Efficient Transport Network Design with Quantum Annealing

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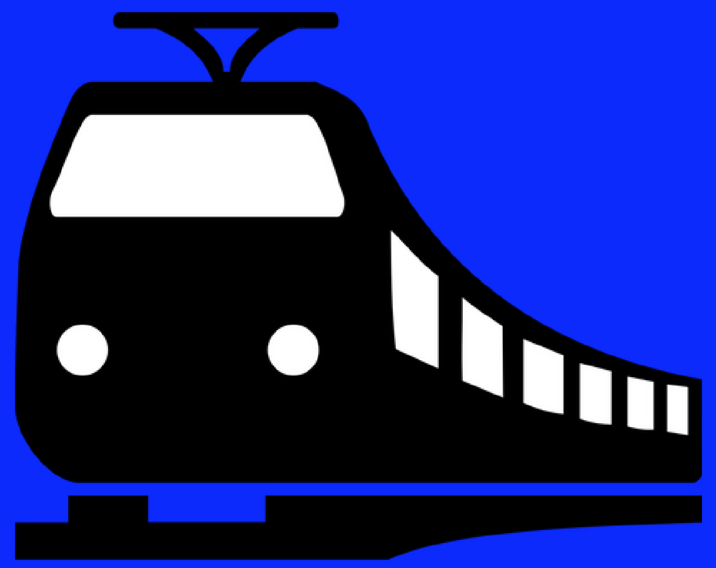
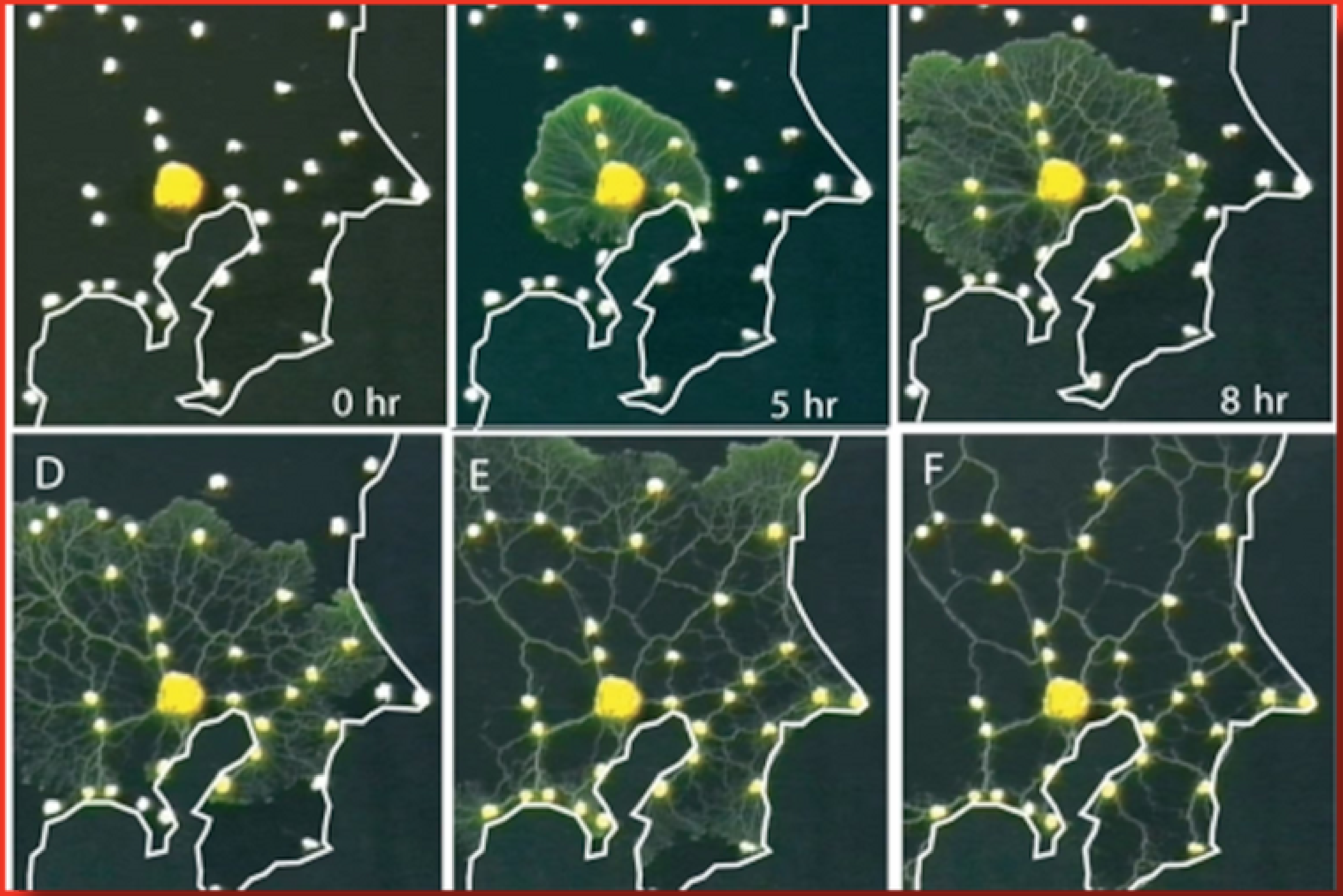
Transport networks such as metro systems, highways, and power transmission are hardly reversible once constructed. Hence, efficient planning of such networks is crucial and can ensure a less costly & sustainable transport system.



Nature is an optimizer?

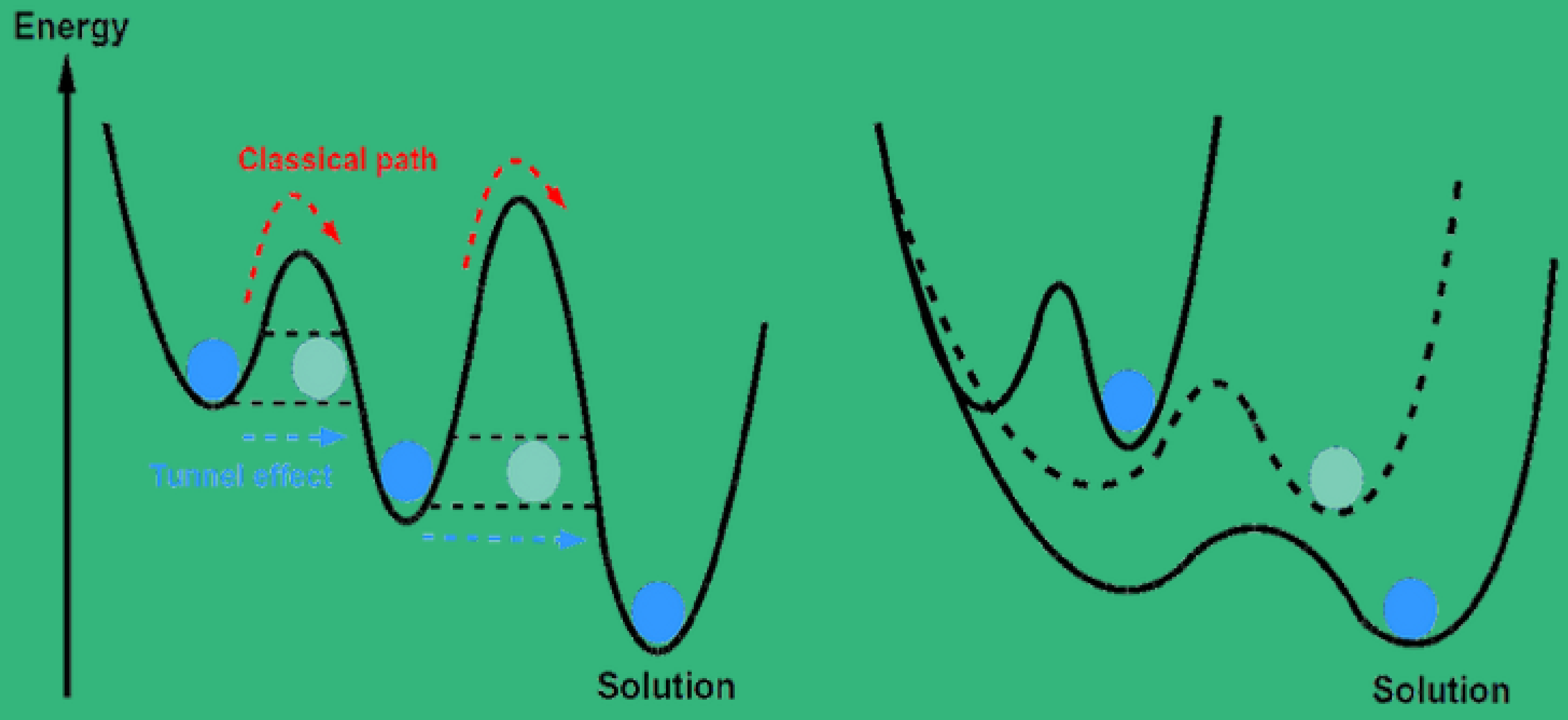


Evolutionary algorithm of biology

A research team from Japan and the UK simulated the Tokyo rail system using a slime mold and some nutrients placed according to the positions of the train stations. The slime mold evolved within a few hours to form a network for transporting nutrients and the pattern was surprisingly similar to that of the intricately designed railway network.

