Recommender Systems: *Interaction-Based*

Mining Massive Datasets

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Sources

- Data Mining, The Textbook (2015) by Charu Aggarwal (Section 18.5) – slides by Lijun Zhang
- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. (Chapter 9) - slides A, B

Interaction-based recommendations

Missing-value estimation/completion

- The matrix is extremely large and sparse

$$M = \begin{bmatrix} \blacksquare & & \blacksquare & & & \blacksquare & \\ & \blacksquare & & \blacksquare & \blacksquare & & \blacksquare \\ \blacksquare & & & \blacksquare & & \blacksquare & \\ & & \blacksquare & & \blacksquare & & \\ & \blacksquare & & \blacksquare & & & \blacksquare \end{bmatrix} \in \mathbb{R}^{n \times d}$$

Only black squares have non-zero values.

Types of algorithms

- Neighborhood-Based Methods
  - User-Based or Item-Based Similarity with Ratings
- Graph-Based Methods
- Clustering Methods
  - Adapting k-Means Clustering or Adapting Co-Clustering
- Latent Factor Models
  - Matrix Factorization, e.g., Singular Value Decomposition

User-based similarity with ratings

- Let $I_{u,v}$ be common ratings between two users
- Similarity: Pearson correlation coefficient

$$\mathrm{sim}(u,v) = \frac{\sum_{i \in I_{u,v}} (u_i - \hat{u}) \cdot (v_i - \hat{v})}{\sqrt{\sum_{i \in I_{u,v}} (u_i - \hat{u})^2 \cdot \sum_{i \in I_{u,v}} (v_i - \hat{v})^2}}$$

$$\hat{u} = \frac{1}{|u|} \sum_{i=1}^{|u|} u_i \qquad \hat{v} = \frac{1}{|v|} \sum_{i=1}^{|v|} v_i$$

Note: averages are taken over all elements, not only ones in common

User-based similarity with ratings (cont.)

$$\mathrm{sim}(u,v) = \frac{\sum_{i \in I_{u,v}} (u_i - \hat{u}) \cdot (v_i - \hat{v})}{\sqrt{\sum_{i \in I_{u,v}} (u_i - \hat{u})^2 \cdot \sum_{i \in I_{u,v}} (v_i - \hat{v})^2}}$$

- Score of recommendation

$$\mathrm{score}(u,i) = \hat{u} + \frac{\sum_{v : v_i \neq \mathrm{NULL}} \mathrm{sim}(v,u) \cdot (v_i - \hat{v})}{\sum_{v : I_{u,v} \neq \emptyset} |\,\mathrm{sim}(v,u)|}$$

Note: for efficiency one can take only the most similar users

Exercise

	2			4	5	
	5		4			1
			5		2	
		1		5		4
u			4			2
	4	5		1		

$$\mathrm{sim}(u,v) = \frac{\sum_{i \in I_{u,v}} (u_i - \hat{u}) \cdot (v_i - \hat{v})}{\sqrt{\sum_{i \in I_{u,v}} (u_i - \hat{u})^2 \cdot \sum_{i \in I_{u,v}} (v_i - \hat{v})^2}}$$

$$\mathrm{score}(u,i) = \hat{u} + \frac{\sum_{v: v_i \neq \mathrm{NULL}} \mathrm{sim}(v,u) \cdot (v_i - \hat{v})}{\sum_{v: I_{u,v} \neq \emptyset} |\mathrm{sim}(v,u)|}$$

Complete yellow cells in spreadsheet:

1. Similarities *sim(u,v)*
2. Predicted rating of all movies that user *u* has not seen yet
3. **Which movie is recommended?**

Spreadsheet link:
https://upfbarcelona.padlet.org/sandrabuda1/theory-exercises-tdmvfhddcnvfj5b8

Exercise from ML for Recommender Systems tutorial by Alex Karatzoglou, 2015.

