Your Name
MATH 211
Sunday 5th March, 2017

Pset #1

Problem 1. Show that $\sqrt{2}$ is irrational.

Solution. Assume that $\sqrt{2}$ is even. It can therefore be expressed as a rational fraction of the form $\frac{a}{b}$, where a and b are relatively prime integers. It follows that $\frac{a^2}{b^2} = 2$, and therefore that $a^2 = 2b^2$, so a^2 and a are even. Now let a be expressed as $2c$. This shows that $b^2 = 2c^2$, and thus that both b^2 and b are even. However, two relatively prime integers cannot both be even, as they share a common factor of 2. Therefore, $\sqrt{2}$ must be irrational. $\square$

Problem 2 (Extra Note Goes Here)**.** Show that there are infinitely many primes.

Solution. Let $\mathbb{U}$ be the finite set of all primes $\{p_1, p_2, p_3, \ldots p_n\}$. Let

$$N = \prod \mathbb{U} + 1 = p_1 p_2 p_3 \cdots p_n + 1.$$

N is not divisible by any element of $\mathbb{U}$ since it will necessarily have a remainder of 1. Therefore, N must be divisible by some prime $p \notin \mathbb{U}$. Thus, for every finite set of prime numbers $\mathbb{U}$, there will always exist some prime p which is not an element of $\mathbb{U}$, and therefore there are infinitely many primes. $\square$