Assignment 1. Social Norms and Strategy Behavior

Submission deadline: Nov 3, 2017

Question 1

Consider the following variant to a two-person guessing game. Two players simultaneously choose a number. Both players have their own target value: p_1 and p_2, respectively, and their own set of strategies: $x_1 \in X_1$ and $x_2 \in X_2$, respectively. Player i's payoff is decreasing in the difference between the number x_i chosen and p_i times the number x_j chosen by the other player: $\pi_1 = 1000 - |x_1 - p_1x_2|$, $\pi_2 = 1000 - |x_2 - p_2x_1|$. Suppose $p_1 = 0.7$ and $p_2 = 1.5$; $X_1 = [300, 500]$ and $X_2 = [100,900]$.

a) Suppose that player 1 is a level-1 player (who thinks that player 2 is level-0). What number would player 1 choose?

b) Suppose that player 2 is a level-1 player (who thinks that player 1 is level-0). What number would player 2 choose?

c) Suppose that player 1 is a level-2 player (who thinks that player 2 is level-1). What number would player 1 choose?

d) Suppose that player 2 is a level-2 player (who thinks that player 1 is level-1). What number would player 2 choose?

e) Suppose that player 1 is a level C-2 believing that player 2 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 1 choose?

f) Suppose that player 2 is a level C-2 believing that player 1 is a mixture of level-0 and level-1 with $\tau = 2$. What number would player 2 choose?

g) What is the Nash equilibrium for the game?

h) Run an experiment with some of your friends or family and record what numbers they choose in this game.

i) Which of the different behavioral models best describes the observed choices under i)?

Question 2

Consider a first-price sealed-bid auction with 2 bidders who bid for an item with a common value $v = (s_1 + s_2)/2$ where s_i $(i = 1,2)$ is the signal of bidder. Signals are drawn randomly and independently from a uniform distribution on the interval [0,1].

(a) Check that the strategies $b_i = s_i/2$, for i = 1,2 constitute a Nash Equilibrium.

(b) What will be the bidding strategy of a Level-1 player, when he thinks that Level-0 players bid randomly.

(c) What will be the bidding strategy of a Level-1 player, when she thinks that Level-0 players bid their signal.

(d) Give the intuition for why the bidding strategy under (b) is higher than the bidding strategy under (c).

Question 3

Consider a first-price sealed-bid auction with 2 bidders with independent private values (v_i, i = 1,2) drawn from a uniform distribution on the interval [0,1].

(a) Suppose bidder 1 is type Level-1 (thinking that bidder 2 is type Level-0). What will bidder 1's bidding function be, that is, how will bidder 1's bid b_1 depend on his or her private value v_1?

(b) Suppose bidder 1 is type Level-2 (thinking that bidder 2 is type Level-1). What will bidder 1's bidding function be, that is, how will bidder 1's bid b_1 depend on his or her private value v_1?

(c) In a second-price sealed-bid auction it is a dominant strategy to bid your private value. [You may have learned this in another course.] What does this tell you immediately about the behavior of Level-1 or Level-2 bidders?

Question 4

Consider the following 2-player game, where $x > 1$:

		Cathy	
		L	R
Ron	U	x, 1	0, 0
	D	0 , 0	1, 2

Denote the probability that Ron will play up by q_U and the probability that Cathy will play left by p_L.

(a) What are the Nash equilibria of this game (there are three; two in pure, one in mixed strategies)?

(b) Write down the two equations that determine the Quantal Response Equilibrium (see Section 6.2.5 of Cartwright), that is, write q_U as a function of p_L and write p_L as a function of q_U.

(c) Use a computer program (like excel, matlab, swp, or whatever will do the job) to draw graphs of the two functions under (b) for the case λ=1, and determine the intersection of the two graphs. This gives you the QRE.

(d) Compare the Quantal Response Equilibrium to the mixed strategy Nash Equilibrium. Give the intuition for why they are different.

Question 5.

Investor X and investor Y simultaneously decide how much to invest in a new project. Payoffs are increasing in how much the other investor invests. To be precise the payoffs to X and Y are:

$U^X(x,y) = x(½y + 75 – x)$

$U^Y(x,y) = y(½x + 75 – y)$

where x and y denote the investment level of X and Y, respectively. For both X and Y, the minimum investement level is 0 and the maximum level is 200.

(a) What are the Nash equilibrium investment levels?

(b) Suppose investor X is type level-1. How much will investor X invest?

(c) Suppose investor X is type C-2 (believing that investor Y is a mixture of level-0 and level-1, with proportions determined by τ=1.5). How much will investor X invest?

(d) Compare how the investment levels of type level-0, type level-1, and type level C-2 compare to the Nash equilibrium.

Question 6.

Investor X and investor Y simultaneously decide how much to invest in a new project. Payoffs are decreasing in how much the other investor invests. To be precise the payoffs to X and Y are:

$U^X(x,y) = x(-½y + 125 – x)$

$U^Y(x,y) = y(-½x + 125 – y)$

where x and y denote the investment level of X and Y, respectively. For both X and Y, the minimum investement level is 0 and the maximum level is 200.

Answers the same questions (a)-(d) as in Question 5.

(e) Can you explain why the answers under (d) differ for Question 5 and Question 6?

Question 7.

Consider the following weakest link game (see also Table 6.17 in Cartwright).

	Minimum effort in the group						
	7	6	5	4	3	2	1

	7	13	11	9	7	5	3	1
	6		12	10	8	6	4	2
	5			11	9	7	5	3
Your effort	4				10	8	6	4
	3					9	7	5
	2						8	6
	1							7

(a) Do the Nash equilibria of this game depend on the number of players?

(b) Do you think actual play of the game depends on the number of players? If so, how? And if not, why not?

(c) Does a level-k analysis of the game predict that play will depend on the number of players? If so, how? And if not, why not?

(d) How do you think actual play of the game will be affected when the payoffs are determined by the median effort in the group rather than the minimum effort?

(e) Can you think of a setting in which the minimim effort determines team output and one in which median effort determines team output?

Question 8

Consider a variation of the information cascade game in which each individual only observes the choice of her or his immediate predecessor but not the choices of the individuals before that. Argue whether or not an information cascade can form in this game.

Question 9

You visit an unfamiliar town which has two identical looking restaurants: A and B. You ask a local and she thinks restaurant A is the better one. You see that currently one table is occupied in restaurant A, and three tables in restaurant B. You suspect that all of the current patrons have also asked a local about her or his preferred restaurant. Which restaurant would you choose? Which restaurant should you choose? Would it make a difference if you knew in which order the patrons had entered the two restaurants? Should it make a difference if you knew in which order the patrons had entered the two restaurants?