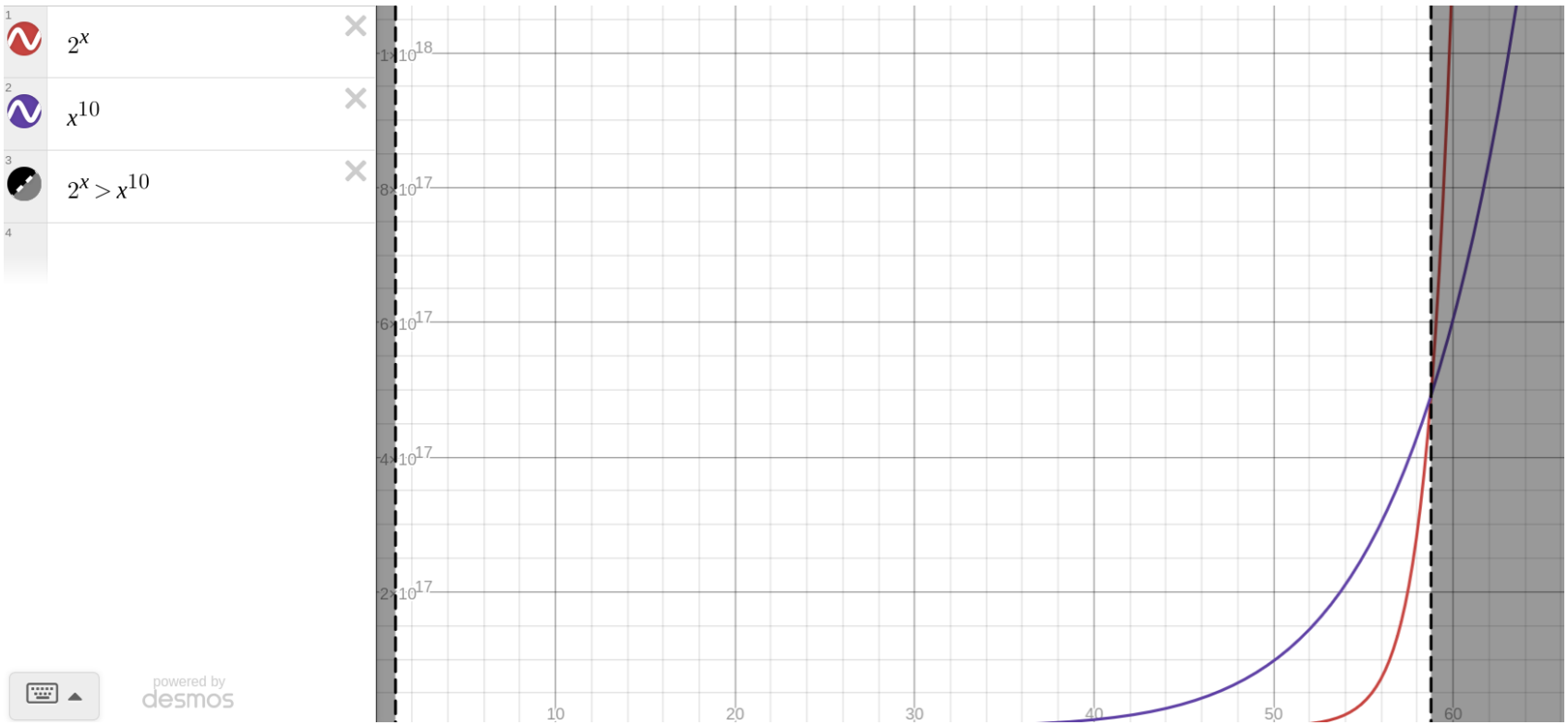
Intractability

Textbook: Chapter 9

Tractability vs. Intractability

Def: A language A is **tractable** ("realistically" solvable) if $A \in P$. It is **intractable** (theoretically solvable, but not in "realistic" time) if $A \notin P$.

- In reality, there are plenty of language $\in P$ that are infeasible to decide IRL: $O(n^{999999})$ is technically "tractable"



- Exponential cost will eventually overtake **any** polynomial

Space Constructability

- Recall: A function runs with deterministic **space** complexity $O(f(n))$ if it uses space $f(n)$ on input length n
- **Def:** A function f is **space-constructable** if a TM can map any string of length x to $f(x)$ in space $O(f(x))$
- Space-constructable function examples
  - $\lg(n)$
  - $n \lg(n)$
  - n^2

Space Hierarchy Theorems

- Clearly, $SPACE(n) \subseteq SPACE(n^2)$ since n is $O(n^2)$
- How can we prove that two space complexity classes are different, e.g. $SPACE(n) \subsetneq SPACE(n^2)$?

Thm: Space hierarchy theorem. For any space-constructable f, a language A exists that is decidable in $O(f(n))$ space but **not** in $o(f(n))$ space.

- A is decidable with space bounded by $f(n)$, but cannot be decided with space insignificant to $f(n)$

Corollary: For any nonnegative integers $\epsilon_1, epsilon_2$ where $0 \leq \epsilon_1 < \epsilon_2$,

$$SPACE(n^{\epsilon_1}) \subsetneq SPACE(n^{\epsilon_2})$$

- Finally, a proper hierarchy! This is very useful!

Space Hierarchy Proof

Time Constructability

- Time complexities give us a lot more trouble than space ones
- If we could prove that $P \subsetneq NP$ like we can prove $PSPACE = NPSPACE$, we could settle the P/NP debate
- Thus, we try to do the same with time

Def: A function t where $t(n)$ is at least $O(n \log n)$ is called **time constructible** if a TM can map input of length n to $t(n)$ in deterministic time $O(t(n))$.

- Examples: $n \log n$, $n\sqrt{n}$, n^2, 2^n

Time Hierarchy Theorems

Thm: Time hierarchy theorem. For any time constructible function t, a language A exists that is decidable in $O(t(n))$ time but not in time

$$o\left(\frac{t(n)}{\log t(n)}\right)$$

- Subtly different, and therefore less strong!

Corollary: For functions t_1, t_2 where t_2 is time constructable and t_1 is $o\left(\frac{t_2(n)}{\log t_2(n)}\right)$,

$$TIME(t_1(n)) \subsetneq TIME(t_2(n))$$

Time Hierarchy Proof

EXPSPACE-Completeness

- Recall: *EXPSPACE* is the set of all problems solvable given exponential space
- $EXPSPACE = \bigcup_k SPACE(a^{n^k})$

Relativization

Circuit Complexity

Next up: Advanced Complexity Analysis