Game Theory 1: Problem Set 2

Group 10: J. Bouten; T. Vissers; F. Strik; J. Bonthuis; M. Kroes

22 September, 2017

Problem 1

Homogeneous cost levels:
$\pi_i = (100 - q_i - q_{-i} - 20)q_i = 80q_i - q_i q_{-i} - q_i^2$
FOC: $\frac{\partial \pi_i}{\partial q_i} = 80q_i - q_{-i} - 2q_i = 0 \rightarrow q_i = 40 - \frac{1}{2}q_{-i}$
Symmetry: $q_i = q_j \rightarrow q_i = 40 - \frac{1}{2}(q_i) \rightarrow q_i = \frac{80}{3}$
Thus the Nash equilibrium in the homogeneous cost model is $(q_1, q_2) = (\frac{80}{3}, \frac{80}{3})$
$HHI_{old} = \left(\frac{1}{2}\right)^2 \cdot 2 = \frac{1}{2}$

Heterogeneous cost levels:
$\pi_1 = 70q_1 - q_1 q_2 - q_1^2$
FOC: $\frac{\partial \pi_1}{\partial q_1} = 70q_1 - q_2 - 2q_1 = 0 \rightarrow q_1 = 35 - \frac{1}{2}q_2$
$\pi_2 = 90q_2 - q_1 q_2 - q_2^2$
FOC: $\frac{\partial \pi_2}{\partial q_2} = 90 - q_1 - 2q_2 = 0 \rightarrow q_2 = 45 - \frac{1}{2}q_1$
$q_1 = 35 - \frac{1}{2}(45 - \frac{1}{2}q_1) \rightarrow q_1 = \frac{50}{3}$
$q_2 = 45 - \frac{1}{2} \cdot \frac{50}{3} \rightarrow q_2 = \frac{110}{3}$
Thus the Nash equilibrium in the heterogeneous cost model is $(q_1, q_2) = (\frac{50}{3}, \frac{110}{3})$
$HHI_{new} = \left(\frac{5}{16}\right)^2 + \left(\frac{11}{16}\right)^2 = \frac{73}{128} > \frac{1}{2}$

The Herfidahl-Hirschmann Index is now larger than it was before, the market concentration has become larger.

Problem 2

Problem 3

(a) $\pi_i = p_i q_i - F = q_i - q_i^2 - \theta q_i \sum_{j \neq i} q_j - F$
FOC:$\frac{\partial \pi_i}{\partial q_i} = 1 - 2q_i - \theta \sum_{j \neq i} q_j = 0 \rightarrow q_i = \frac{1}{2} - \frac{\theta}{2} \sum_{j \neq i} q_j$
Symmetry: $q_i = q_j \rightarrow q_i = \frac{1}{2} - \frac{\theta}{2}(n-1)q_i \rightarrow q_i = \frac{1}{2+\theta(n-1)}$
Symmetric Nash equilibrium: $(q_i^*) = (\frac{1}{2+\theta(n-1)})$

(a) Firms will enter as long as they can generate positive profits by entering the market. This then determines the number of firms that will enter.
We find the equilibrium number of firms (n^*) by setting profits ($\pi_i(q_i, q_j, n, \theta) = \pi_i(q_i^*, q_i^*, n, \theta)$)

equal to 0 and then solving for n.
$\pi_i = \left(\frac{1}{2+\theta(n-1)}\right)\left(1-(\theta(n-1)+1)\left(\frac{1}{2+\theta(n-1)}\right)\right) - F = 0 \rightarrow n =$

Problem 4