

Methods: Game Theory 1

Fall 2017

Problem Set 2: Static Games of Complete Information

Problem 1: Consider a market with inverse demand function $P = 100 - Q$. There are two firms, both with cost function $C_i(q_i) = 20q_i$. What is the Cournot-Nash equilibrium? Now suppose that firm 1's costs increase to $C_1(q_1) = 30q_1$, while firm 2's cost decrease to $C_2(q_2) = 10q_2$. What is the new equilibrium? Calculate the Herfindahl-Hirshman Index (HHI) of market concentration for this equilibrium. Is it smaller or larger than it was before? (For a definition of the HHI and another index of market concentration, please see the last page of this document.)

Problem 2: Consider a market consisting of four firms, each of which produces the same product. The inverse market demand for this product is $P = 100 - Q$, where P is price and Q is aggregate output. The production costs for firms 1, 2, and 3 are identical and given by $C(q_i) = 20q_i$ ($i = 1, 2, 3$), where q_i is the output of firm i . This means that for each of these firms, variable costs are constant at 20 per unit. The production costs for firm 4 are $C(q_4) = (20 + \gamma)q_4$, where $\gamma > -20$ is some constant. Note that if $\gamma > 0$, then firm 4 is a high-cost firm, while if $\gamma < 0$, firm 4 is a low-cost firm. Note also that $Q = q_1 + q_2 + q_3 + q_4$. Assume that the firms each choose their outputs to maximise profits given that they each act as Cournot competitors.

- (a) Compute the individual output and the profit for each firm as well as the product price in the Nash equilibrium. For this to be a “true” equilibrium in which all four firms are active, all of the firms must at least be covering their variable costs. Identify the constraints that γ must satisfy for this to be the case.
- (b) Assume now that firms 1 and 2 merge (so that firms 1 and 2 maximize their joint profits) and that all firms (including the merged entity) continue to act as Cournot competitors after the merger. Is this merger profitable? (Hint: Compare the joint pre-merger profits of firms 1 and 2 with their profit as a merged entity after the merger.) Clearly show the reasoning and the computations that lead to your answer to this question.
- (c) Now assume that firms 1 and 4 merge. Can this merger be profitable if γ is positive so that firm 4 is a high-cost firm? What are the conditions on γ so that the profits of firm 2 rise as a result of this merger?

Problem 3: Suppose there are n firms selling differentiated products. Inverse demand for firm i is $p_i = 1 - q_i - \theta \sum_{j \neq i} q_j$, where $\sum_{j \neq i} q_j$ is the sum of the quantities of all $n - 1$ firms other than firm i . Suppose the cost function for each firm is $C_i(q_i) = F$ (only fixed cost).

- (a) What is the (symmetric) Nash equilibrium if the firms compete in quantities?
- (b) Suppose now that there is free entry. What determines the number of firms that will enter the market? Compute the number of firms that will eventually be in the market under the assumption that market demand and cost functions do not change! Argue why and how the number of firms will be affected by the parameter θ .

Problem 4: Find all pure and mixed-strategy Nash equilibria of the following game!

		Player 2		
		L	M	R
Player 1	T	2, 2	0, 3	1, 3
	B	3, 2	1, 1	0, 2

Measuring market structure

- The Concentration Ratio, CR_n , is defined as the aggregate market share of the largest n firms. See below for an example.
- The Herfindahl-Hirschman Index (HHI) for an industry with N firms is defined as

$$HHI = \sum_{i=1}^N (s_i)^2,$$

where where $s_i = \frac{q_i}{Q}$ is the market share of the i th largest firm:

- It is more common to measure the shares in percentage terms such that the $HHI = \sum_{i=1}^N (100s_i)^2$ attains its maximum of 10,000 in case of monopoly ($100^2 = 10,000$).

Example:

Firm Rank	Market Share in %	Squared Market Share
1	25	625
2	25	625
3	25	625
4	5	25
5	5	25
6	5	25
7	5	25
8	5	25
Concentration Index	$CR_4 = 80$	$HHI = 2,000$

- Advantage of HHI over a measure such as CR_n : It reflects the combined influence of both unequal firm sizes and the concentration of activity in a few large firms. It provides a “better” picture of the overall distribution of firms sizes in the industry. (See one of the homework problems.)
- If you have problems understanding the above definitions or the examples, please consult a textbook on Industrial Organization!