

## Kodutöö nr. 5

### 1. variant

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Vektorruumil  $\mathbb{R}^4$  üle  $\mathbb{R}$  on defineeritud skalaarkorrutamine järgmise valemiga:

$$\langle (x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4) \rangle = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2 + x_3y_3 + 2x_3y_4 + 2x_4y_3 + 5x_4y_4.$$

Ortonormeerida Gram-Schmidti protsessiga vektorsüsteem

$$a_1 = (2, 4, 3, 4), \quad a_2 = (0, 1, 0, 2), \quad a_3 = (5, 4, 1, 4)$$

selle skalaarkorrutamise suhtes.

#### Lahendus:

Kõigepealt võtan uue süsteemi esimeseks vektoriks vana süsteemi esimese vektori:  $b_1 = a_1 = (2, 4, 3, 4)$ . Teine vektor esitub kujul  $b_2 = k_1b_1 + a_2$ , kuna ta peab kuuluma algse võrrandisüsteemi lineaarkattesse, ning et ta oleks vektoriga  $b_1$  ortogonaalne, peab kehtima  $\langle b_1, k_1b_1 + a_2 \rangle = 0$ . Leian sealt kordaja  $k_1$ :

$$\begin{aligned} k_1 &= -\frac{\langle b_1, a_2 \rangle}{\langle b_1, b_1 \rangle} = -\frac{\langle (2, 4, 3, 4), (0, 1, 0, 2) \rangle}{\langle (2, 4, 3, 4), (2, 4, 3, 4) \rangle} \\ &= -\frac{2 + 2 \cdot 4 + 2 \cdot 6 + 5 \cdot 8}{4 + 8 + 8 + 2 \cdot 16 + 9 + 2 \cdot 12 + 2 \cdot 12 + 5 \cdot 16} \\ &= -\frac{62}{209} \end{aligned}$$

Seega teine vektor on

$$b_2 = -\frac{62}{209}b_1 + a_2 = \left(-\frac{124}{209}, -\frac{39}{209}, -\frac{186}{209}, -\frac{170}{209}\right) = -\frac{1}{209}(124, 39, 186, 170).$$

Viimane vektor esitub sarnaselt kujul  $b_3 = k_2b_2 + a_3$  ning kordaja saab leida:

$$\begin{aligned} k_2 &= -\frac{\langle b_2, a_3 \rangle}{\langle b_2, b_2 \rangle} \\ &= -\frac{-\frac{1}{209}\langle (124, 39, 186, 170)(5, 4, 1, 4) \rangle}{\left(-\frac{1}{209}\right)^2 \langle (124, 39, 186, 170)(124, 39, 186, 170) \rangle} \\ &= \frac{209\langle (124, 39, 186, 170)(5, 4, 1, 4) \rangle}{\langle (124, 39, 186, 170)(124, 39, 186, 170) \rangle} \\ &= \frac{209 \cdot 7037}{333666} = \frac{1470733}{333666} \end{aligned}$$

Seega kolmas vektor on

$$\begin{aligned} b_3 &= \frac{1470733}{333666}b_2 + a_3 = \left(\frac{795742}{333666}, \frac{1060221}{333666}, -\frac{975216}{333666}, -\frac{138374}{333666}\right) \\ &= \frac{1}{333666}(795742, 1060221, -975216, -138374) \end{aligned}$$

Seega sain ortonormeeritud süsteemiks

$$b_1 \sim (2, 4, 3, 4), \quad b_2 \sim (124, 39, 186, 170), \quad b_3 \sim (795742, 1060221, -975216, -138374)$$