	Incident Wave @ Obstac	les
	Not centered a The origin	
T / Yin	in Global coordinates)	- V
	Recall: We define the incident wave uin	nc Cuhich is
P	a vector displacement) in terms of two potentials the compressional and shear potentials, respective	Is Dinc and Yin,
-	the compressional and shear potentials respective	ly, as fellows:
	Tuinc = Vr. & Pinc + Vr. 0,2 × Yinc ?	7 2
	For an specific problem, we define these of $(r, \theta) = e^{ik_{\theta} \times \cdot \frac{\partial}{\partial r}} = e^{ik_{\theta}(r \cos \theta, r \sin \theta)}$. For an specific problem, we define these of the problem of th	potentials as
	$\oint_{inc} (r, \theta) = e^{ik_{\theta} \times \cdot d} = e^{ik_{\theta} (r \cos \theta, r \sin \theta)}$	(cos 8, sin 8)
9	$\Rightarrow \varphi_{n}(r, \sigma) = e^{ik\varphi r\cos(\theta - \delta)}$	Y A C
	where 8 is the "angle of incidence" of	Pinc
	in the global-polar-coordinate system; // pa	Global Axis
	and (r,0) we global polar coordinates.	2/
	The some of the southern	10
	Su of the	3
.	(2) Yinc (1,0) = 0	J
9	- AJ	
9	Thus, in global polar coordinates, we can write [Uinc(r, 0) = [Uinc)r(r, 0)] = Proprinc = [opinion of the coordinates]	Mine as
	(Uinc(r, 0) = [Uinc)r(r, 0)] = Tropinc = opinc (n, 0)	$ = (\varphi(s, s, s, s) \varphi(s, s, s) $
	[Line) a (r, +)] I district (r)	$-ik_{\theta}\sin(\theta-\delta)\phiin(r,\theta)$
	The state of the s	(compressions)
	(Here, the notation (unc) , means "the displacement	in the radial direction",
	and (unc) a means "the displacement in the angular	direction".)
T		
11. (Wh	at about local coordinates?	1 Part
6.00		
talvens.	In our multiple-scattering problem not all ob	istacles
- who	are centered at the origin. So we introd	
	coordinate systems centered at the middle	of
	each obstacles we write these as Crm,	
)	for the local polar coordinates for obstacle	m.
	<u>'</u>	

where we call write (rm om) as functions of crops or vice
versa à far example, we call wite
$(r, \theta) := (r(r^m, \theta^m), \theta(r^m, \theta^m)).$
The same of the standard of th
Globel axis
In order to talk about
"hard" boundary conditions, we want to
decompose Uine into a component in
the Crm-radial direction & a component
in the One angular direction Cratha
than in the r & 0 -directions).
so we need to find a new way
to decompose uinc in this new coordinate
system; that is, we
Usinc = $\begin{bmatrix} (uinc)n \end{bmatrix}$ $\begin{bmatrix} (uinc)nm \end{bmatrix}$ $\begin{bmatrix} (uinc)am \end{bmatrix}$ $\begin{bmatrix} (uinc)am \end{bmatrix}$ $\begin{bmatrix} (uinc)am \end{bmatrix}$ $\begin{bmatrix} (uinc)am \end{bmatrix}$
to get this "mapping", exentrally
10 yet 4vis mapping, exemitally
*KEY IDEA= * We will decompose Uinc in the (rmom) -local
Tookinate system, and was find a mapping from (mgm) +> (r, 0)
that will allow a scitable Change - of-coordinates transformations
THE WIN SHIP THE STATE OF THE S
Steps
[Find a way to express (r, 0) as a function of (rmom)
· Let (x,y) = (r cos 0, rsin 0) be any point in global conditions
· Let (xc, yc) be the global cartesian coordinates of the center
of obstacle m
· Let (xmym) = (rmcosom, rmsinom) be the same point
as (x,y), but expressed as m-local coordinates.