Can we do this with the evolution of quantum states?

Quantum Annealing



Quantum Tunnelling

Adiabatic evolution

$$H_c = \sum_i h_i s_i^z + \sum_i \sum_{j>i} J_{ij} s_i^z s_j^z$$

$$H_I = \sum_i h_i' s_i^x$$

$$H = (1-s)H_I + sH_c$$

Quantum annealing is a quantum computing approach that encodes the cost function as the Hamiltonian of an Ising model, and then evolves the initial quantum state adiabatically to the ground state of this Hamiltonian.

The evolution of a quantum system can optimize real-world problems!

Problem to be solved:

Given a region with some potential stations, how can we design an efficient transport network considering the information of distance and the traffic volume of each station?

Our goal:

Design a transport network with low overall distance, low average travel time, and efficient arrangement of lines

Tools:

D-Wave Leap's CQM Hybrid Solver

Scalability : >5000 qubits

Noise is not so problematic

Can easily impose quadratic constraints

Procedure

Divide the network into smaller subnetworks by minimizing the sum of the overall distance of each subnetwork

Extract a set of connected networks from each subnetwork while minimizing the overall distance

Compute the average distance between any two stations of a network and select the best one of the set

Assign several lines to the transport network based on the transfer frequency or lengths of the lines

2

Minimize the overall distance while preserving the connectivity of the network

Solve for the adjacency matrix of a connected graph

The entries of the adjacency matrix are represented by binary variables Xij

The overall distance is encoded as the cost Hamiltonian.

Necessary but not sufficient conditions are encoded in quadratic forms

Objective: $H_c = \sum_{j>i} X_{ij} W_{ij}$

Constraints:

1.Each station has to be connected at least once:

$$\sum_{j} X_{ij} \geq 1$$

2. For any pair of connected stations, at least one of them has to connect with another station:

$$X_{ij} - \sum_{\substack{s \neq j \\ k \neq i}} (X_{is} + X_{jk}) \leq 0$$

2

To effectively reduce traffic congestions and distribute resources wisely, stations with higher passenger load should have more connections, and vice versa

Eventually, we need to check if the adjacency matrix represents a connected network. The valid solutions will be kept as good candidates of the ideal network.

Additional Constraints:

1. Top 10% of the busiest stations have at least 4 connections:

$$\sum_j X_{ij} \geq 4 \;\; \forall i \in L$$

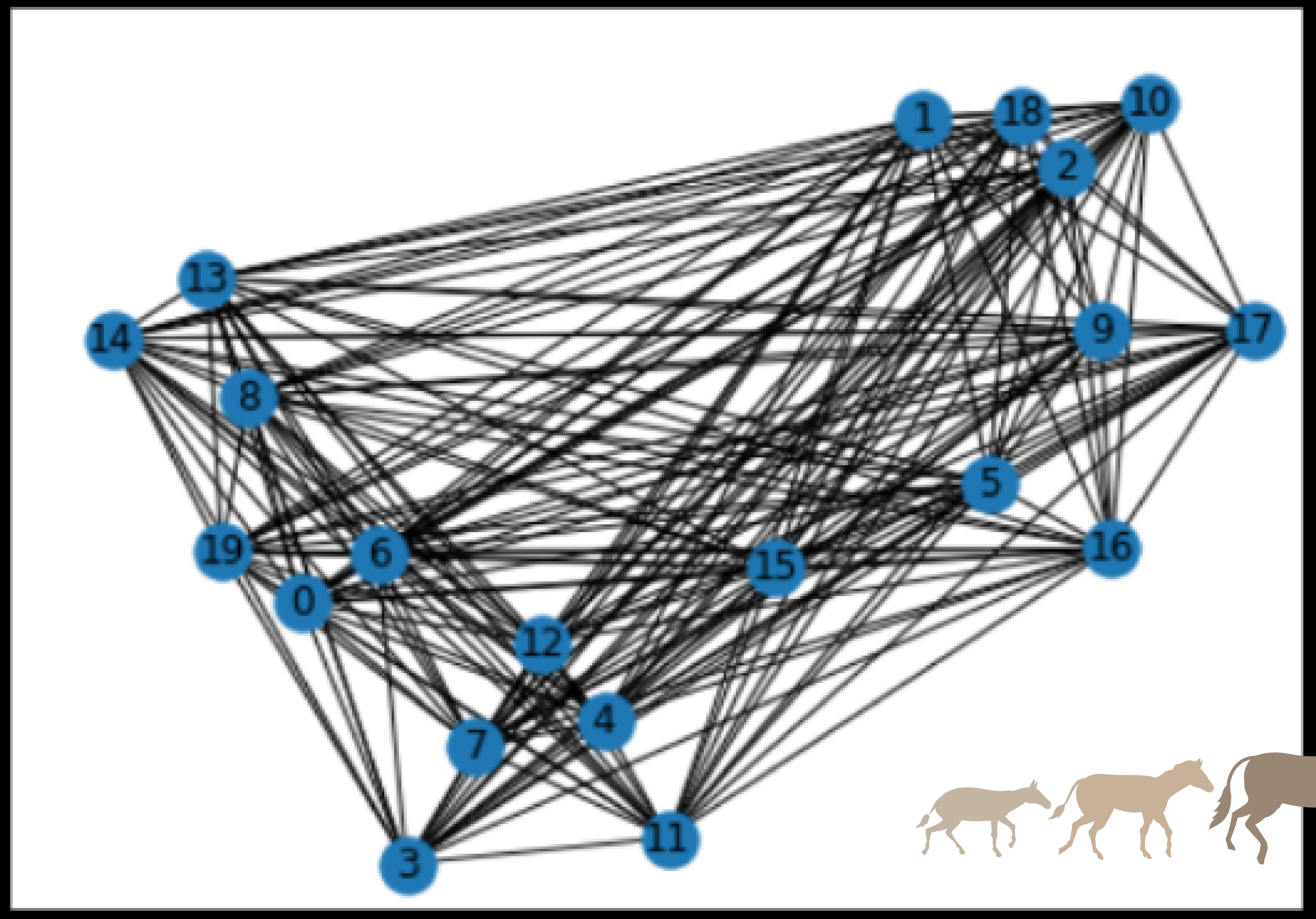
2. Bottom 10% of the busiest stations have at most 2 connections:

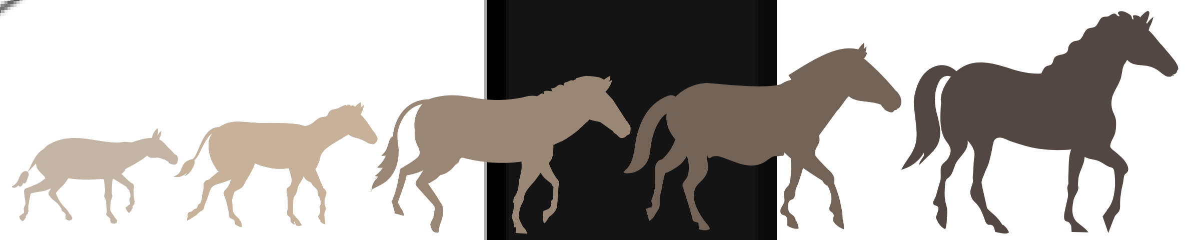
$$\sum_j X_{ij} \leq 2 \;\; \forall i \in S$$