Answer

							avg(v)	sim(u,v)
	2			4	5		3.67	NULL
	5		4			1	3.33	0.87
			5		2		3.50	1.00
		1		5		4	3.33	-1.00
u	**3.51**	**3.81**	4	**2.42**	**2.48**	2	3.00	
	4	5		1			3.33	NULL

$$\mathrm{sim}(u,v) = \frac{\sum_{i \in I_{u,v}} (u_i - \hat{u}) \cdot (v_i - \hat{v})}{\sqrt{\sum_{i \in I_{u,v}} (u_i - \hat{u})^2 \cdot \sum_{i \in I_{u,v}} (v_i - \hat{v})^2}} \qquad \mathrm{score}(u,i) = \hat{u} + \frac{\sum_{v: v_i \neq \mathrm{NULL}} \mathrm{sim}(v,u) \cdot (v_i - \hat{v})}{\sum_{v: I_{u,v} \neq \emptyset} |\mathrm{sim}(v,u)|}$$

[Exercise from ML for Recommender Systems tutorial by Alex Karatzoglou, 2015.]

You can do the same with items!

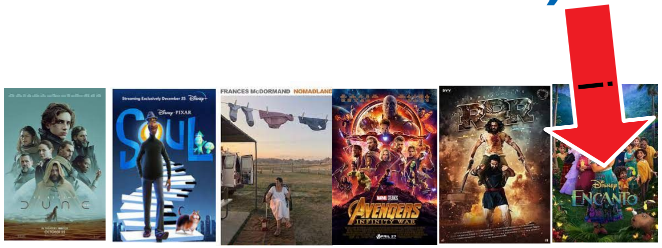
- Item-based similarities with ratings

$$\mathrm{sim}(i,j) = \frac{\sum_{u \in I_{i,j}} (u_i - \hat{i}) \cdot (u_j - \hat{j})}{\sqrt{\sum_{u \in I_{i,j}} (u_i - \hat{i})^2 \cdot \sum_{u \in I_{i,j}} (u_j - \hat{j})^2}}$$

- Item-based recommendations

$$\mathrm{score}(u,i) = \hat{i} + \frac{\sum_{j : u_j \neq \mathrm{NULL}} \mathrm{sim}(i,j) \cdot (u_j - \hat{j})}{\sum_{j : I_{i,j} \neq \emptyset} |\,\mathrm{sim}(i,j)|}$$

(Do it at home)



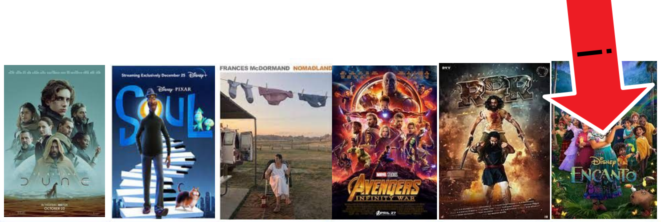
2			4	5	
5		4			1
		5		2	
	1		5		4
		4			2
4	5		1		

1. Compute avg(j) for all items
2. Compute sim(i,j) for all items for which there is some intersection with i
3. Compute score(u,i) for all users who have not seen i yet

$$\mathrm{sim}(i,j) = \frac{\sum_{u \in I_{i,j}} (u_i - \hat{i}) \cdot (u_j - \hat{j})}{\sqrt{\sum_{u \in I_{i,j}} (u_i - \hat{i})^2 \cdot \sum_{u \in I_{i,j}} (u_j - \hat{j})^2}}$$

$$\mathrm{score}(u,i) = \hat{i} + \frac{\sum_{j: u_j \neq \mathrm{NULL}} \mathrm{sim}(i,j) \cdot (u_j - \hat{j})}{\sum_{j: I_{i,j} \neq \emptyset} |\mathrm{sim}(i,j)|}$$

Exercise from ML for Recommender Systems tutorial by Alex Karatzoglou, 2015.



