



Frame Coodinate Transformation

심주용
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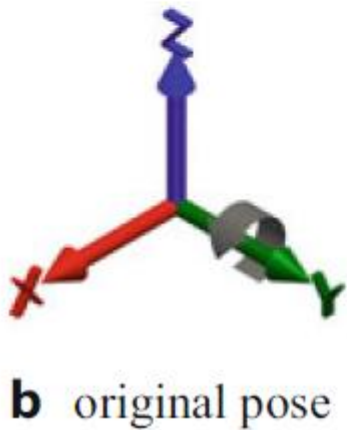
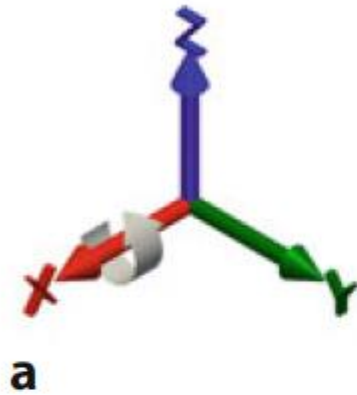
Coordinate Frames



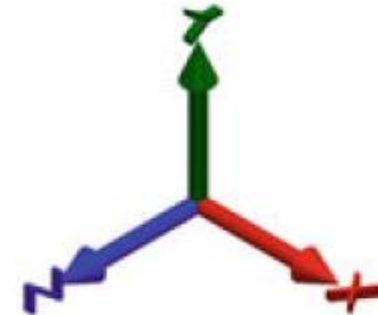
3D Coordinate Frames



Noncommutativity of rotation



after first rotation



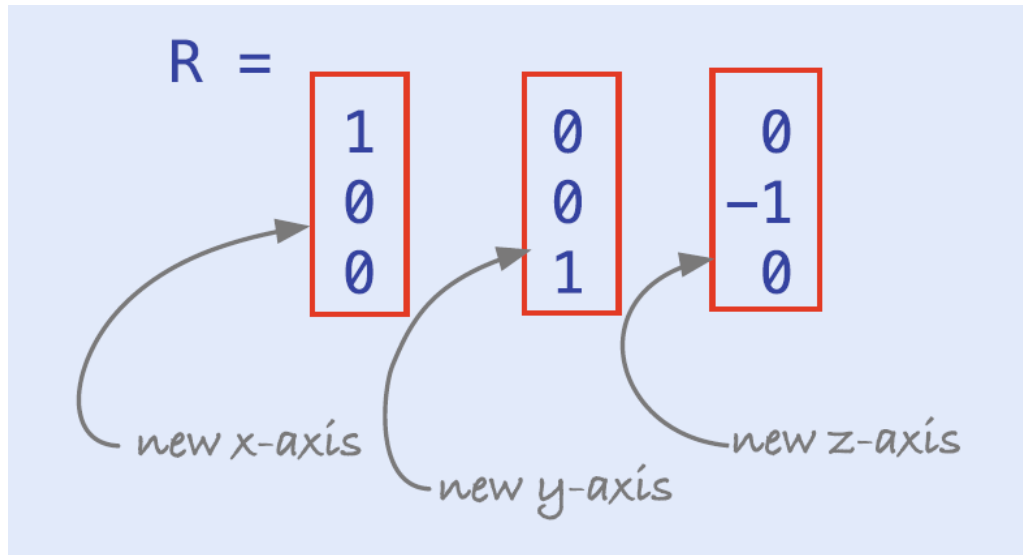
after second rotation

3D Rotation Matrix



Frame B to Frame A

$$\begin{pmatrix} {}^A p_x \\ {}^A p_y \\ {}^A p_z \end{pmatrix} = {}^A \mathbf{R}_B \begin{pmatrix} {}^B p_x \\ {}^B p_y \\ {}^B p_z \end{pmatrix}$$