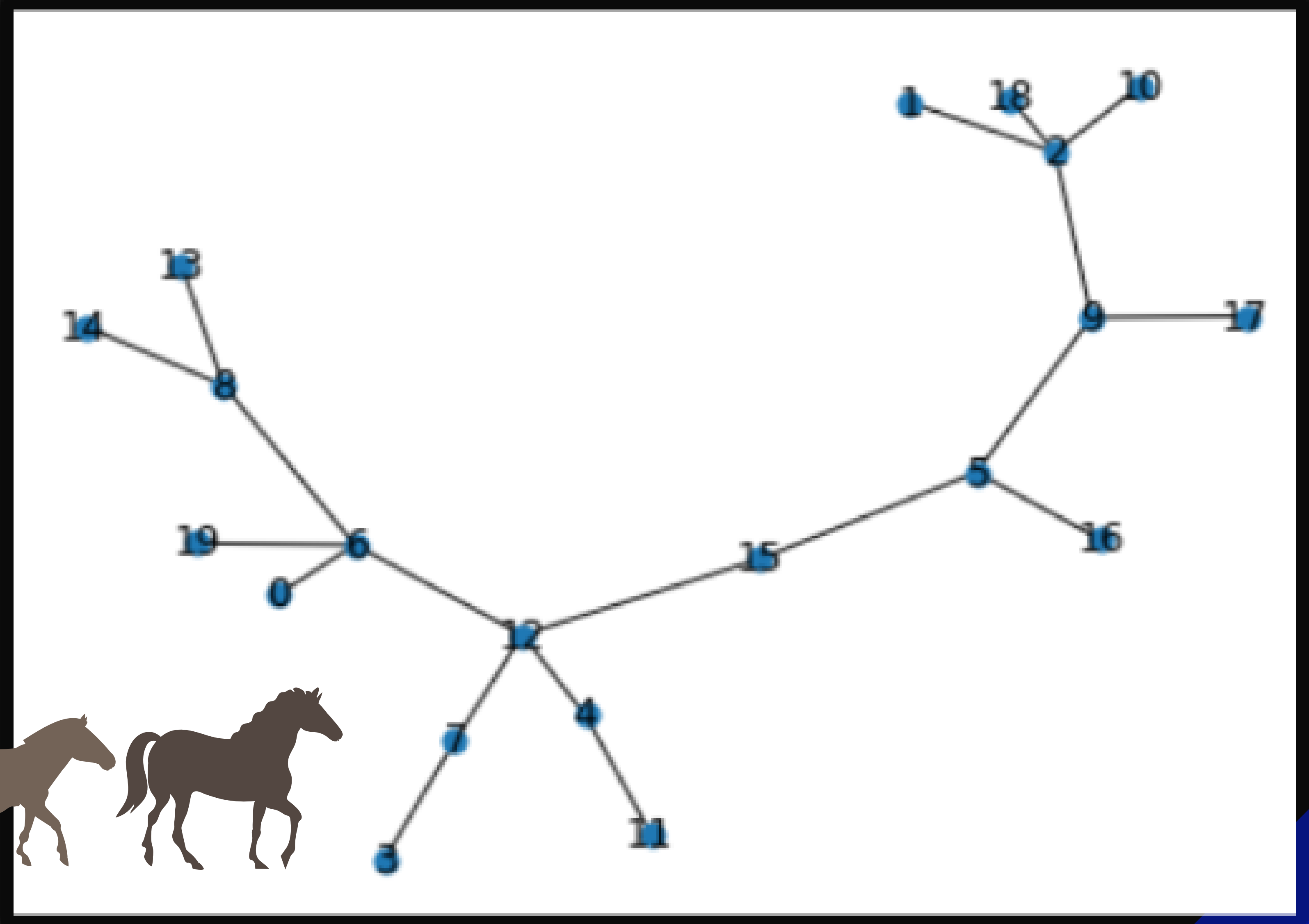
3. Check if the network is connected:

$$\sum_{k=1}^{n-1} A^k \quad \text{has no zero element}$$

Quantum state evolution







Complete graph

Connected graph

1

Divide a network into two subnetworks of equal size by minimizing the sum of the overall distance of each network

Represent the class of the stations as binary variables Xi

The sum of the overall distance is encoded as the cost Hamiltonian.

Each subnetwork has the same number of stations

Repeat this step until each subnetwork is sufficiently small

Objective:

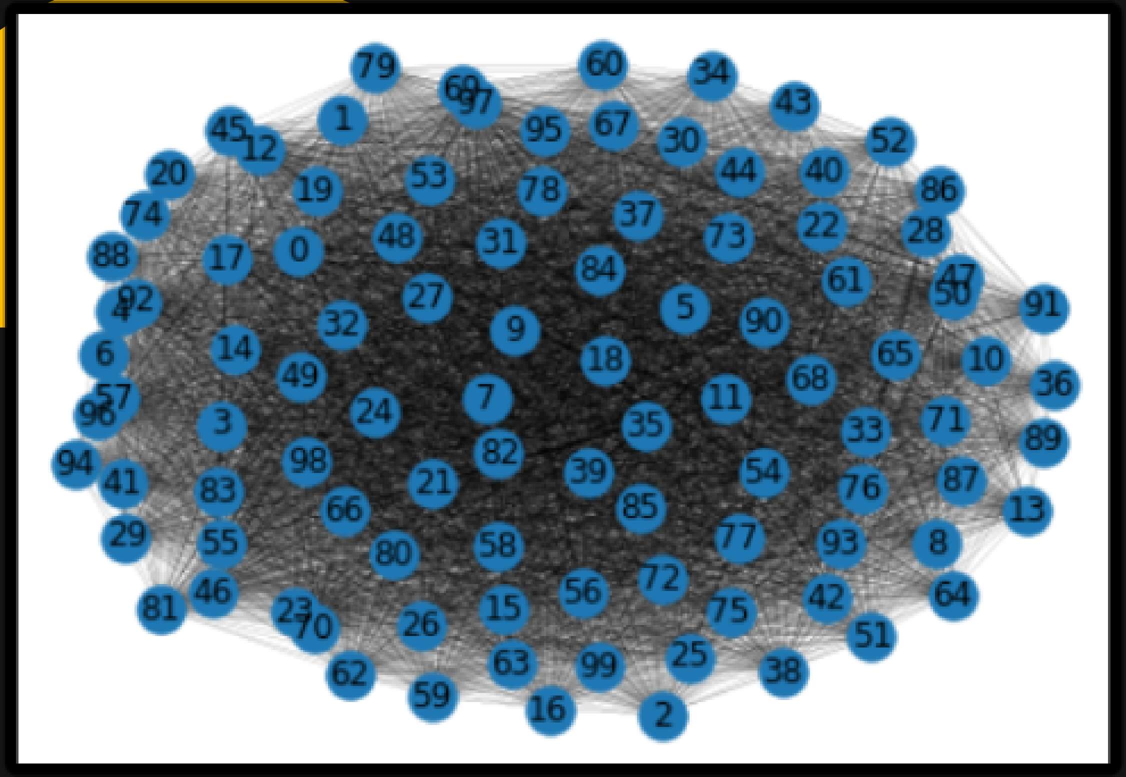
$$Hc = \sum_{j>i} [(1 - X_i)(1 - X_j) + X_i X_j] W_{ij}$$

Constraints:

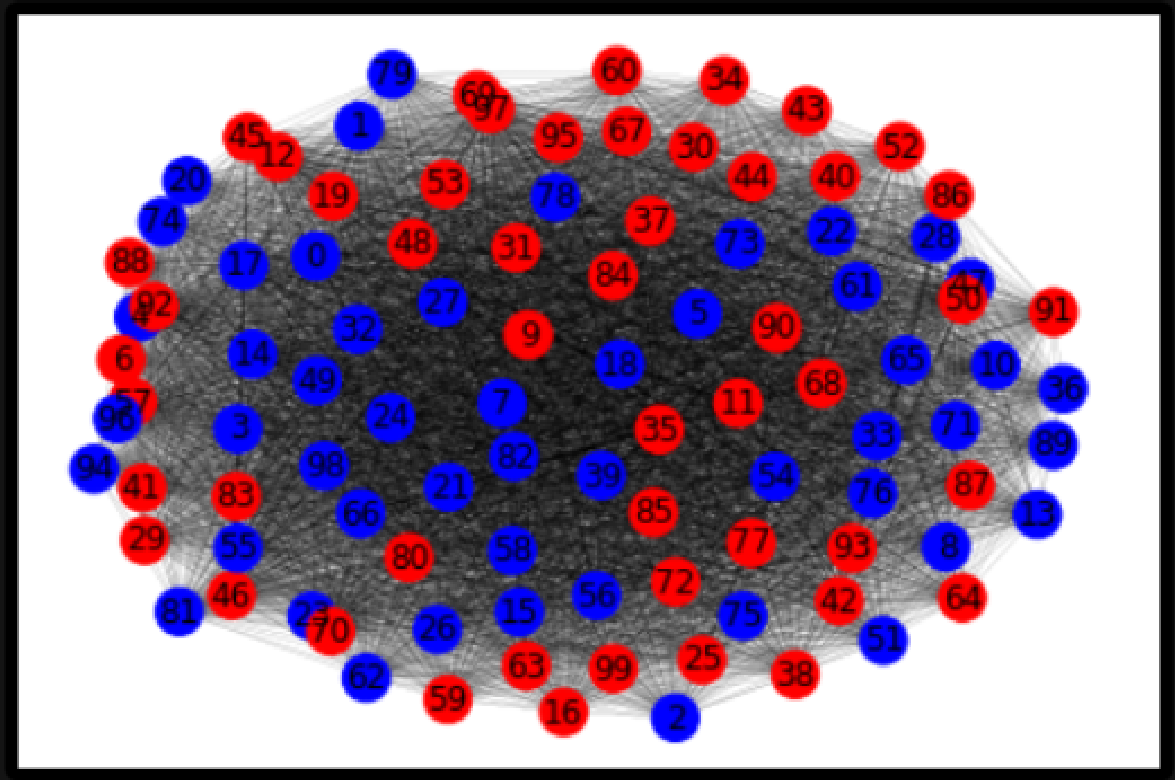
1. Each subnetwork has the same number of stations:

