# Game Theory 1: Problem Set 1

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**Problem 1**

Firstly we delete vectors **A** (dom. by **B** & **C**), **a** (dom. by **b**), **E** (dom. by**B**), **d** (dom. by **b**) and **D** (dom. by **B**). These vectors are all strictly dominated.

The following matrix now remains:

| | |
|---|---|
| 88, 130 | 88, 130 |
| 58, 160 | 128, 90 |

From this game we can see that **c** is weakly dominated by **b**, after deleting **c** we find that **C** is strictly dominated by **B**. The remaining cell is [**B,b**], based on deleting dominated strategies the recommended strategy would be to play **B** for player 1 and **b** for player 2.

**Problem 2**

If first weakly dominated strategy **T** is deleted, the outcome will be one in row **B** (meaning the outcome will be either cell [**B,L**] or cell [**B,R**]).

On the other hand, if weakly dominated strategy **L** is deleted first, the outcome will be one in column **R** (meaning the outcome will be either cell [**T,R**] or cell [**B,R**]). The second mover has no incentive to choose as the outcome for the second mover will always equal 0.

In the case that weakly dominated strategies can be deleted simultaneously, the outcome will be [**B,R**].

**Problem 3**

One solution is first deleting strictly dominated strategy $\mathbf{H_1}$ (dom. by $\mathbf{O_1}$) for player one, which leaves the matrix

| | |
|---|---|
| -1, 1 | 1, -1 |
| 3, 1 | 2, 1 |

In this matrix, we can now delete weakly dominated strategy $\mathbf{T_2}$ (dom. by $\mathbf{H_2}$) for player two. Lastly we can delete strictly dominated strategy $\mathbf{T_1}$ (dom. by $\mathbf{O_1}$) for player one, leaving the cell **[O,H]** as our first possible solution.

Another solution is starting with deleting strictly dominated strategy $\mathbf{T_1}$ (dom. by $\mathbf{O_1}$) for player one. Then deleting weakly dominated strategy $\mathbf{H_2}$ (dom. by $\mathbf{T_2}$) for player two and lastly deleting strictly dominated strategy $\mathbf{H_1}$ (dom. by $\mathbf{O_1}$) for player one, leaving the second possible solution **[O,T]**.

### Problem 4

In column **L**, the highest outcome for player one is given by playing strategy **T**. In column **M** the highest outcome for player one is given by playing strategy **B**. In column **R** the highest outcome for player one is given by playing strategy **M**. From the viewpoint of player one we now have three candidates for Nash equilibria: **[T,L]**, **[B,M]** and **[M,R]**.

We now have to check whether player two would deviate from these cells, for this we run the same procedure for player two. In row **T**, the highest outcome for player two is given by playing strategy **L**. In row **M** the highest outcome for player two is given by playing strategy **M**. in row **B** the highest outcome for player two is given by playing strategy **M**. From this information we can see that **[M,R]** is not a Nash equilibrium, because player two would deviate.There are two Nash equilibria: cell **[T,L]** and cell **[B,M]**.

### Problem 5

#### (a)

If **[A,a]** is played, player 2 has an incentive to change to **[A,b]**,
If **[A,b]** is played, player 1 has an incentive to change to **[B,b]**,
If **[B,B]** is played, player 2 has an incentive to change to **[B,a]**,
If **[B,a]** is played, player 1 has an incentive to change to **[A,a]**,
If **[A,a]** is played, ...
And so forth: the game is never in equilibrium as there is always a possible deviation leading to a higher outcome to one of the two players. The best responses of player 1 and player 2 never intersect, therefore there is no cell which is a Nash equilibrium in pure strategies.

#### (b)

To check whether $[(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})]$ is a Nash equilibrium in mixed strategies we will solve the following system of equations:

Player 2's strategy: $\sigma_2 = (q, 1-q), 0 \leq q \leq 1$.

$u_1(A, \sigma_2) = 1 \cdot q - 1(1-q) = 2q - 1$
$u_1(B, \sigma_2) = -1 \cdot q + 1(1-q) = 1 - 2q$

If there is a Nash equilibrium then player 1 randomizes between both of her pure strategies. Player 1 must be indifferent between both of her strategy choices:

If $2q - 1 = 1 - 2q$ then $q = \frac{1}{2}$

Player 1's strategy: $\sigma_1 = (p, 1-p), 0 \leq p \leq 1$.

$u_2(\sigma_1, a) = -1 \cdot p + 1(1-p) = 1 - 2p$
$u_2(\sigma_1, b) = 1 \cdot p - 1(1-p) = 2p - 1$

If there is a Nash equilibrium then player 2 randomizes between both of her pure strategies. Player 2 must be indifferent between both of her strategy choices:

If $2p - 1 = 1 - 2p$ then $p = \frac{1}{2}$

Therefore indeed, $\sigma = (\sigma_1, \sigma_2) = ((\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}))$ is a Nash equilibrium in mixed strategies.

**Problem 6**

**(a)**

$P(Q) = a - b \cdot Q$ with $MC = 0$

$Q = q_1 + ... + q_n$

Due to symmetry $q_i = q_1 = q_2 = ... = q_n$

$\pi_i = (a - b \cdot Q)q_i - cq_i$

We set the derivative of this profit function equal to 0 to find the best response function of firm i.

$\frac{\partial \pi_i}{\partial q_i} = a - 2q_i - bQ_{n-i} - c = 0$

From this equation we can get the best response of firm i:

$q_i^* = \frac{a-c}{b(1+n)}$

This is a unique Nash equilibrium, deviation from $q_i^*$ of any firm can only lead to a negative effect on individual profits. Cournot equilibria are always unique due to convergence of $q_i$ towards the Cournot equilibrium from any starting point of $q_i$. It is a unique Nash equilibrium because it is the only point where the best response functions of all players intercept.

**(b)**

As $n \to \infty$,
$q_i^* = \frac{a-c}{b(1+n)} \to 0$
$P = \frac{a+nc}{1+n} \to c$ (marginal costs)
$\pi_i = \frac{(a-c)^2}{b(1+n)^2} \to 0$

Economic intuition:
An increase in the number of firms increases competition. $n \to \infty$ is the outcome of the perfectly competitive market with prices equal to marginal costs. No firm has the market power to obtain any profits. The outcomes under a cournot market with infinitely many firms are equal to the outcomes in a market with Bertrand competition, where firms compete in prices until the price equals their marginal cost level. (Assuming homogeneous products and cost levels)