	2			4	5	2.94
	5		4			1
			5		2	2.48
		1		5		4
			4			2
	4	5		1		1.12
avg(j)	3.66	3	4.33	3.33	3.5	2.33
sim(i,j)	-1	-1	0.86	1	NULL	

$$2.33 + \frac{-1 \cdot (2 - 3.66) + 1 \cdot (4 - 3.33)}{|-1| + |-1| + |0.86| + |1|} = 2.94$$

$$2.33 + \frac{0.86 \cdot (5 - 4.33)}{|-1| + |-1| + |0.86| + |1|} = 2.48$$

$$2.33 + \frac{-1 \cdot (4 - 3.66) - 1 \cdot (5 - 3) + 1 \cdot (1 - 3.33)}{|-1| + |-1| + |0.86| + |1|} = 1.12$$

$$\mathrm{sim}(i,j) = \frac{\sum_{u \in I_{i,j}} (u_i - \hat{i}) \cdot (u_j - \hat{j})}{\sqrt{\sum_{u \in I_{i,j}} (u_i - \hat{i})^2 \cdot \sum_{u \in I_{i,j}} (u_j - \hat{j})^2}}$$

$$\mathrm{score}(u,i) = \hat{i} + \frac{\sum_{j : u_j \neq \mathrm{NULL}} \mathrm{sim}(i,j) \cdot (u_j - \hat{j})}{\sum_{j : I_{i,j} \neq \emptyset} |\mathrm{sim}(i,j)|}$$

Exercise from ML for Recommender Systems tutorial by Alex Karatzoglou, 2015.