$$\sum_i X_i = \frac{N}{2}$$

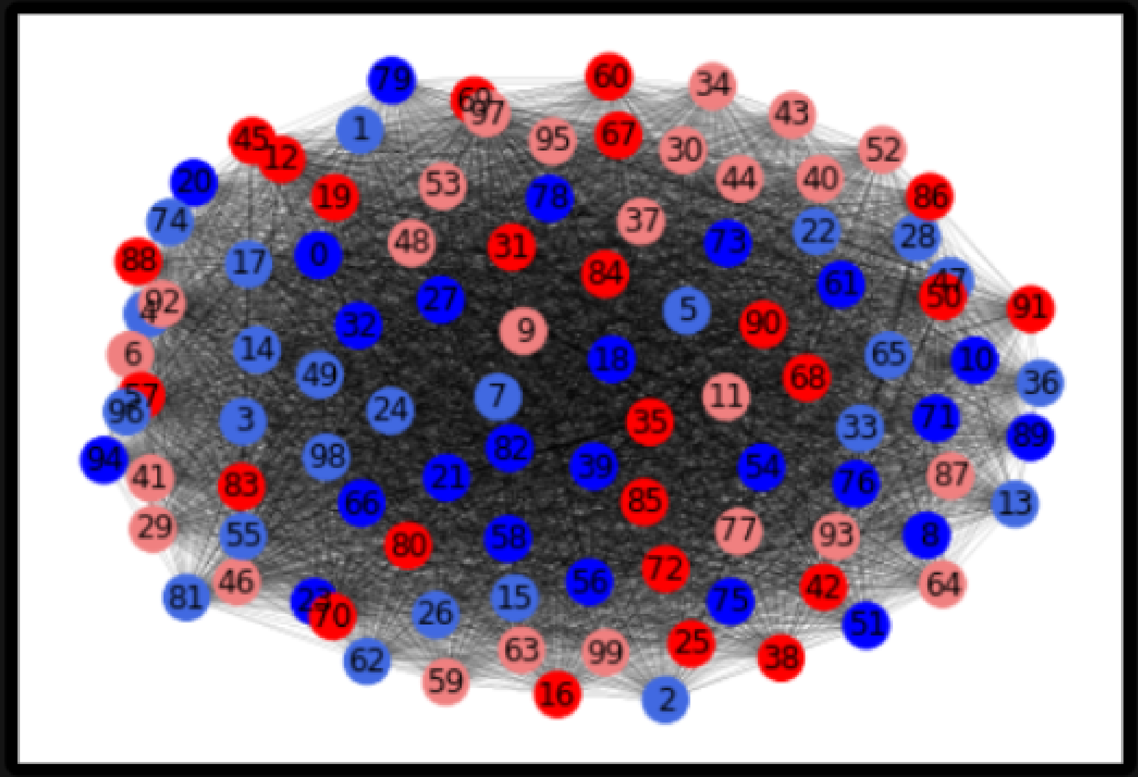
100 stations divided into 4 groups



full network



50 stations



25 stations

3

Compute the average distance between any two stations of a network

Design a quantum shortest-path algorithm to compute the shortest distance between two nodes

Each station is represent as a binary variable Y. if the station is on the path then Y = 1

The sum of the distance between any two connected stations on the path is encoded as the cost Hamiltonian.

A valid path is a connected subgraph including the start and the end points

Objective:

$$Hc = \sum_{k>s} W_{ks} A_{ks} Y_k Y_s$$

Constraints:

1.The start and the end points are on the path:

$$Y_{start} = Y_{end} = 1$$

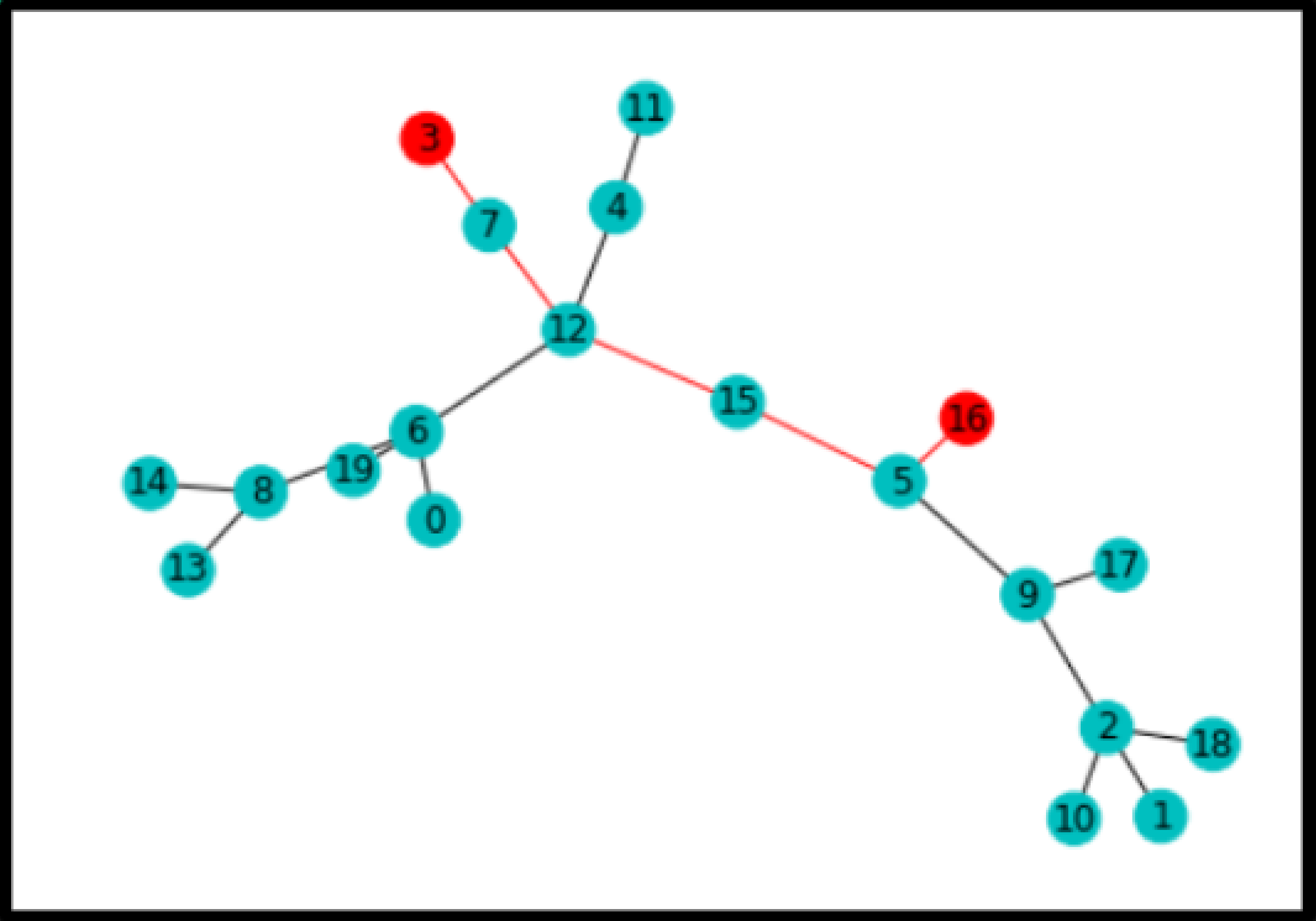
2. Both start and end points need to be connected exactly once:

$$\sum_k A_{start,k} Y_k = 1 \qquad \sum_k A_{end,k} Y_k = 1$$

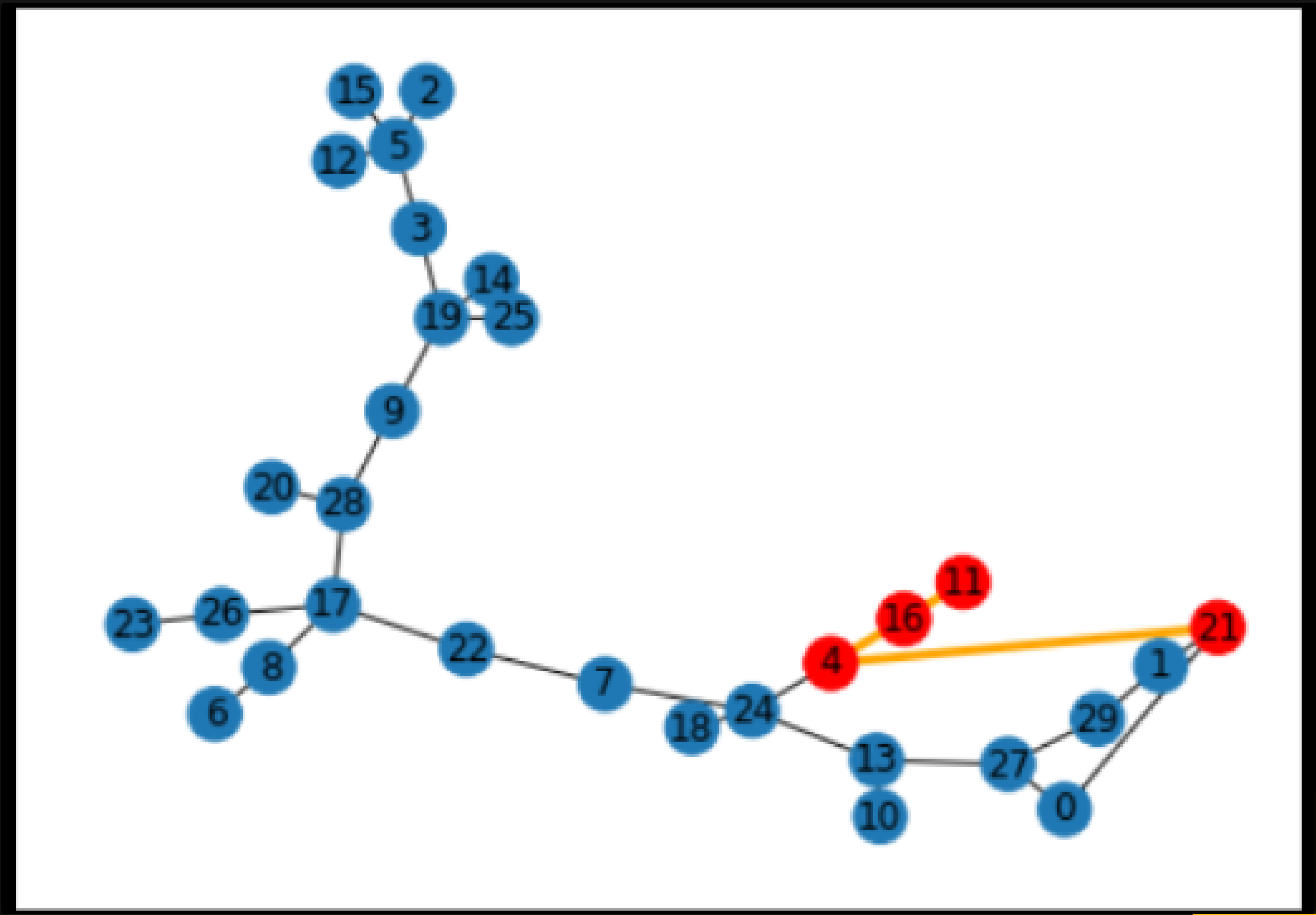
2. Number of connected edges = number of nodes(stations) -1 :

