Game Theory 1: Problem Set 1

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Problem 3

(a)
- If $k_2 = k_H$,

Player 2

		C	D
Player 1	C	$0, 0$	$\underline{0}, \underline{R}$
	D	$\underline{R}, \underline{0}$	$\frac{R}{2} - k_H, \frac{R}{2} - k_H$

- If $k_2 = k_L$,

Player 2

		C	D
Player 1	C	$0, 0$	$\underline{0}, \underline{R}$
	D	$\underline{R}, 0$	$\frac{R}{2} - k_H, \underline{\frac{R}{2} - k_L}$

(b)
- If $k_2 = k_H$,
  The game has two pure strategy Nash equilibria:

$$(D, C), (C, D)$$

Checking for a mixed strategy equilibrium:

$$u_1(C) = 0$$

$$u_1(D, \sigma_2) = \sigma_{2C} \cdot R + (1 - \sigma_{2C}) \cdot \left(\frac{R}{2} - k_H\right)$$

$$u_1(C) = u_1(D, \sigma_2) \iff \sigma_{2C} = \frac{2k_H - R}{2k_H + R}$$

Similarly for player 2:

$$u_2(C) = u_2(\sigma_1, D) \iff \sigma_{1C} = \frac{2k_H - R}{2k_H + R}$$

Thus there is one mixed strategy nash equilibruim:

$$(\sigma_{1C}, \sigma_{1D}), (\sigma_{2C}, \sigma_{2D}) = \left(\frac{2k_H - R}{2k_H + R}, \frac{2R}{2k_H + R}\right), \left(\frac{2k_H - R}{2k_H + R}, \frac{2R}{2k_H + R}\right)$$

- If $k_2 = k_L$,

  We can see that for player 2, the strategy C is strictly dominated by D. The following payoff matrix remains after deleting strategy C for player 2.

  Player 2

		C
Player 1	C	$0, R$
	D	$\frac{R}{2} - k_H, \frac{R}{2} - k_L$

  We can now see that for player 1 in this matrix, strategy D is strictly dominated by strategy C.

  This means that there is only one unique pure strategy Nash equilibrium:

  $$(C, D)$$

  And there are no mixed strategy Nash equilibria.

(c) If player 2 has an airbag, player 2 always prefers driving head on versus swerving to the right as the potential losses from crashing do not outweigh the potential benefits in terms of respect that are generated by driving head on if both players drive head on. Player 2 will thus always gain from driving head on and player 2 will therefore do so.

If player 2 has an airbag, player 1 will know that player 2 will always drive head on. Knowing this, player one will always gain from swerving to the right, as otherwise an accident will occur of which the costs outweigh the benefits. Player 1 will thus always choose to swerve to the right and have a payoff of 0 instead of a negative payoff.

(d) Type spaces:

$\Theta_1 = \{k_H\}$
$\Theta_2 = \{k_L, k_H\}$

Action spaces:

$A_1 = \{\textit{head on},\ \textit{swerve right}\}$
$A_2 = \{\textit{head on},\ \textit{swerve right}\}$

(e) A strategy for player 1 in this game is a function s_1 that specifies with which probabilities to drive head on or swerve right given its type k_H.

A strategy for player 2 in this game is a function s_2 that specifies with which probabilities to drive head on or swerve right for each of its possible types k_L and k_H.

That is,

$$s_i : \Theta_i \to A_i$$

(f)