new semantic assigning property
17
18     // regular methods
19     default void doBenchwork() {
20         System.out.println(name() + " doBenchwork in the " + lab());
21     }
22 }
23
24 class FacultyImpl extends PersonImpl implements Faculty {
25     // nothing new needed, so just extends PersonImpl
26 }
27
28 interface Student extends Person {
29     default String dorm() {return addr();} // new semantic assigning property
30
31     // regular methods
32     default void takeRest() {
33         System.out.println(name() + " takeRest in the " + dorm());
34     }
35 }
36
37 class StudentImpl extends PersonImpl implements Student {
38     // nothing new needed, so just extends PersonImpl
39 }
40
41 interface ResearchAssistant extends Student, Faculty {
42     // factory method
43     static ResearchAssistant make() {
44         ResearchAssistant ra = new ResearchAssistantImpl();
45         return ra;
46     }
47 }
48
49 class ResearchAssistantImpl implements ResearchAssistant {
50     // define three fields: NOTE: totally independent to those fields in PersonImpl, StudentImpl, and FacultyImpl
51     String _name;
52     String _faculty_addr;
53     String _student_addr;
54
55     ResearchAssistantImpl() { // constructor
56         _name = "ResAssis";
57         _faculty_addr = "lab";
58         _student_addr = "dorm";
59     }
60
61     // property methods
62     @Override public String name() { return _name; }
63     @Override public String addr() { return dorm(); } // use dorm as addr
64     @Override public String dorm() { return _student_addr; }
65     @Override public String lab() { return _faculty_addr; }
66 }
67
68
69 public class MI {
70     public static void main(String[] args) {
71         ResearchAssistant ra = ResearchAssistant.make();
72         Faculty f = ra;
73         Student s = ra;
```

