Post's Correspondence Problem (PCP)

Textbook: Chapter 5.2

Intro

- Imagine we have some set of dominos, each with a top and bottom section
- We want to arrange them (repetition allowed) so that the top and bottom match when read left to right

$$\left\{ \left[\frac{ab}{a}\right], \left[\frac{c}{bc}\right] \right\}$$

$$\left[\frac{ab}{a}\right]\left[\frac{c}{bc}\right] \rightarrow \frac{abc}{abc}$$

- Some sets of dominos have possible matches, and some don't

Def. Post's Correspondence Problem. Does a given set of dominos have a possible match?

As a language:

$$PCP = \{w : w \texttt{ encodes a set of dominos which match }\}$$

Undecidability

Thm. *PCP* is undecidable.

Pf. By reduction from the decision problem A_{TM}. We will assume *PCP* is decidable and show that this implies A_{TM} is decidable (a contradiction).

- We can construct an instance of *PCP* which simulates a TM
  - Can design it so it matches iff the TM accepts
- Therefore a *PCP* decider would decide A_{TM}

Next up: Nondeterministic Turing Machines