> > C

(Notice): $(x,y) = (x^m, y^m) + (x_c, y_c)$ • (ROWERDO) = (xm + xc, ym + yc) -6 = (rmcosom + xc, rmsinom+yc) · Converting this back to global polar coordinates: $r(r^{m}, \theta^{m}) = \sqrt{x^{2} + y^{2}}$ = $\sqrt{(f^m(0)\delta^m + x_c)^2 + (f^m sin \delta^m + y_c)^2}$ = V (rm)2cos20m +2rmxccos0m +x2 +(rm)2sin20m+2rycsin6m+yc2 = $\sqrt{(r^m)^2(\cos^2\theta^m + \sin^2\theta^m)} + 2r^m(x_c\cos\theta^m + y_c\sin\theta^m) + (x_c^2+y_c^2)$ (r(m, 0m) = V(rm) = +2rm(xccos 0m +ycsin 0m) +(x2+y2) & O(rm, om) = atan2(y,x) (O(rm, om) = atan 2(rmsin om + yc, rmcosom + xc) where $(atan2(y,x)) 8 = (arctan(\frac{y}{x}))$ if x>0arctan () + IT if x < 0 and y > 0 arctan (y) - IT if x 40 and y 40 + \$\frac{1}{2} if x=0 and 470 --# if x=0 and y L D if x=0 and y=0undefined . is the function which gives a prease, angle for any $point(x,y) \neq (90).$ Now we have expressed (n, 0) as a function of Crm, om). (2) Decompose line in the m-local cocydinale systems 6 Uinc (r,0) = [luinc) pm (r,0) = Vrmom Dinc = [3 m Pincher, 0) in Den Ocaclino) (uinc) pm(r, b) The same was the server of

```
(3) Expand out total derivatives:
   (unc)m (n,0) = 3m Pinc (r(m,om), A(m,om))
   (Min) om (n, o) 10 pine (r(m, om), O(m, om))
                = \left(\frac{\partial \phi_{inc}}{\partial r}\right) \left(\frac{\partial r}{\partial r}\right) + \left(\frac{\partial \phi_{inc}}{\partial \theta}\right) \left(\frac{\partial \phi}{\partial r}\right)
                    Im [danc)(dr) + (danc)(de)]
  (4) Compre all required derivatives
  (Pincly 0) = euror (0518-8)
         + doine = iko cos(0-8)e ikoros(0-8) = iko cos (0-8) dinc(1,18)
       ) doinc = -ikpr sinla-d)e ikprosto-o) = -ikpr sin (0-d) dine (170)
  C(r^m \theta^m) = \sqrt{(r^m)^2 + 2r^m (x_c(0)\theta^m + y_c(0)\theta^m) + (x_c^2 + y_c^2)}
       -> dr = 2rm +2(xccosem +ycsinem)
                    2 V (m)2 + 2 m (x closom + 4 cs mom) + (x2+42)
                = rm + xccosom + ycsin om
                       r (mam)
     Lydr = 2rm (-xcsinom + yccosom)
                     7 V (rm)2+2rm(xc650m+yc5M0m) +(x2+y2)
               = rm(-xcsinom+yccosom)
                          r(mgm)
(O(r", bm) = atan2 (rmsin bm +yc, rmc056"+xc)
     arm (dx) (dx) + (datan 2(y,x)) (dy)
              = \left(\frac{-y}{x^2 + y^2}\right) (\cos \theta^m) + \left(\frac{x}{x^2 + y^2}\right) (\sin \theta^m)
    - Wole in Hore,
       x2+y2 = (rm sps om + xc)2 + (rmsin om +4c)e
                = (rm)2520m + 2rmxcos 0m +x2 + (rm)25120m +2rmycsinom +y2
               = (r^n)^2 + 2r^m(x_c cos \theta^m + y_c sin \theta^m) + (x_c^2 + y_c^2)
                = [r(rm, 6m)]2
```

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= \frac{\partial \theta}{\partial r^{m}} = \left( -r^{m} \sin \theta^{m} - y c \right) (\cos \theta^{n}) \left( r^{m} \cos \theta^{m} + x c \right) (\sin \theta^{m}) 
= \left( \frac{\partial \theta}{\partial r^{m}} - \frac{\partial \theta}{\partial r^{m}} \right) \left( r^{m} \cos \theta^{m} + x c \right) (\sin \theta^{m}) 
= \left( \frac{\partial \theta}{\partial r^{m}} - \frac{\partial \theta}{\partial r^{m}} \right) \left( r^{m} \cos \theta^{m} + x c \right) (\sin \theta^{m}) 
-8
-8
                                                 = - msinom cosom-yc cosom + mshow us 6 m + x csinom
_
[r[m,om]]
= 12 (xcsin om - yccos om)
3
