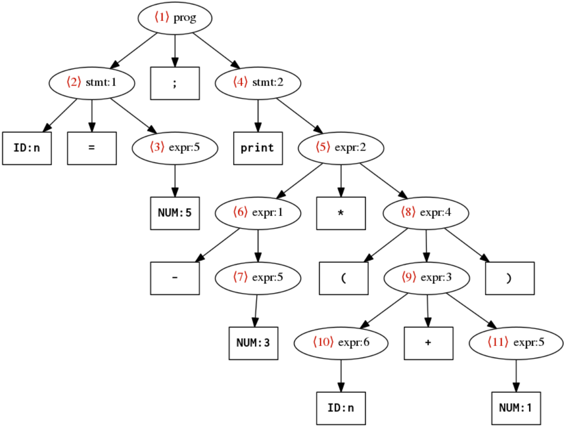
Grammars and CFG



Textbook: 2.1

Grammar example

Def: Reverse Polish Notation. The following are **production rules**. If a statement can be derived using these rules, it is a member of RPN.

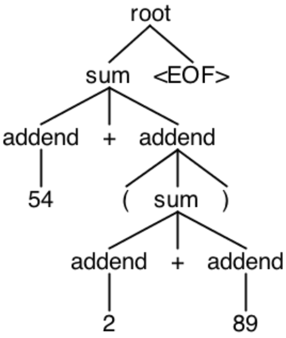
1. A valid RPN string is an `EXPRESSION`
2. An `EXPRESSION` is a number literal or `FORMULA`
3. A `FORMULA` is of the form `a b o`, where `a, b` are `EXPRESSION`s and `o` is an `OPERATOR`
4. An `OPERATOR` is `+`, `-`, `*`, or `/`

- So `1` and `2` are valid `EXPRESSION`s, since they are number literals
- `+` is a valid `OPERATOR`
- Therefore `1 2 +` is a valid `EXPRESSION`
- Then we could derive `3 1 2 + *` or `4 3 1 2 + * /` or `4 3 1 2 + * / 1 5 + -` or infinitely many others

A set of production rules is called a grammar.

Using grammars

- Grammars are commonly seen in parsing
- `bison` is a common tool
- Builds a "parse tree" for a given language that is then transformed and compiled



Formal definition of a context-free grammar

Def: Context-free grammar.

A **context-free grammar** is a 4-tuple (V, Σ, R, S) where:

1. V is a finite set called the **variables** (these will be replaced by rules before we are done)
2. Σ is a finite set $V \cap \Sigma = \emptyset$ called the **terminals** (these cannot be replaced by rules in a CFG)
3. R is a finite set of **rules**, each rule being **of the form** $A \to \alpha$ for some $A \in V, \alpha \in (V \cup \Sigma)^*$
4. $S \in V$ is the start variable

Important: The difference between CFG and other grammars is in the restriction of rules to the form $A \to \alpha$. Other grammar classes don't have this restriction and are more powerful! This distinction is not made clear in the textbook.

Grammar languages

Def: Derivation. If $u, v, w \in (V \cup \Sigma)^*$ and $(A \to w) \in R$, we say that uAv **yields** uwv, written $uAv \Rightarrow uwv$. Say that u **derives** v, written $u \stackrel{*}{\Rightarrow} v$ iff:

1. $u = v$ **or**
2. A sequence $u_1, u_2, \cdots, u_k$ exists for $k \geq 0$ and

$$u \Rightarrow u_1 \Rightarrow u_2 \Rightarrow \cdots \Rightarrow u_k \Rightarrow v$$

Def: Language of a grammar. The language of a grammar G is the set of all strings $\in \Sigma^*$ which can be derived by G.

$$\mathcal{L}(G) = \{w \in \Sigma^* : S \stackrel{*}{\Rightarrow} w\}$$

EBNF (via UCI)

- Just as regular expressions describe regular languages, Extended Backus-Naur Form (EBNF) describes context-free languages
- Also like regex, there is a formal and informal definition
- We will look at the informal definition as provided by GNU `bison`

```
lhs: some_name;
some_name: first_option | second_option | third_option ;
recursive: "hi there" | "hi there" recursive ;
```

EBNF RPN

- Using `bison` EBNF, our RPN grammar becomes:

```
// Entry rule
EXPRESSION: NUMBER_LITERAL | FORMULA ;

FORMULA: EXPRESSION EXPRESSION OPERATOR ;
OPERATOR: '+' | '-' | '*' | '/' ;
NUMBER_LITERAL: DIGITS | DIGITS '.' DIGITS ;
DIGIT: '0' | '1' | '2' | '3' | '4' |
       '5' | '6' | '7' | '8' | '9' ;
DIGITS: DIGIT | DIGIT DIGITS ;
```

Ambiguity

Def: Leftmost derivation. A derivation of a string via a grammar is **leftmost** if, on each step, the leftmost remaining variable is the one replaced.

Def: Ambiguity. If a string can be derived in two separate ways, we say that it is **derived ambiguously**. A string w is derived ambiguously on grammar G if it has two or more different leftmost derivations. Any grammar with one or more ambiguous strings is said to be ambiguous.

Next up: PDA, non-context-free languages, the Chomsky hierarchy