$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

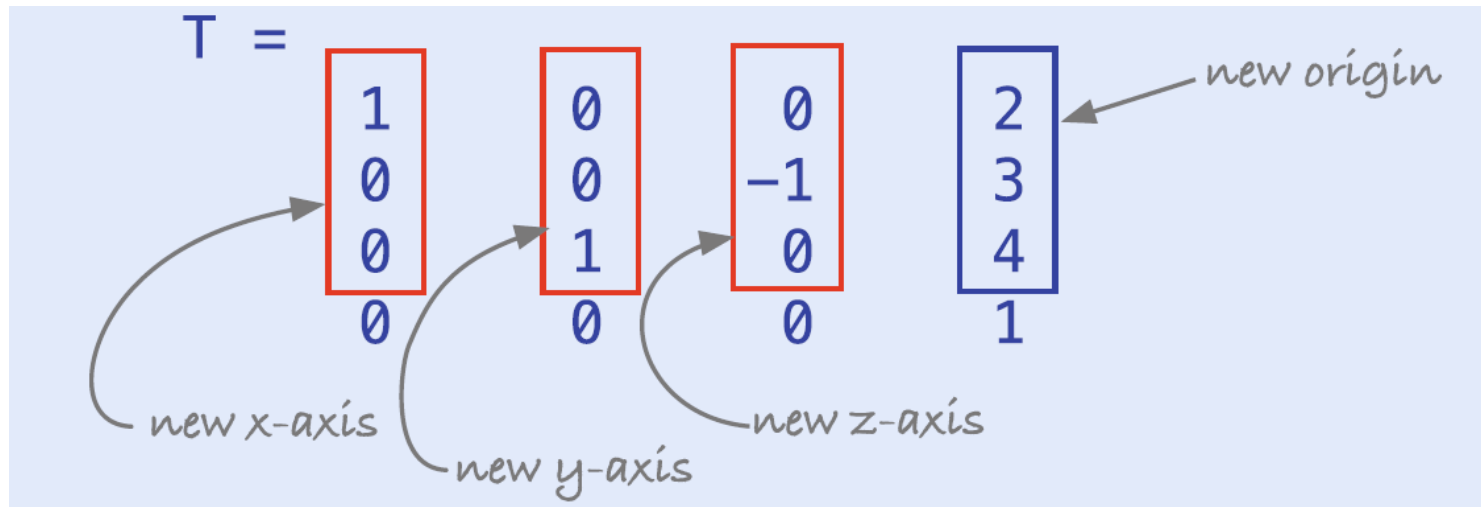
$$\mathbf{R}_y(\theta) = \begin{pmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{pmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Homogeneous Transformation Matrix



$$\begin{pmatrix} A_x \\ A_y \\ A_z \\ 1 \end{pmatrix} = \begin{pmatrix} {}^A\mathbf{R}_B & {}^A\mathbf{t}_B \\ \mathbf{0}_{1 \times 3} & 1 \end{pmatrix} \begin{pmatrix} B_x \\ B_y \\ B_z \\ 1 \end{pmatrix}$$