$$\sum_{k>s} A_{ks} Y_k Y_s = \sum_k Y_k - 1$$

Quantum Shortest Path Algorithm



1 possible path



multiple possible paths

3

Compute the average distance between any two stations of a network

The average shortest distance between any two stations can be computed either classically or with the quantum shortest-path algorithm

Classically, we can use the all-pair-shortest - paths algorithm (APSP) to calculate the distance between any pair of stations and then take the average

Alternatively, we can compute the distance one by one with the quantum algorithm and take the average, but this is less efficient

Objective:

$$\bar{d} = \frac{1}{n(n-1)} \sum_{<u,v>} d(u,v)$$

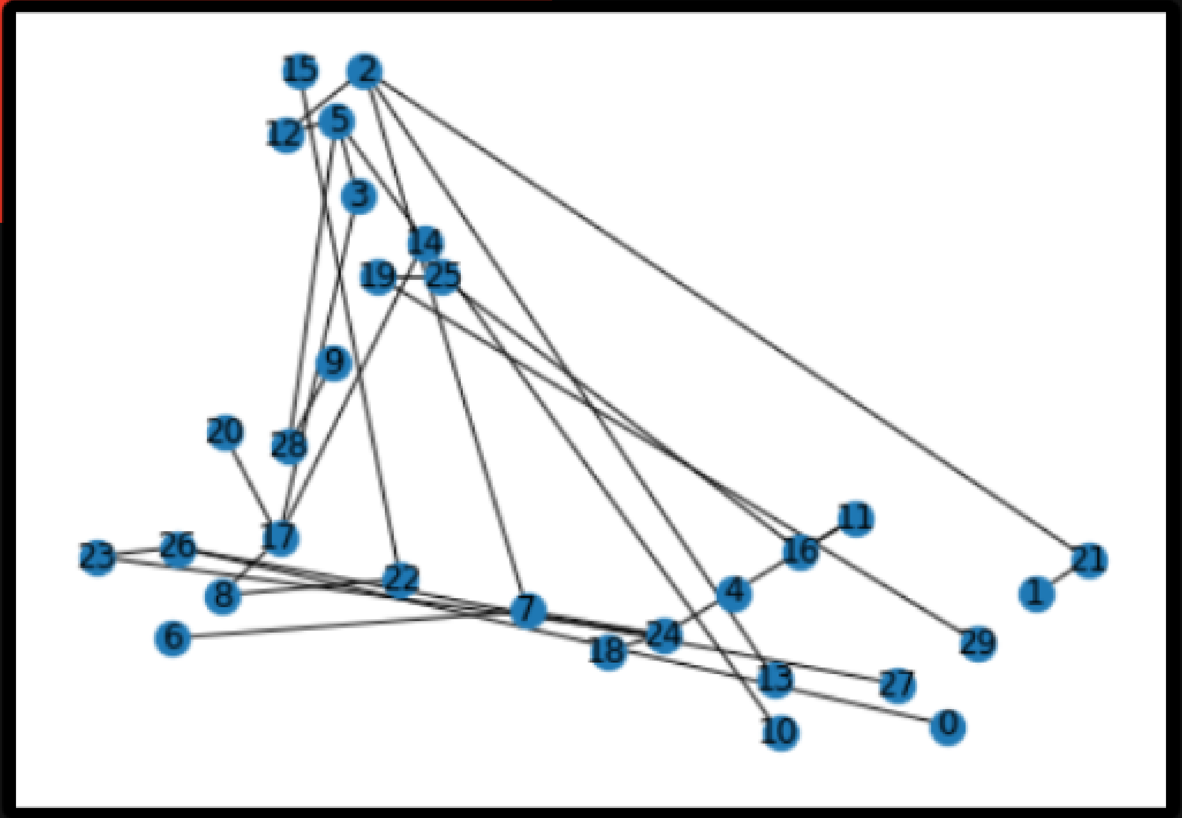
Selection:

1.Compute the average travel distance for each connected networks sampled from Step 2 and select the optimal network considering both the overall length and average travel distance.

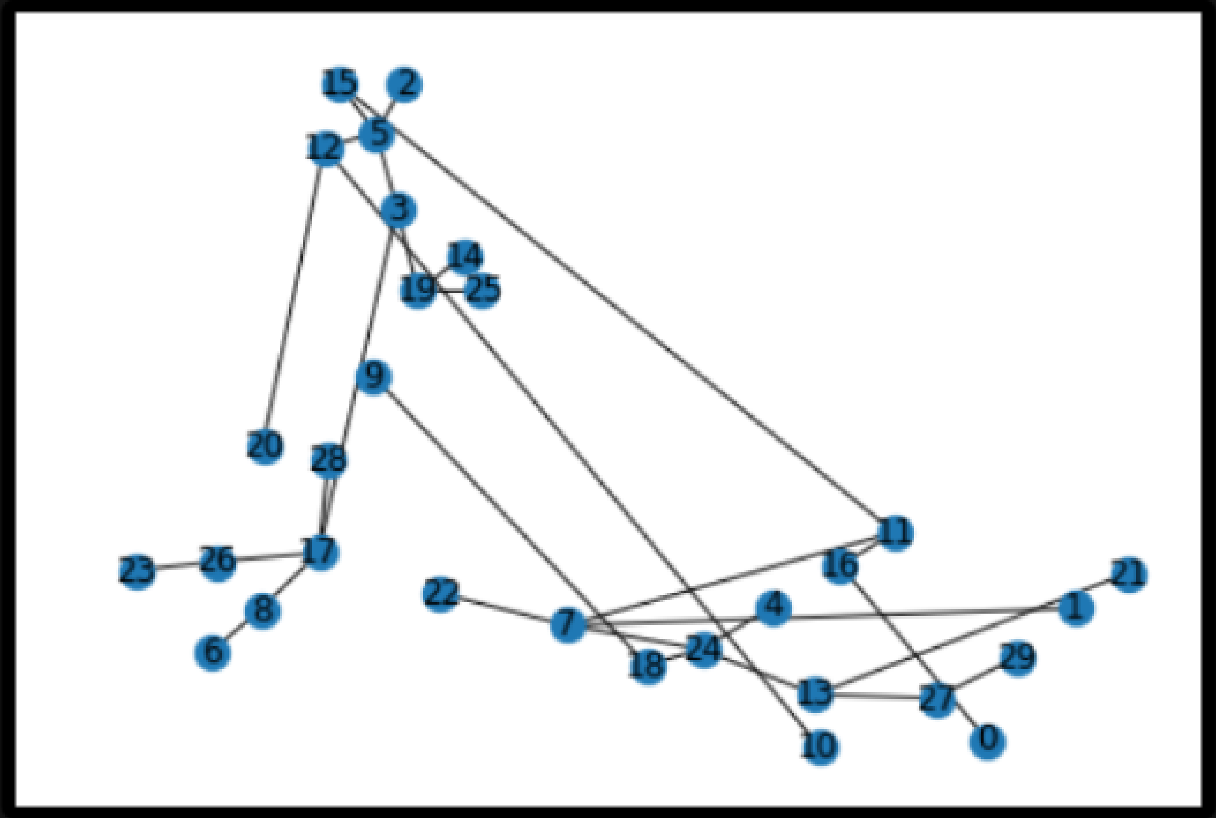
2. Connect the optimal subnetworks generated in Step 1 and 2 by choosing a connection for each pair of subnetworks that again minimizes the average travel distance

Optimal Transport Network!

