Game Theory 1: Problem Set 3

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Problem 1

(a) Backward induction:

Second stage (t=2)

$q_2 = b(q_1, q_3) = \frac{1}{2}(a - c_2 - q_1 - q_3)$
$q_3 = b(q_1, q_2) = \frac{1}{2}(a - c_3 - q_1 - q_2)$

$\hookrightarrow q_2 = b(q_1) = \frac{1}{2}(a - c_2 - q_1 - (\frac{1}{2}(a - c_3 - q_1 - q_2)))$
$\frac{3}{4}q_2 = \frac{1}{4}(a + c_3 - q_1 - 2c_2)$

$$q_2(q_1) = \begin{cases} \frac{1}{3}(a + c_3 - q_1 - 2c_2) & \text{if } q_1 \leq a - 2c_2 + c_3 \\ 0 & \text{if } q_1 > a - 2c_2 + c_3 \end{cases}$$

Similarly: $q_3(q_1) = \begin{cases} \frac{1}{3}(a + c_2 - q_1 - 2c_3) & \text{if } q_1 \leq a - 2c_3 + c_2 \\ 0 & \text{if } q_1 > a - 2c_3 + c_2 \end{cases}$

First stage (t=1)
$\pi_1(q_1) = (a - c_1 - q_2(q_1) - q_3(q_1))q_1$
$\pi_1(q_1) = (a - c_1 - \frac{1}{3}(a + c_3 - q_1 - 2c_2) - \frac{1}{3}(a + c_2 - q_1 - 2c_3))q_1$
$\pi_1(q_1) = \frac{1}{3}(a - q_1 + c_3 + c_2 - 3c_1)q_1$
$FOC : \frac{\partial \pi_1}{\partial q_1} = \frac{1}{3}(a - 2q_1 + c_3 + c_2 - 3c_1) = 0$
$\hookrightarrow q_1^* = \frac{1}{2}(a - 3c_1 + c_2 + c_3)$
This function along with the reaction functions of q_2 and q_3 as specified above is the SPNE of this Stackelberg game.

$q_2^* = \frac{1}{3}(a + c_3 - (\frac{1}{2}(a - 3c_1 + c_2 + c_3)) - 2c_2) = \frac{1}{6}(a + c_3 - 5c_2 + 3c_1)$
Similarly: $q_3^* = \frac{1}{6}(a + c_2 - 5c_3 + 3c_1)$

Thus the outcome of the SPNE is:
$(q_1, q_2, q_3) = (\frac{1}{2}(a - 3c_1 + c_2 + c_3), \frac{1}{6}(a + c_3 - 5c_2 + 3c_1), \frac{1}{6}(a + c_2 - 5c_3 + 3c_1))$

(b) Backwards induction:

Second stage (t=2)

$$q_3 = b(q_1, q_2) = \begin{cases} \frac{1}{2}(a - c_3 - q_1 - q_2) & \text{if } q_1 + q_2 \leq a - c_3 \\ 0 & \text{if } q_1 + q_2 > a - c_3 \end{cases}$$

First stage (t=1)

$\pi_1 = (a - c_1 - q_1 - q_2 - \frac{1}{2}(a - c_3 - q_1 - q_2))q_1 = \frac{1}{2}q_1(a - 2c_1 + c_3 - q_1 - q_2)$

$FOC : \frac{\partial \pi_1}{\partial q_1} = \frac{1}{2}(a - 2c_1 + c_3 - 2q_1 - q_2) = 0$

$q_1(q_2) = \frac{1}{2}(a - 2c_1 + c_3 - q_2)$

Similarly, $q_2(q_1) = \frac{1}{2}(a - 2c_2 + c_3 - q_1)$

Substituting for q_2 in $q_1(q_2)$ we find:

$q_1 = \frac{1}{2}(a - 2c_1 + c_3 - (\frac{1}{2}(a - 2c_2 + c_3 - q_1)))$

$q_1 = \frac{1}{4}(a - 4c_1 + c_3 + 2c_2 + q_1)$

$\frac{3}{4}q_1 = \frac{1}{4}(a - 4c_1 + c_3 + 2c_2)$

$q_1^* = \frac{1}{3}(a - 4c_1 + c_3 + 2c_2)$

Similarly: $q_2^* = \frac{1}{3}(a - 4c_2 + c_3 + 2c_1)$

These functions along with the reaction function of firm 3 as specified above is the SPNE of this Stackelberg game

$q_3^* = \frac{1}{2}(a - c_3 - \frac{1}{3}(a - 4c_1 + c_3 + 2c_2) - \frac{1}{3}(a - 4c - 2 + c_3 + c_1))$

$q_3^* = \frac{1}{6}(a - 5c_3 + 2c_1 + 2c_2)$

Thus the outcome of the SPNE is:

$(q_1, q_2, q_3) = (\frac{1}{3}(a - 4c_1 + c_3 + 2c_2), \frac{1}{3}(a - 4c_2 + c_3 + 2c_1), \frac{1}{6}(a - 5c_3 + 2c_1 + 2c_2))$

(c)

(d) $c_1 = c_2 = c_3 = c$ and $a > c$

$SPNE_{(a)} = (\frac{1}{2}(a - c), \frac{1}{6}(a - c), \frac{1}{6}(a - c)) \rightarrow Q = \frac{5}{6}(a - c)$

$SPNE_{(b)} = (\frac{1}{3}(a - c), \frac{1}{3}(a - c), \frac{1}{6}(a - c)) \rightarrow Q = \frac{5}{6}(a - c)$

$SPNE_{(c)} = (\frac{1}{2}(a - c), \frac{1}{2}(a - c), \frac{1}{8}(a - c)) \rightarrow Q = \frac{9}{8}(a - c)$

With symmetric cost levels and $a > c$ a welfare maximizing regulator would choose for the market structure defined in question (c). Total output is the greatest under this structure, therefore this is the closest to the perfectly competititve market (maximum total welfare) of all three structures (a), (b) and (c).

$TW_{(c)} > TW_{(a)} = TW_{(b)}$ as: $\frac{9}{8}(a - c) > \frac{5}{6}(a - c) = \frac{5}{6}(a - c)$

Problem 2