The Physics behind the Simulation

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Introduction: What Are We Seeing?

Black holes represent one of the most extreme and captivating phenomena in the known universe. As objects of immense mass concentrated into a single point, they are a cornerstone of modern astrophysics and a subject of immense public fascination.

The simulation this report analyzes provides a direct visual representation of a black hole's immediate environment. We observe the object not by seeing the hole itself, but by witnessing the dramatic effects it has on its surroundings. It is visualized as a central dark shadow, surrounded by a brightly glowing accretion disk, and set against the backdrop of a distorted starry sky.

The goal of this report is to explain the underlying physical phenomena that combine to create this remarkable image. We will explore how extreme gravitation, as described by Einstein's General Relativity, bends the fabric of spacetime. We will then examine how this curvature dictates the path of light, leading to gravitational lensing, and finally, we will analyze the relativistic effects, such as the Doppler shift, that illuminate the accretion disk.

1 The Foundation: Gravitation as Spacetime Curvature

In Newtonian physics, gravitation is a force acting between two masses. In Einstein's General Relativity (GR), this is fundamentally different: gravitation is not a force, but a property of geometry. The presence of mass and energy curves the four-dimensional spacetime.

Other objects (and light as well) then move along the "straightest" possible paths—so-called geodesics — through this curved spacetime.

1.1 The Schwarzschild Metric

To describe this curvature, GR uses a "metric tensor" $g_{\mu\nu}$. This tensor defines how distances and times are measured at every point in spacetime. The **Schwarzschild metric** is the exact and simplest solution to Einstein's field equations that describes the spacetime outside a static, non-rotating, spherically symmetric mass M—making it an ideal model for the black hole in our simulation.

In spherical coordinates (t, r, θ, ϕ), the line element ds^2 (the "distance" between two infinitesimally close spacetime points) is:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Here, G is the gravitational constant, c is the speed of light, and M is the mass of the black hole.

The point where this metric exhibits a singularity is the **Schwarzschild Radius** r_s:

$$r_s = \frac{2GM}{c^2}$$

This is the "event horizon". If we substitute r_s into the metric, it becomes more readable:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

(where $d\Omega^2$ represents the spherical surface term)

2 The Method: The Path of Light

How do we find the path that light takes?

2.1 The Null Geodesic

Light (photons) moves at the speed of light. In relativity, this means the four-dimensional "distance" it travels is always exactly zero. Its path is a **null geodesic**.

$$ds^2 = 0$$

To find the path of a photon, we must therefore set the equation for the Schwarzschild metric to zero. For motion in the equatorial plane ($\theta = \pi/2$, $d\theta = 0$), this simplifies to:

$$0 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\phi^2$$

Solving this equation to get r as a function of ϕ (the path) is the core of the problem.

2.2 Connection to the Simulation

In the simulation this happens in small discrete timesteps (λ to be exact). This results in the following three equations, describing the path of the light:

$$\begin{aligned} \dot{r} &= p_r \\ \dot{\phi} &= \frac{L}{r^2} \\ \dot{p_r} &= \frac{L^2}{r^3} - \frac{3}{2} r_s \frac{L^2}{r^4} \end{aligned}$$

The calculation continues until one of the following three things happens:

The light falls into the black hole.
- The respective pixel is colored black

The light leaves the simulation area.
- The respective pixel is colored in the color of the starry sky.

The photon hits the accretion disk.
- The respective pixel is colored to the corresponding color of the accretion disk

3 The Accretion Disk

The glowing ring of fire around the black hole is known as an **accretion disk**. This is not a solid object, but a disk of gas, dust, and plasma orbiting the black hole at tremendous speeds, much like planets orbit a star.

Due to the immense frictional forces within the accretion disk, the infalling material is heated to temperatures of millions of degrees, resulting in intense radiation, primarily in the X-ray and gamma-ray spectra. These extreme temperatures cause the accretion disk to appear predominantly blue, despite its frequent depiction as red in popular media. In the simulation, the accretion disk is modulated using Perlin noise and a color fade to produce a more realistic and natural appearance.

3.1 The Relativistic Doppler Effect

One of the most striking features of the simulation is the asymmetrical brightness of the accretion disk: one side is dramatically brighter than the other. This phenomenon is caused by relativistic beaming. It occurs because the light in the disk is moving at extreme speeds, reaching a significant fraction of the speed of light. At such high speeds, light is no longer emitted evenly in all directions. Instead, it is strongly "beamed" or "focused" in the direction of motion.

The approaching side: The material moving toward the camera focuses its light in our direction. This focusing of light energy makes the disk appear dramatically brighter and bluer on this side.

The receding side: The material moving away from us beams its light away from us. This "de- beaming" effect causes much less light to reach us and makes the disk appear darker and redder.