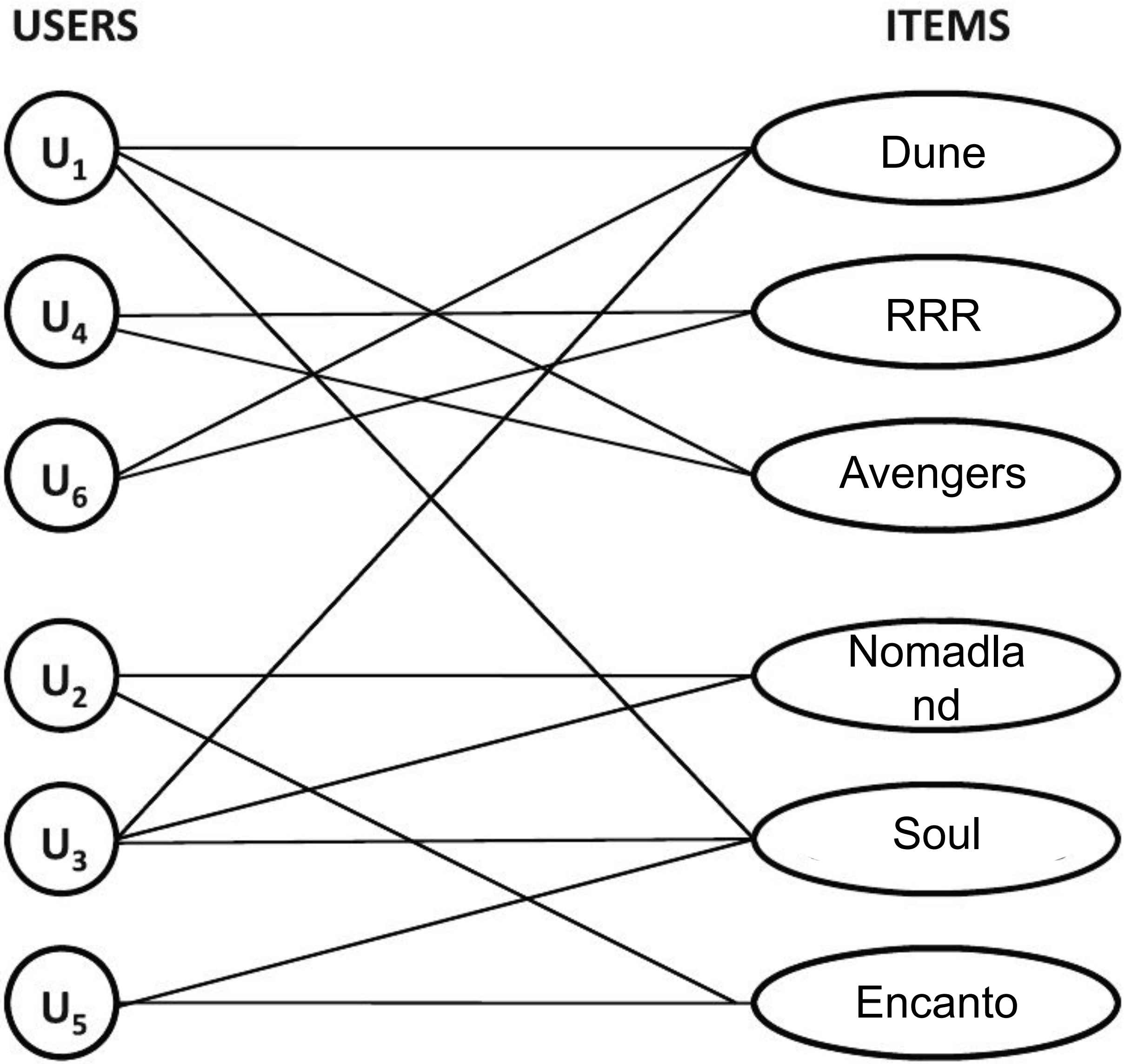
Note

- There are many ways of computing user-based similarity and item-based similarity
- There are many ways of using these to generate recommendations
- The method we have described is aware of the **bias of users**, in the sense of some users being more positive/negative than others in general

Graph- and clustering-based methods

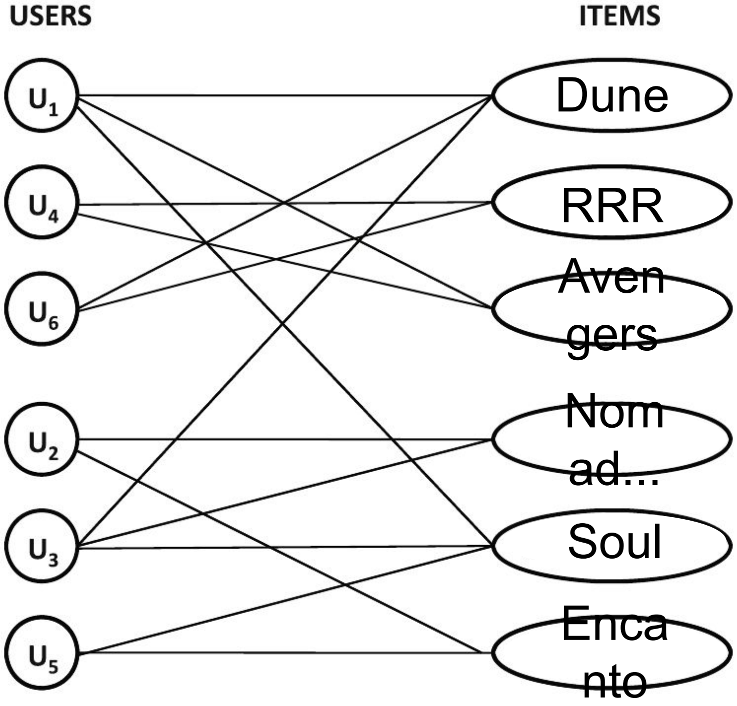
Graph-based methods

- Bipartite user-item graph with nodes N_u U N_i
- N_u users
- N_u items
- Non-zero utility ⇒ edge



