Exercises from Chapter #1

1 Equivalence exercises

1. **(b)** $\lambda xy.xz \equiv \lambda mn.mz$
2. **(c)** $\lambda xy.xxy \equiv \lambda a.(\lambda b.aab)$
3. **(b)** $\lambda xyz.zx \equiv \lambda tos.st$

2 Combinators

1. **Yes,** $\lambda x.xxx$ is a combinator.
2. **No,** $\lambda xy.zx$ is not a combinator since z is a free variable.
3. **Yes,** $\lambda xyz.xy(zx)$ is a combinator.
4. **Yes,** $\lambda xyz.xy(zxy)$ is a combinator.
5. **No,** $\lambda xy.xy(zxy)$ is not a combinator since z is a free variable.

3 Normal form or diverge?

1. **Normal form,** $\lambda x.xxx$ is already fully reduced.
2. **Diverge,** $(\lambda z.zz)(\lambda y.yy)$ diverges.
3. **Normal form,** $(\lambda x.xxx)z$ reduces to zzz.

4 Beta reduce

1. $(\lambda abc.cba)zz(\lambda wv.w) \equiv z$
2. $(\lambda xy.xyy)(\lambda a.a)b \equiv bb$
3. $(\lambda y.y)(\lambda x.xx)(\lambda z.zq) \equiv qq$
4. $(\lambda z.z)(\lambda z.zz)(\lambda z.zy) \equiv yy$
5. $(\lambda xy.xyy)(\lambda y.y)y \equiv yy$
6. $(\lambda a.aa)(\lambda b.ba)c \equiv aac$
7. $(\lambda xyz.xz(yz))(\lambda x.z)(\lambda x.a) \equiv \lambda\beta.za$