Game Theory 1: Problem Set 2

Group 10: J. Bouten; T. Vissers; F. Strik; J. Bonthuis; M. Kroes

22 September, 2017

Problem 1

Homogeneous cost levels:
$\pi_i = (100 - q_i - q_{-i} - 20)q_i = 80q_i - q_iq_{-i} - q_i^2$
FOC: $\frac{\partial \pi_i}{\partial q_i} = 80q_i - q_{-i} - 2q_i = 0 \rightarrow q_i = 40 - \frac{1}{2}q_{-i}$

Therefore: $q_i = 40 - \frac{1}{2}(40 - \frac{1}{2}q_i) \rightarrow q_i = \frac{80}{3}$
Thus the Nash equilibrium in the homogeneous cost model is $(q_1, q_2) = (\frac{80}{3}, \frac{80}{3})$
$HHI_{old} = \left(\frac{1}{2}\right)^2 \cdot 2 = \frac{1}{2}$

Heterogeneous cost levels:
$\pi_1 = 70q_1 - q_1q_2 - q_1^2$
FOC: $\frac{\partial \pi_1}{\partial q_1} = 70q_1 - q_2 - 2q_1 = 0 \rightarrow q_1 = 35 - \frac{1}{2}q_2$
$\pi_2 = 90q_2 - q_1q_2 - q_2^2$
FOC: $\frac{\partial \pi_2}{\partial q_2} = 90 - q_1 - 2q_2 = 0 \rightarrow q_2 = 45 - \frac{1}{2}q_1$
$q_1 = 35 - \frac{1}{2}(45 - \frac{1}{2}q_1) \rightarrow q_1 = \frac{50}{3}$
$q_2 = 45 - \frac{1}{2} \cdot \frac{50}{3} \rightarrow q_2 = \frac{110}{3}$
Thus the Nash equilibrium in the heterogeneous cost model is $(q_1, q_2) = (\frac{50}{3}, \frac{110}{3})$
$HHI_{new} = \left(\frac{5}{16}\right)^2 + \left(\frac{11}{16}\right)^2 = \frac{73}{128} > \frac{1}{2}$

The Herfidahl-Hirschmann Index is now larger than it was before, the market concentration has become larger.

Problem 2