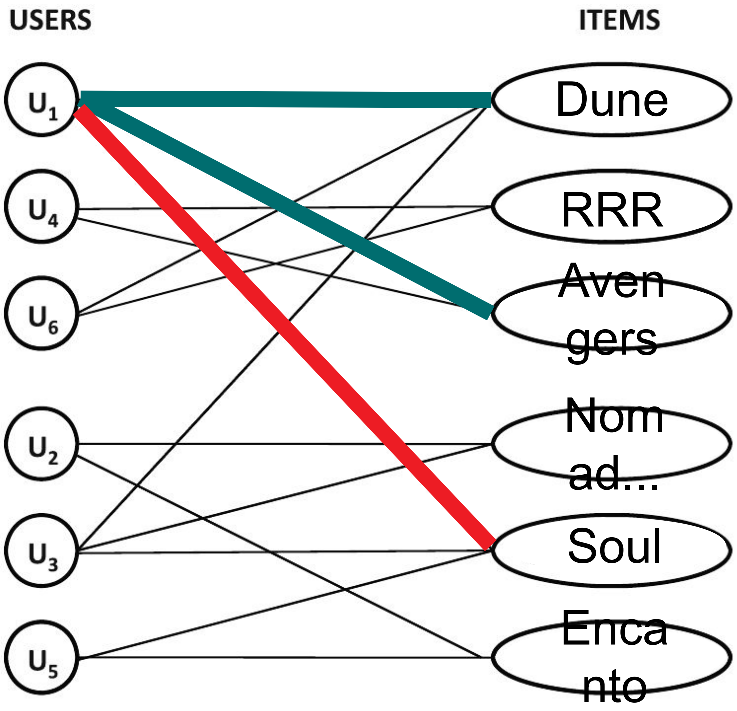
Graph-based methods (cont.)

- Use graph-based methods
  - **Random walk** with restart to a user or item
  - ***SimRank*** (not seen in class)
- Low “random jump” probability might favor popular items



Graph-based methods (cont.)

- Signed networks can be used
  - Remember to interpret ratings with respect to user and item averages
  - Below average rating ⇒ -
  - Above average rating ⇒ +
- Positive link prediction problem



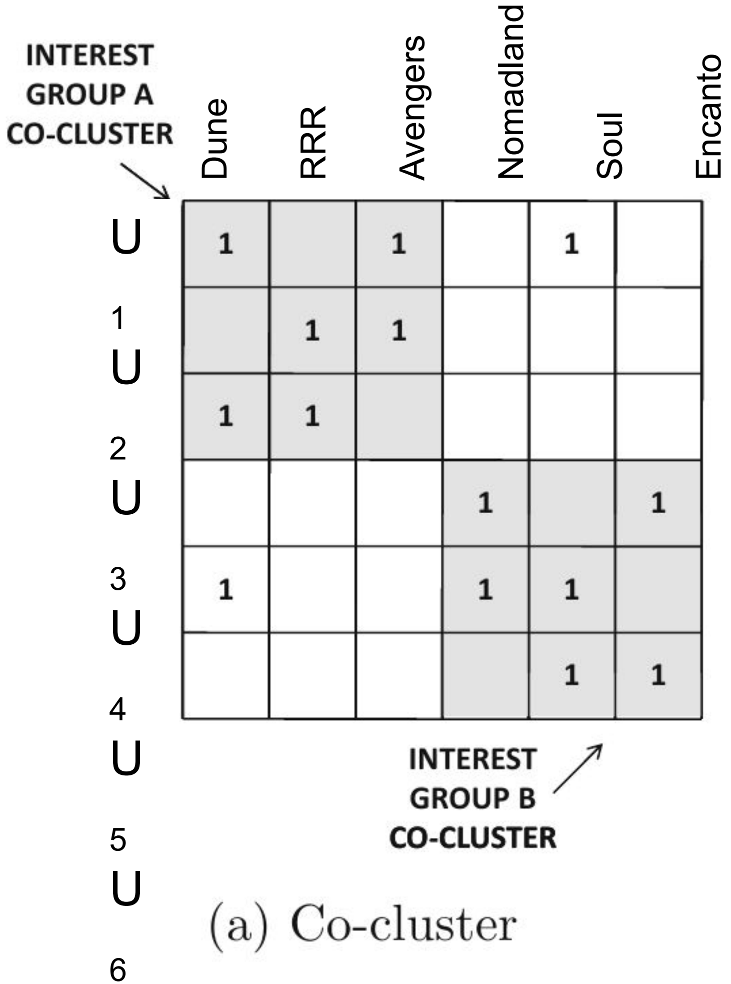
Clustering methods

- Motivations
  - Reduce computational cost
  - To some extent address data sparsity
- Results of clustering
  - Clusters of users for user-user similarity recs.
  - Clusters of items for item-item similarity recs.

