HRR

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When thinking about HRRs, this vector $\vec{U}$ contains Fourier coefficients. The IDFT is then

$$\vec{u} = \mathrm{IDFT}(\vec{U})$$

$$\Rightarrow \hat{u}_n = \frac{1}{N} \sum_{k=0}^{N-1} \underbrace{U_k}_{e^{iu_k}} e^{\frac{2\pi i n k}{N}}, \qquad n = 0, \ldots, N-1.$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} e^{iu_k} e^{\frac{2\pi i n k}{N}}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} e^{i\left(\frac{2\pi n k}{N} + u_k\right)}$$

I will refer to the domain of $\hat{u}$ as the "pseudo-spatial domain", where n is the pseudo-spatial variable. This is the representation most often used in <u>Nengo</u>.

> If we like, we can insist that $\vec{u}$ be chosen so that $u_k = \bar{u}_{N-k}$, $k = 0, \ldots, N-1$ (conjugate-symmetry). Then $\hat{u} = \mathrm{IDFT}(\vec{U})$ will be real-valued,
>
> $\hat{u} \in \mathbb{R}^N$.

We can encode x by raising $\vec{U}$ to the exponent x. This gives us

$$\vec{U}^x = \left(e^{i\vec{u}}\right)^x = e^{i\vec{u}x}$$

$$= \left[e^{iu_0 x}, e^{iu_1 x}, \ldots, e^{iu_{N-1} x}\right]^T$$

The notation $\phi(x)$ is often used to represent the pseudo-spatial version

$$\phi(x) = \mathrm{IDFT}(\vec{U}^x)$$

For this, it is assumed that $\vec{U}$ represents half of the Fourier coefs, and the IDFT is done on a conjugate-symmetric expansion, to yield real-valued $\phi(x)$.