```

74
75     ra.doBenchwork(); // ResAssis doBenchwork in the lab
76     ra.takeRest();   // ResAssis takeRest in the dorm
77
78     f.doBenchwork(); // ResAssis doBenchwork in the lab
79     s.takeRest();    // ResAssis takeRest in the dorm
80 }
81 }

```

6 RELATED WORKS

6.1 Compare our method with Scott Meyers's Item 33

In Scott Meyers's book "More Effective C++" [Meyers 1995] there is Item 33: "Make non-leaf classes abstract". While this advice has some similarity as our method, actually they are different: firstly this advice does not emphasize the importance of decoupling of data-interface and data-implementation as we do, and it can solve the diamond problem cleanly. Also for example, `PersonImpl` seems to be a leaf class for `Person`, but in Listing 4 `StudentImpl` inherited from `PersonImpl`, making it a non-leaf class, and we cannot make `PersonImpl` abstract. So in our method: fields (implementation) inheritance is still an option (when it helps to reduce code duplication). Our method maximizes the chance that developer can achieve code reuse.

6.2 Compare our method with MI via composition

With the technique introduced in this paper, there are some boilerplate property implementation code needed for each virtual property. However for any non-trivial program, typically the number of regular class methods is far more than the number of fields. So our approach is better than MI via composition in terms of the needed supporting boilerplate code. More importantly, with MI via composition the programmers still need to solve the field joining problem. While with our approach, the fields joining or separation problems are solved perfectly by overriding the corresponding virtual property methods to read / write the same (e.g. `_name`) or different (e.g. `_faculty_addr`, or `_student_addr`) fields in the data implementation class.

6.3 Compare our method with mixins / traits

The programmers need to learn the new language concept and rules associated with mixins or traits; while in our method, DDIFI only use the existing language mechanisms, which the programmers have already mastered.

7 DISCUSSIONS AND FUTURE WORKS

7.1 The ResearchAssistant inheritance problem challenge

In Section 1 we introduced the ResearchAssistant inheritance Problem 1 as a working example of our method. We believe if one can solve this problem, then one can solve *all* the problems that can arise in multiple inheritance. We would like to propose the following challenges to the whole programming language research community:

- (1) Design another alternative cleaner solution (than ours) to the diamond problem of `<Person, Student, Faculty, ResearchAssistant>`.
- (2) Devise another multiple inheritance problem (be it diamond problem or not), which *cannot* be solved by using the method in this paper.

7.2 Advantages and disadvantages of DDIFI

we would like to summarize the advantages and disadvantages of DDIFI as the following:

Advantages:

- Clean: completely solved the diamond problem cleanly.
- General: works in not only in native multiple inheritance languages like C++, Python, OCaml, Lisp, and Eiffel etc., but also works in single inheritance languages like Java, C#, D, Scala etc. (I.e. DDIFI achieved clean multiple inheritance in these native single inheritance languages).

Disadvantages:

- Each class now split into two classes: one as data-interface (also contains regular methods implementation), and the other as data-implementation.

However we think:

- The concept of *program to interface* is a good practice in almost any serious software project already, which is well-understood by the developers.
- The regular methods must access fields using property methods which will incur lots of virtual function call cost in performance.

However we think:

- "Premature optimization is the root of all evil": making the multiple inheritance logic correct is more important than micro optimizations.
- Virtual methods is the corner-stone of OOP (since its start in 1960s), it is well optimized by modern compilers. If one do not want use virtual functions, probably s/he does not want to use OOP in the first place.
- Also we can always use local temp vars to reduce the number of virtual property method calls needed in regular methods.

7.3 Cfront 2.0 challenge

In retrospect, when C++ first introduced multiple inheritance in its version 2.0 (then called Cfront Release 2.0 [Stroustrup 1989a]), all the native language mechanisms to achieve DDIFI are available. Since the original source code [Stroustrup 1989b] only works on the old system back in 1980's, we would like to propose the following challenge: if we can port the source code onto a modern Unix/Linux system, then we can try DDIFI in C++ 2.0!

7.4 Programming paradigms evolution: procedural, OOP, DDIFI

In the following table, we compare three different ways of programming using C++ side by side:

- (1) Procedural programming, where data and functions are separate.
- (2) Object oriented programming (OOP), where data and methods are bundled together in one unit (class).
- (3) OOP with Decoupling Data Interface From data Implementation (DDIFI), where each class is split into a *data-interface* class and a *data-implementation* class.

Procedural programming	Object oriented programming	OOP with DDIFI
<pre>struct Person { String name; String addr; }; void a_function(Person* p) { print(p->addr); }</pre>	<pre>class Person { String name; String addr; public: void a_regular_method() { print(this->addr); } };</pre>	<pre>class Person { public: virtual String name() = 0; virtual String addr() = 0; void a_regular_method() { print(this->addr()); } }; class PersonImpl : Person { private: String _name; String _addr; public: virtual String name() { return _name; } virtual String addr() { return _addr; } };</pre>

7.5 Integrate into existing language compilers

The new programming rules we introduced in Section 4 can also be added to existing OOP language compilers (e.g. C++/Python/Java/C#/Lisp/OCaml/Eiffel/Scala/D etc.), maybe with a new command-line option, to help the programmers to achieve clean multiple inheritance in these languages.

It is our hope that the success of DDIFI will guide the future OOP languages design and implementation.

ACKNOWLEDGMENTS

A APPENDIX

The supplementary DDIFI.tgz contains the implementation source code of this design pattern DDIFI in the following 9 languages: C++, Java, C#, Python, OCaml, Scala, Eiffel, Lisp (CLOS [Steele 1990]) and D etc.

A.1 Our approach demo in C#

C#'s (\geq v8.0) default interface methods are essentially the same as Java's [Microsoft 2022]. The following is the equivalent C# program of our Java example:

Listing 9. MI in C#