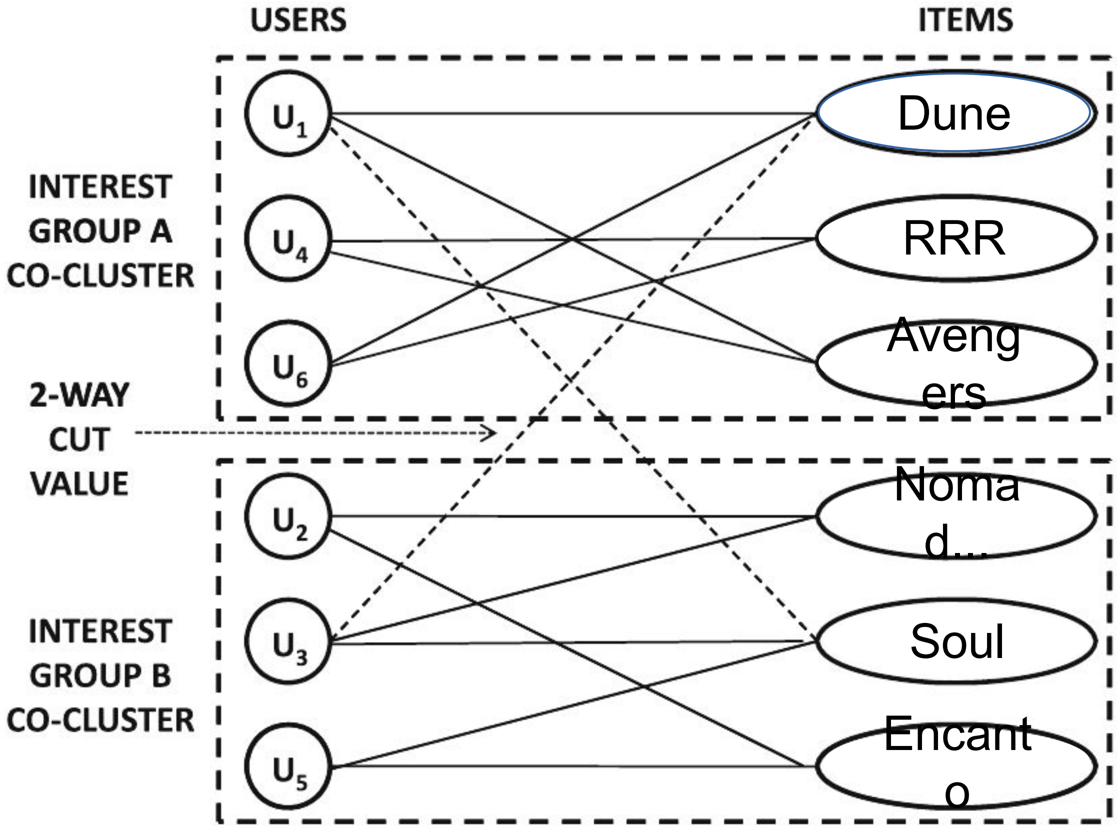
Clustering methods (cont.)

- User-user recommendation approach
  - Cluster users into groups
  - For any user u, compute average normalized rating for each item i the user has not seen
  - Report these ratings for (u,i)
- Same with item-item recommendations
- Neighborhoods will be smaller

Co-Clustering Approach



(a) Co-cluster



(b) User-item graph

Summary

Things to remember

- Interaction-based recommendations
  - User-based
  - Item-based
- Graph-based / clustering-based recommendations

Exercises for TT16-TT18

- Mining of Massive Datasets 2nd edition (2014) by Leskovec et al. Note that some exercises cover advanced concepts:
  - Exercises 9.2.8
  - Exercises 9.3.4
  - Exercises 9.4.6