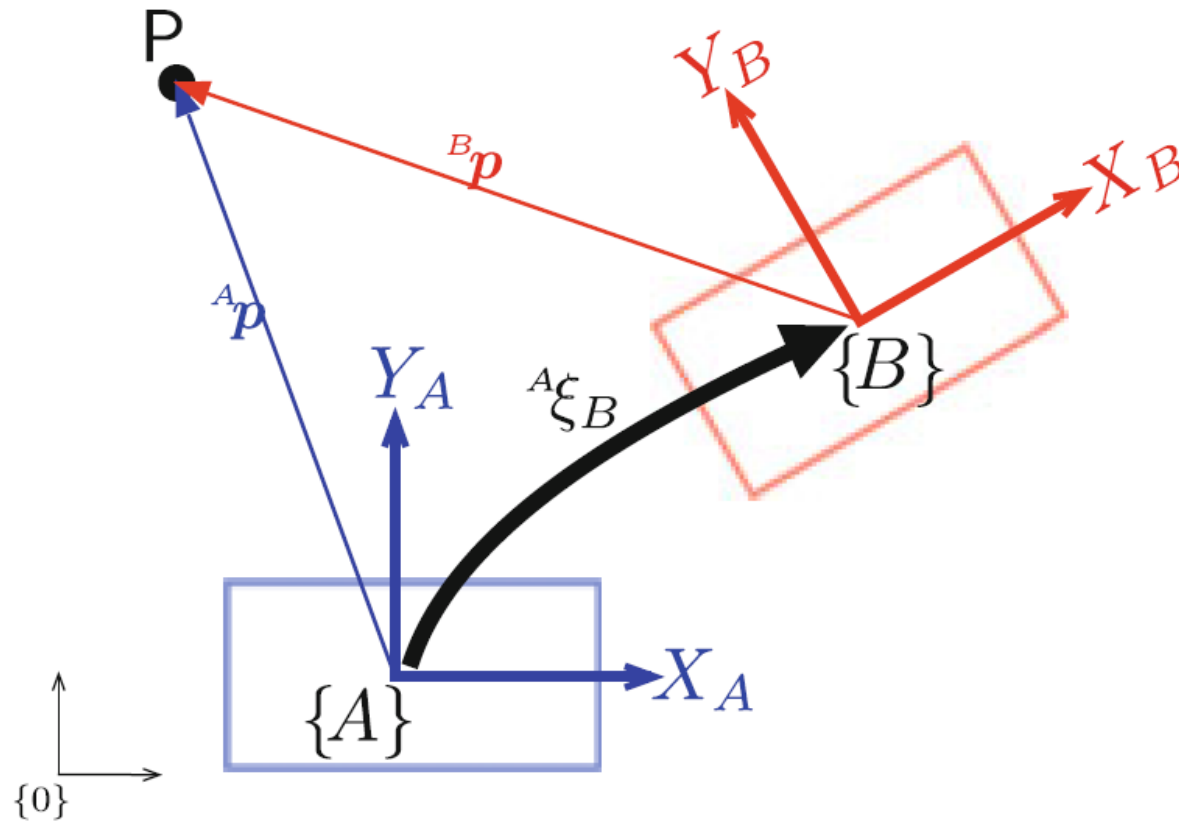


Coordinate Frames



Point P can be described by coordinate vectors expressed in frame {A} or {B}

${}^A\xi_B$: the pose of {B} relative to {A}



$${}^Ap = {}^A\xi_B \cdot {}^Bp$$

$${}^X\xi_Z = {}^X\xi_Y \oplus {}^Y\xi_Z$$

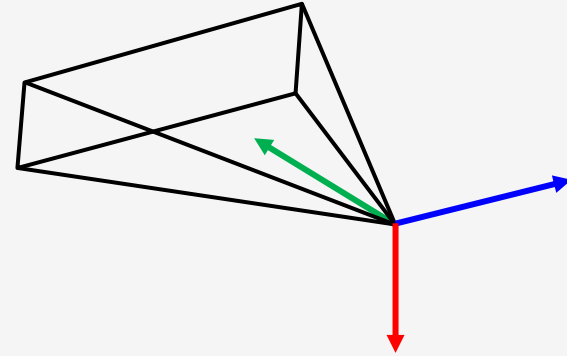
$$\textcircled{X}\xi\textcircled{Y} \oplus \textcircled{Y}\xi\textcircled{Z} = \textcircled{X}\xi\textcircled{Z}$$

same

Frame Coordinate Transformation



```
1 import numpy as np
2 from scipy.spatial.transform import Rotation as R
3 from numpy.linalg import inv
4
5 def create_pose_matrix(x, y, z, heading, pitch, roll):
6     """Create a 4x4 pose matrix from translation and Euler angles."""
7     rot = eul2rot(heading, pitch, roll)
8     pose_matrix = np.eye(4)
9     pose_matrix[:3, :3] = rot
10    pose_matrix[:3, 3] = [x, y, z]
11    return pose_matrix
12
13 # Example DataFrames with data (please ensure prame_data and sel_data are loaded properly)
14 # Generate pose matrices for each dataset
15 team1_poses = [create_pose_matrix(row['x'], row['y'], row['z'], row['heading'], row['pitch'], row['roll']) for index, row in team1_data.iterrows()]
16 sim_poses = [create_pose_matrix(row['x'], row['y'], row['z'], row['heading'], row['pitch'], row['roll']) for index, row in sim_data.iterrows()]
17
18 # Compute individual transformations
19 transformations = []
20
21 for team1_pose, sim_pose in zip(team1_poses, sim_poses):
22     # Compute the transformation  $T_i$  for each pair ( $T_i * sim\_pose = team1\_pose$ )
23     # Rearrange to  $T_i = team1\_pose * inv(sim\_pose)$ 
24     T_i = team1_pose @ inv(sim_pose)
25     transformations.append(T_i)
26
27 # Average the transformations to find the final matrix
28 average_transformation = sum(transformations) / len(transformations)
```



Reality Capture Frame Coordinate

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