                                         \frac{\partial \theta}{\partial \theta^{m}} = \left(\frac{\partial a \tan 2(y \times)}{\partial x}\right) \left(\frac{\partial x}{\partial \theta^{m}}\right) + \left(\frac{\partial a \tan 2(y \times)}{\partial y}\right) \left(\frac{\partial y}{\partial \theta^{m}}\right)
= \left(\frac{-y}{x^{2} + y^{2}}\right) \left(-r^{m} \sin \theta^{m}\right) + \left(\frac{x}{x^{2} + y^{2}}\right) \left(r^{m} \cos \theta^{m}\right)
4
-
                                                   = \frac{1}{r^2} \left[ m \left( -r^m \sin \theta^m - yc \right) \left( -r^m \sin \theta^n \right) + \left( r^m (\cos \theta^m + xc) (r^m \cos \theta^m) \right) \right]
-
                                                  = 1 (m)2sin20m + rmycsin6m + (rm)2cos26m + rmxccoom)
-40
                                                  = rm + (xccoom +ycsnom)
1
-40
-
                                     (5) Substitute & Simplify
                                    => (uinc)rm(r,6) = { (ikpcos(0-2) Quc)[-(rm+xccos0m+ycsn6m)]
                                                                 + (-iKpfsin (+-8) pinc)[ + (xcsinom-yccosom)]}
            recal):
                                                  = + ikp Buch, +) [cos(+->) (rm +xccosom +ycsin 6m)
1
                                                                                       - Sin (0-8) (xesin 0m - yecosom)
         · (05/6) (05/6) - sh(a) sh(b) <
                = cos(a+b)
                                                = - ikp dinc(n) [rmcos(0-0) +xccos(0-0)cos on +yc cos(0-0)sin on
         · (05(9) sin(b) + sin(a)(01(b)
                                                                               - + yesin(0-8) smom + yesin(0-8) carom
          ( = Sin(4+b)
                                    [luix)m= -iKp Bine (0,6) [mcos(0-8) +xccos(0m+0-8) +ycsin(0m+0-8)]
-3
-
                               · (line) pmlr, (1) = = (ikp cos (0-0) acre) [ + (r/ (-xcgin 0m + yclosom))]
4
4
                                                   + (-ikp of sin (0-8) pinc) [ for (rm + (xccos6m + ycsin6m))] }
4
                                           = + ikp ancho) [cos(0-8) (-xcsinom + xccosom)
12
                                                                           - sin(0-i) (rm + xcusom +ycsinom)]
              = - iKp pinc(n,0)[-xc(os(0+) sinom +yccos(0-8) cosom
                                                                            -rmsin(0-8) -xcsin(0-8) coop -ycsin(0-8) sin(0-8)
                        => (Unitation - Like place(170) [-xcsin(0m+0-0) +yccos(com+0-0) +msin(0-0)]
```

		•
	CONCLUSION	2
		2
	$V \subset (M, PRESENT TO ADMINISTRATION U.S. (WYOND) IV$	9
	(rm, pm) - local coordinates as	F
•	Man	
[H . =	(11 :ac) -m (ac) = 1 1 16 dia (ma) ("cos(0-8) +xcos 10 70 0) +gc smile +0-1)	
-ne	(((((((((((((((((((Œ.
		4
	The state of the s	6
	1 CI / CI / CI I'VE WAY (CONTINUE OF THE WORLD OF CARCATEL)	e e
		6
	· Quacing & englicated in global coordinates, and	-
	and · (xc, yc) is the global contenion coordinate of the content obstacle in.)	
	Alabra TC M Act ala > L. H. Harto Has	
	[[(as (a-b) + (a) + (a)]	
y 1		
		6
	1-00 Fore 1112 1) =	
	which exactly natches the original formula given in global	
	polar coordinates.	
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	the property of the contract o	

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