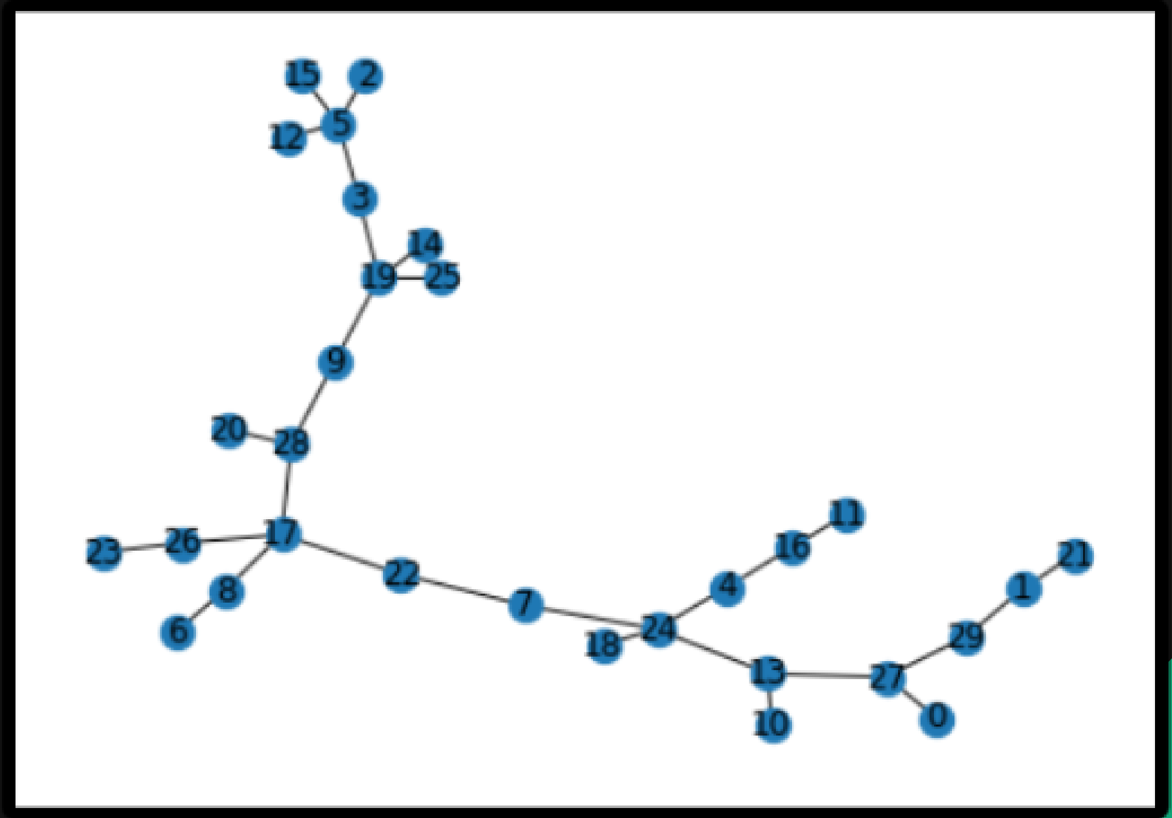
Overall length of the complete network : 3582



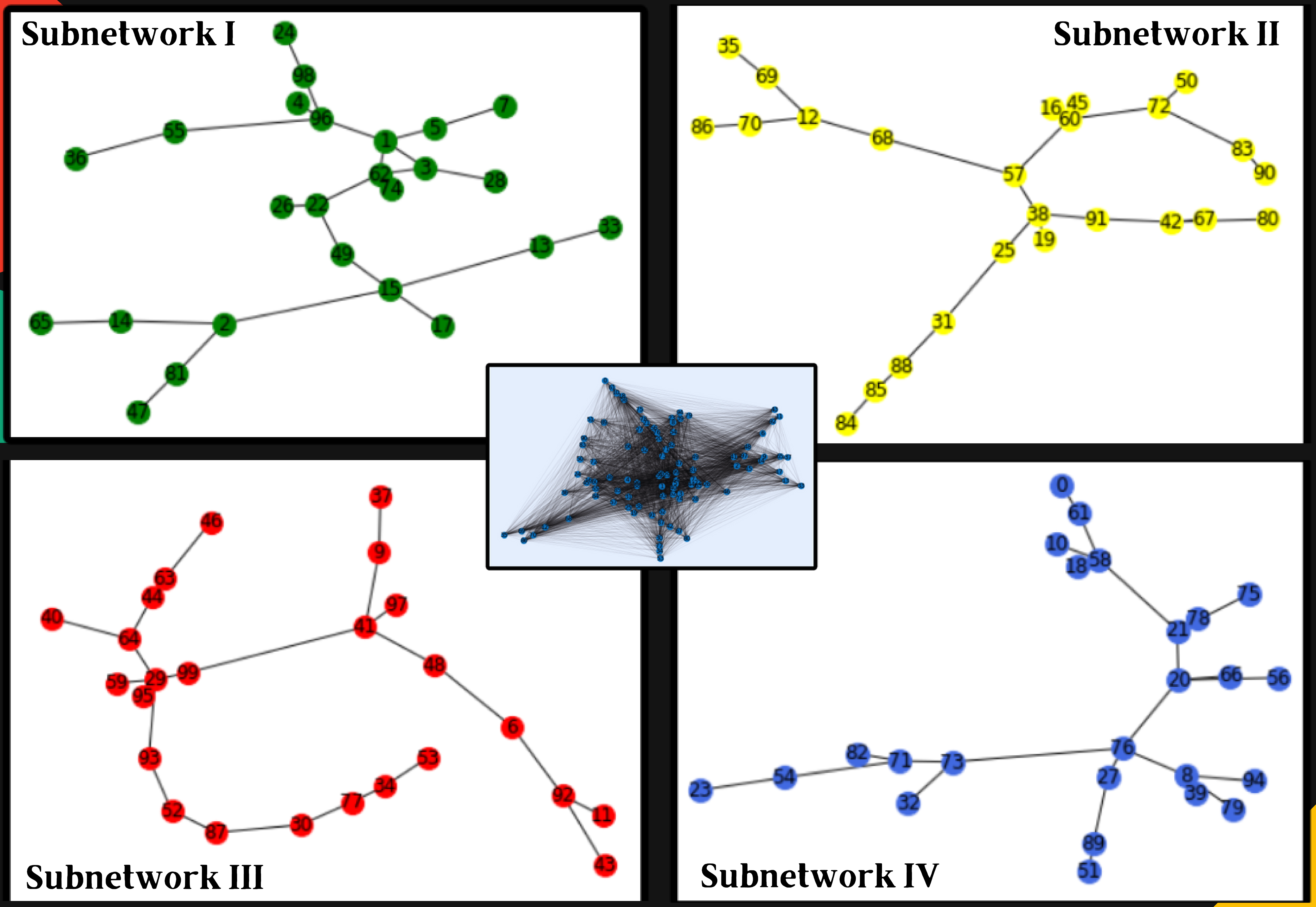
Overall length : 68
Average distance : 13.6