```
1  using System;
2
3  interface Person {
4      public string name(); // abstract property method, to be implemented
5      public string addr(); // abstract property method, to be implemented
6      // no actual field
7  }
8
9  class PersonImpl : Person {
10     // only define fields and property methods in data implementation class
11     string _name = null;
12     string _addr = null;
13     public string name() { return _name; }
14     public string addr() { return _addr; }
15 }
16
17 interface Faculty : Person {
18     string lab() {return addr();} // new semantic assigning property
19
20     // regular methods
21     void doBenchwork() {
22         Console.WriteLine(name() + " doBenchwork in the " + lab());
23     }
24 }
25
26 class FacultyImpl : PersonImpl, Faculty {
27     // nothing new needed, so just extends PersonImpl
28 }
29
30 interface Student : Person {
31     string dorm() {return addr();} // new semantic assigning property
32
33     // regular methods
34     void takeRest() {
35         Console.WriteLine(name() + " takeRest in the " + dorm());
36     }
37 }
38
39 class StudentImpl : PersonImpl, Student {
40     // nothing new needed, so just extends PersonImpl
41 }
42
43 interface ResearchAssistant : Student, Faculty {
44     // factory method
45     public static ResearchAssistant make() {
46         ResearchAssistant ra = new ResearchAssistantImpl();
47         return ra;
48     }
49 }
50
51 class ResearchAssistantImpl : ResearchAssistant {
52     // define three fields: NOTE: totally independent to those fields in PersonImpl, StudentImpl, and FacultyImpl
53     string _name;
54     string _faculty_addr;
55     string _student_addr;
56
57     public ResearchAssistantImpl() { // constructor
58         _name = "ResAssis";
59         _faculty_addr = "lab";
60         _student_addr = "dorm";
61     }
62
63     // property methods
64     public string name() { return _name; }
```

```

65 public string addr() { return dorm(); } // use dorm as addr
66 public string dorm() { return _student_addr; }
67 public string lab() { return _faculty_addr; }
68 }
69
70
71 public class MI {
72     public static void Main(string[] args) {
73         ResearchAssistant ra = ResearchAssistant.make();
74         Faculty f = ra;
75         Student s = ra;
76
77         ra.doBenchwork(); // ResAssis doBenchwork in the lab
78         ra.takeRest();    // ResAssis takeRest in the dorm
79
80         f.doBenchwork();  // ResAssis doBenchwork in the lab
81         s.takeRest();     // ResAssis takeRest in the dorm
82     }
83 }

```

A.2 Our approach demo in Python

The following is the equivalent Python program of our Java example:

Listing 10. MI.py

```

1  import abc
2
3  class Person:
4      @abc.abstractmethod
5      def name(self): # abstract property method, to be implemented
6          pass
7
8      @abc.abstractmethod
9      def addr(self): # abstract property method, to be implemented
10         pass
11
12     # no actual field
13
14
15     class PersonImpl(Person):
16         # only define fields and property methods in data implementation class
17         def __init__(self):
18             self._name = "name";
19             self._addr = "addr";
20
21         def name(self): return self._name;
22         def addr(self): return self._addr;
23
24
25     class Faculty(Person):
26         def lab(self): return self.addr(); # new semantic assigning property
27
28         # regular methods
29         def doBenchwork(self):
30             print(self.name() + " doBenchwork in the " + self.lab());
31
32
33     class FacultyImpl(PersonImpl, Faculty):
34         # nothing new needed, so just: PersonImpl
35         pass
36
37
38     class Student(Person):
39         def dorm(self): return self.addr(); # new semantic assigning property
40
41         # regular methods
42         def takeRest(self):
43             print(self.name() + " takeRest in the " + self.dorm());
44
45
46     class StudentImpl(PersonImpl, Student):
47         # nothing new needed, so just: PersonImpl
48         pass
49
50
51     class ResearchAssistant(Student, Faculty):
52         # factory method
53         @staticmethod

```

```
54     def make():
55         ra = ResearchAssistantImpl();
56         return ra;
57
58
59 class ResearchAssistantImpl(ResearchAssistant):
60     # define three fields: NOTE: totally independent to those fields in PersonImpl, StudentImpl, and FacultyImpl
61     def __init__(self): # constructor
62         self._name = "ResAssis";
63         self._faculty_addr = "lab";
64         self._student_addr = "dorm";
65
66     # property methods
67     def name(self): return self._name;
68     def addr(self): return self.dorm(); # use dorm as addr
69     def dorm(self): return self._student_addr;
70     def lab(self): return self._faculty_addr;
71
72
73 def main():
74     ra:ResearchAssistant = ResearchAssistant.make();
75     f:Faculty = ra;
76     s:Student = ra;
77
78     ra.doBenchwork(); # ResAssis doBenchwork in the lab
79     ra.takeRest();    # ResAssis takeRest in the dorm
80
81     f.doBenchwork(); # ResAssis doBenchwork in the lab
82     s.takeRest();    # ResAssis takeRest in the dorm
83
84
85 if __name__ == '__main__':
86     main()
```

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