CS031 Fall 2023

Lab 09: Regression

Correlation

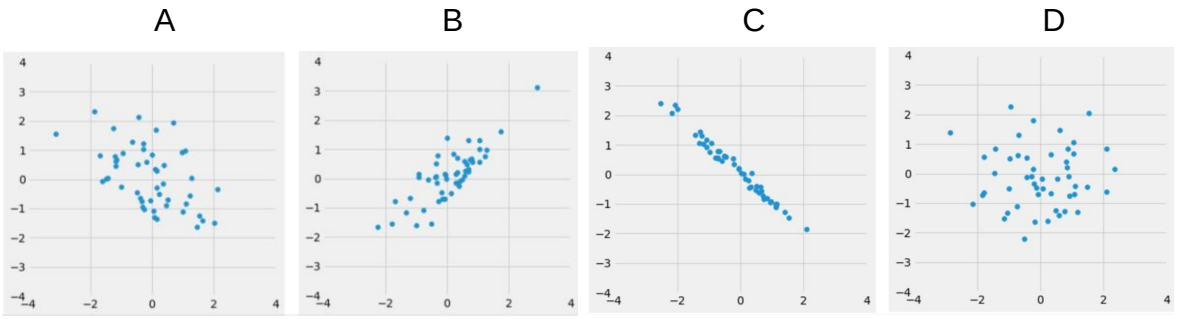
An important aspect of data science is using data to make *predictions* about the future based on the information that we currently have. A question one might ask would be "Given the US GDP of every year of the previous decade, how can we predict the US GDP for next year?" In order to answer this question, we will investigate a method of using one variable to predict another by looking at the *correlation* between two variables.

Question 1. Function: Let's start by writing a function called `correlation_coefficient` that takes in two arrays `x` and `y` of the same length and returns the correlation coefficient between the two.

Hint: Assume you have a function called `convert_su` defined, that converts an array to standard units (we did this last week).

```
def correlation_coefficient(x, y):
    x_su = convert_su(x)
    y_su = convert_su(y)
    return np.mean(x_su*y_su)
```

Question 2. Comparing Correlation: Look at the following four datasets. Rank them from weakest correlation to strongest correlation. Remember that a *strong* correlation has |r| close to 1.



correlation ranking:

D > A > B > C

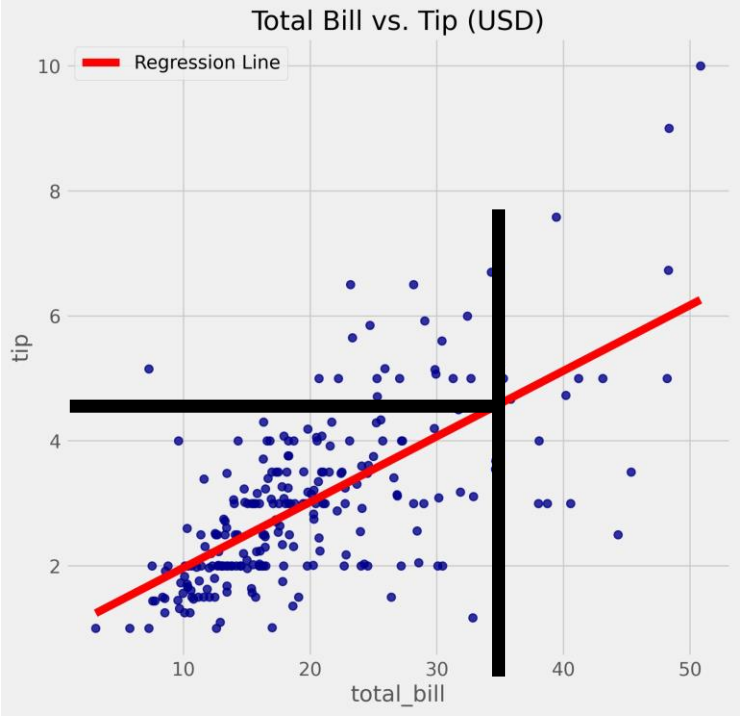
We have introduced correlation as a way of quantifying the *strength* and *direction* of a linear relationship between two variables. However, the correlation coefficient can do more than just tell us about how clustered the points in a scatter plot are about a straight line. It can also help us define the straight line about which the points (in original units) are clustered, also known as the *regression line*.

The formulae for the *slope* and *intercept* for the regression line are shown below. In fact, by a remarkable fact in mathematics, the line uniquely defined by the slope and intercept below is *always* the best possible straight line we could use.

$$\textbf{slope of the regression line} = r \cdot \frac{\text{SD of } y}{\text{SD of } x}$$

$$\textbf{intercept of the regression line} = \text{average of } y - \text{slope} \cdot \text{average of } x$$

Question 3. Restaurants: Suppose you are given the scatter diagram shown below that shows the relationship between the total bill versus tip at American restaurants. You have calculated the line of best fit (shown in red). Suppose your friend Alice goes out to dinner and tells you her total bill was $35. Based on the regression line, what would we predict Alice's tip to be?



We would expect her tip to be about $4.50, as it is somewhat less than $5 but greater than $4.

Question 4. Meggy's Coffee: We want to investigate the correlation between the daily ounces of coffee consumed by an individual and the number of hours the individual stayed awake. It is our intention to use the ounces of coffee consumed to predict the number of hours the individual stayed awake. The data from our sample of 500 people has the following characteristics:

- The number of ounces of coffee consumed has a mean of 12 ounces and SD of 4
- The number of hours stayed awake has a mean of 16 and an SD of 2
- The correlation between the number of ounces of coffee consumed and number of hours spent awake is 0.5.
- Suppose the scatter plot is roughly linear.

a) What is the slope of the regression line?

0.5*2/4 = 1/4 = 0.25 hours/ounce

b) What is the intercept of the regression line?

16 - 0.25*12 = 13 hours

c) Suppose your friend Matthew is in this population (and not in the sample). He told you that he consumed 16 ounces of coffee that morning. Use your line of best fit to predict how many hours Matthew will stay awake today.

13 + 0.25*16 = 17 hours

d) Your other friend, Meghan, is also in the population and not in the sample. She confesses that she drank 80 ounces of coffee that day (wow!). Based on the information above, would the regression line we computed in parts (a) and (b) be appropriate to predict the number of hours Meghan stayed awake? Explain.

The regression line we computed in parts a and b would not be appropriate to predict the number of hours Meghan stayed awake because the prediction would yield 13 + 0.25*80 = 33 hours, which is unrealistic because in most cases the human body does not do well after 24 hours, and the variability in hours stayed awake past that point is not accounted for by the data. Moreover, since Meghan is over 5 standard distributions above the mean, this data is not reflective of large amounts of coffee consumption to that degree (and the relationship between hours awake and coffee consumption is not necessarily linear past a certain point), so this regression line would not be a good predictor of how long Meghan will stay awake.