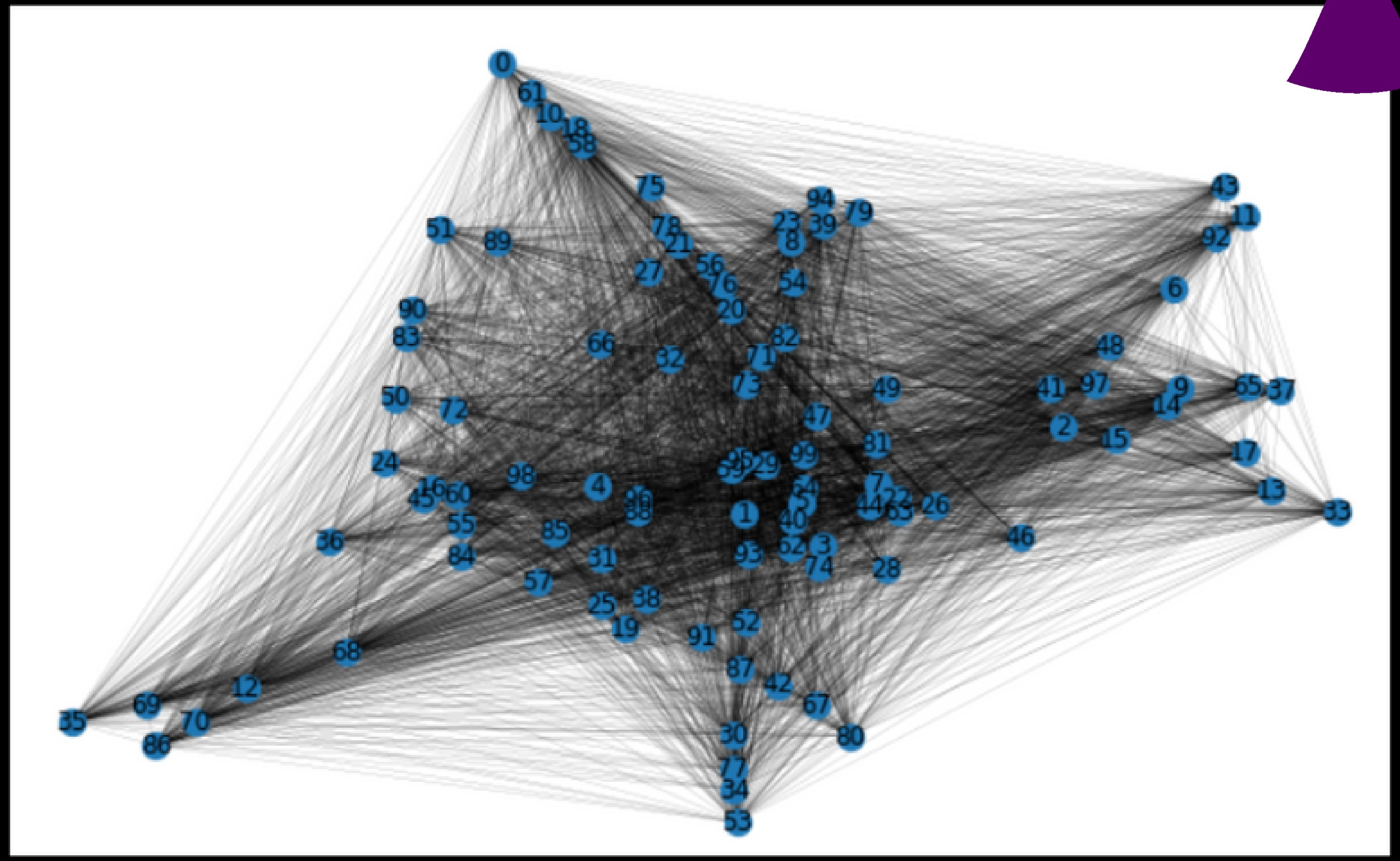
Overall length : 63
Average distance : 13.0

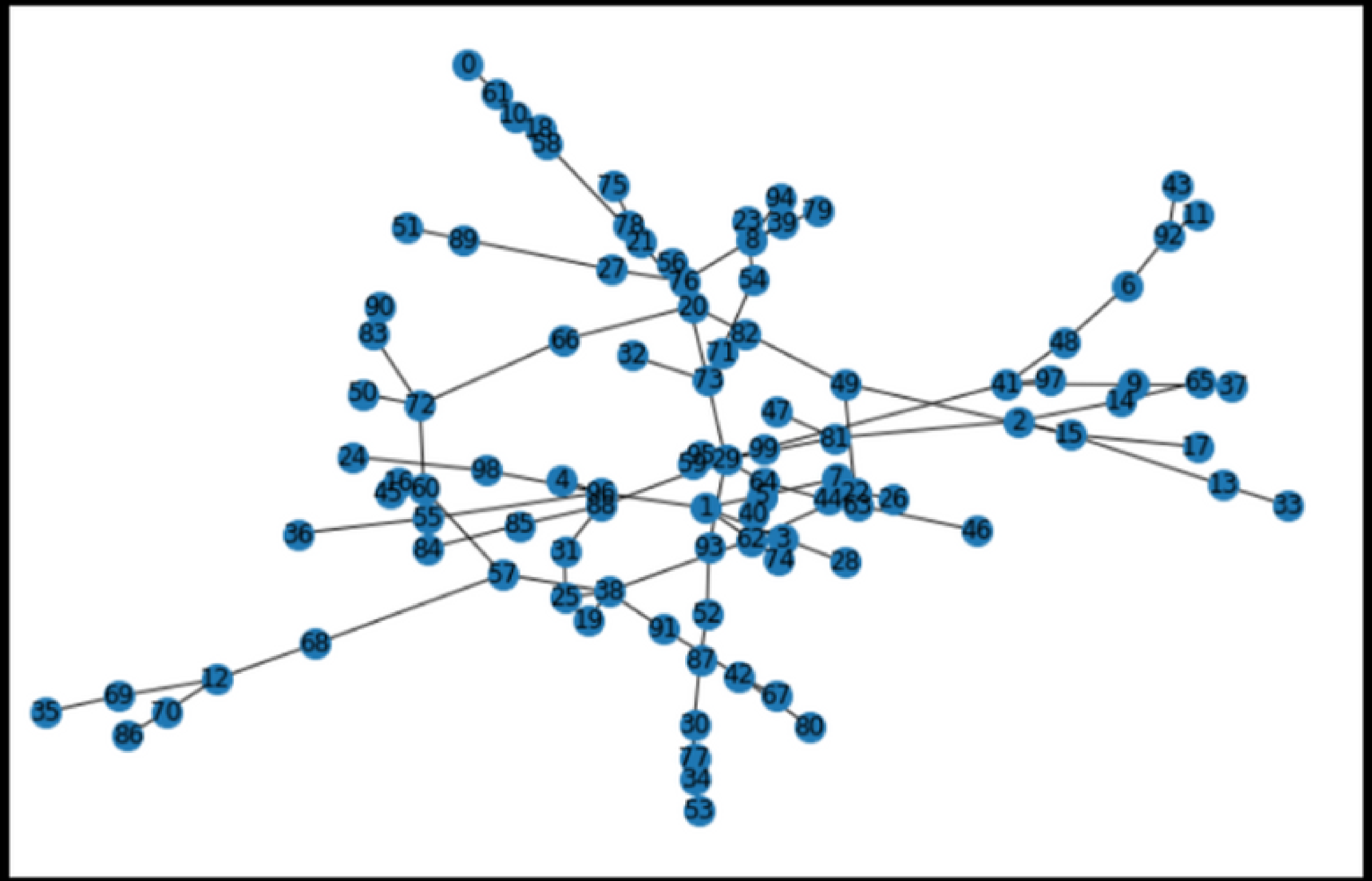


Overall length : 61
Average distance : 11.4


Subnetwork I
Subnetwork II
Subnetwork III
Subnetwork IV

Magic of quantum annealing





2^{4950}

Overall distance : 253425

Average travel distance : 6.9

1/570

4

1

Overall distance : 444

Average travel distance : 27.8

Assignment of Metro Lines

Strategy 1 :

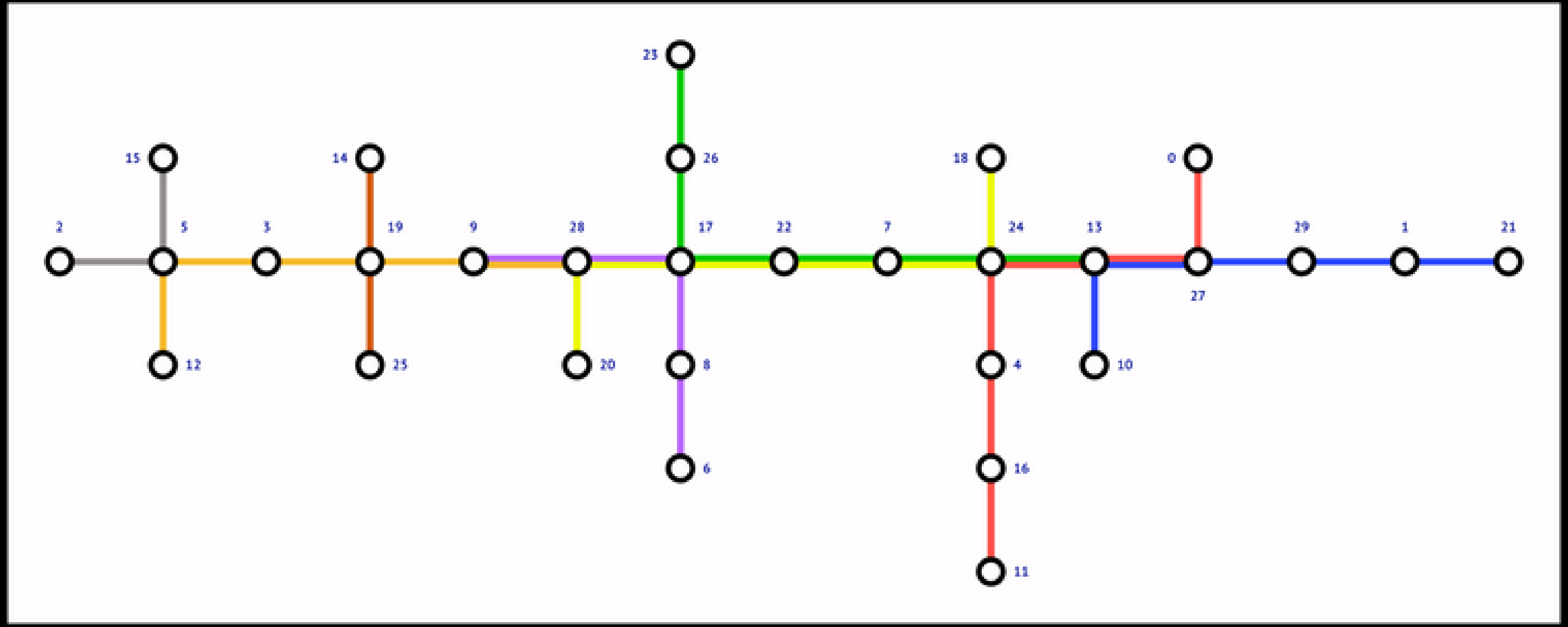
Transferring in a metro is often time-consuming and inconvenient. Hence, a well-arranged set of lines that minimize the average transfer frequency without inducing too many overlaps between lines (increase in overall network length) ensures an efficient and sustainable transportation system

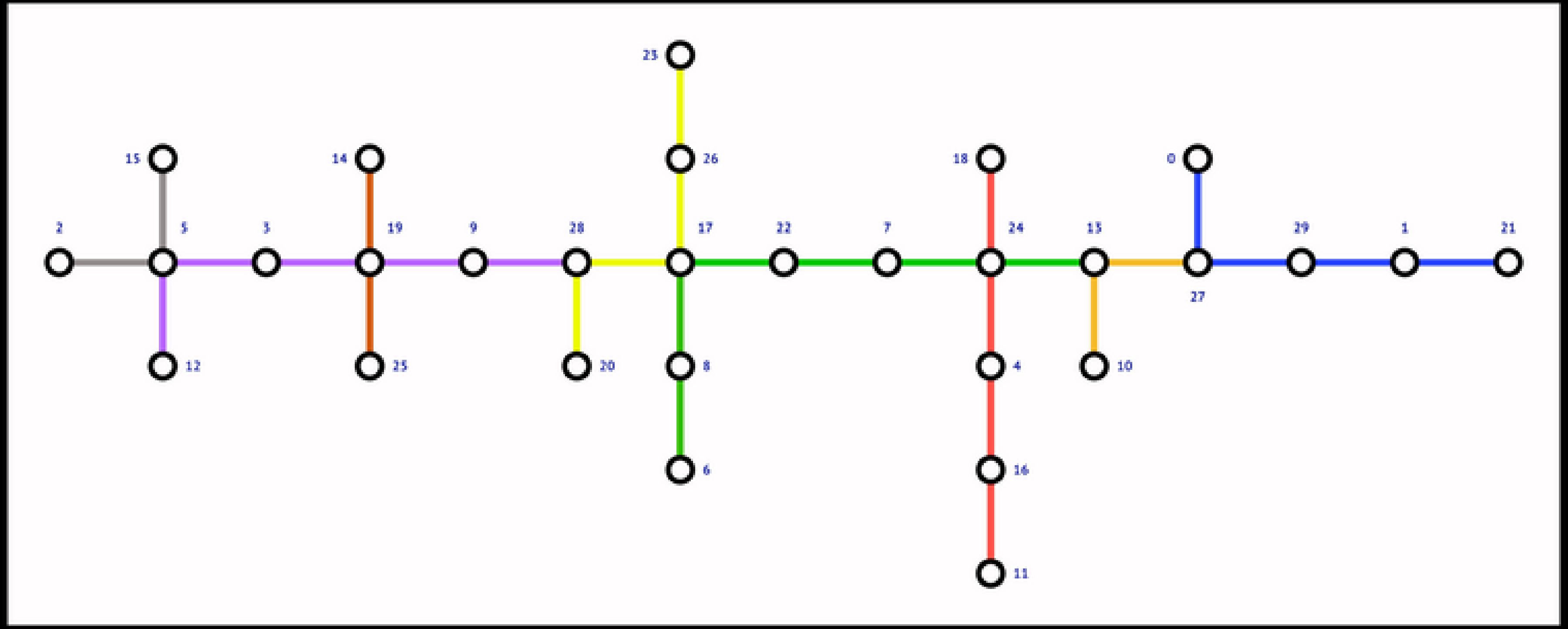
Assignment of Metro Lines

Strategy 2 :

A large discrepancy in the overall length of different lines might not be ideal from the perspective of reducing energy expenditure and risks of operation failure. An excessively long line requires bigger vehicles to accommodate a larger traffic volume while driving a much longer distance, which costs a lot of energy and has a greater impact to transportation should the line fail to operator. Hence, minimizing the variance of length among lines is one of the strategies for arranging metro lines

Metro system with & without overlapping lines :



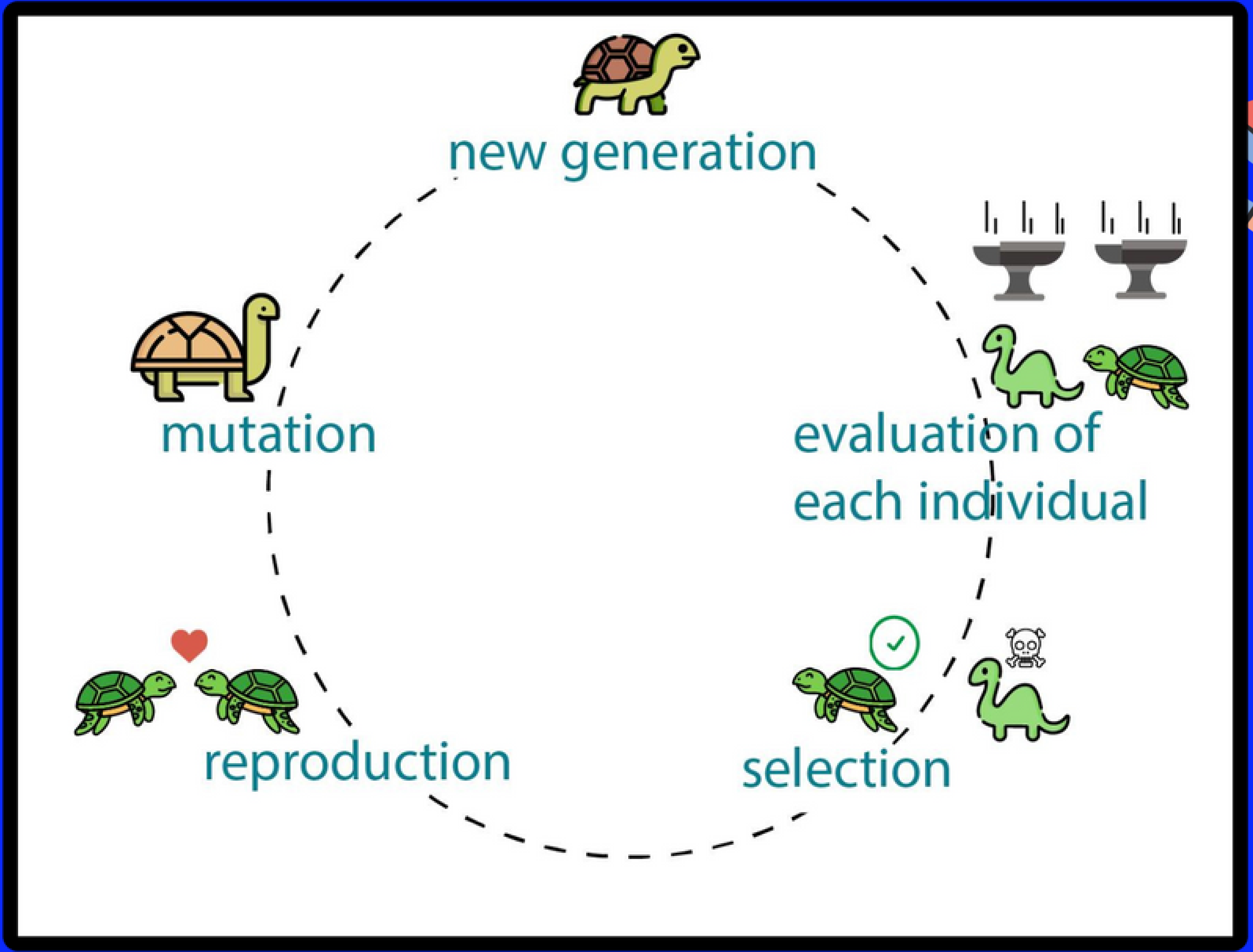


overlap: 21.3% overall length
avg transfer frequency: 1.35 times

overlap: 0% overall length
avg transfer frequency: 1.92 times

Intuitively, There's a trade-off between the transfer frequency and overlaps between lines

Genetic algorithm



Genetic algorithm is a classical evolutionary algorithm for solving optimization problems. It mimics the evolution theory of nature and evolves the population by selecting the fittest individuals according to the target function.

Population generation → Fitness evaluation → Parents Selection → Crossover → Mutation

4 -1

Chromosome:

[(2,21),(12,15),(14,25),(20,28),(6,23),(11,18),(10,13),(0,27)]

Minimize the transfer frequency without inducing too many overlaps between lines

A chromosome is characterized by pairs of terminal stations. A pair of terminal stations constitutes a metro line

The fitness function takes into account both the transfer frequency and the increase in overall length.

In the selection phase, top 30% of the population are selected

Crossover/mutation takes place within a chromosome by randomly changing pairings of terminal stations

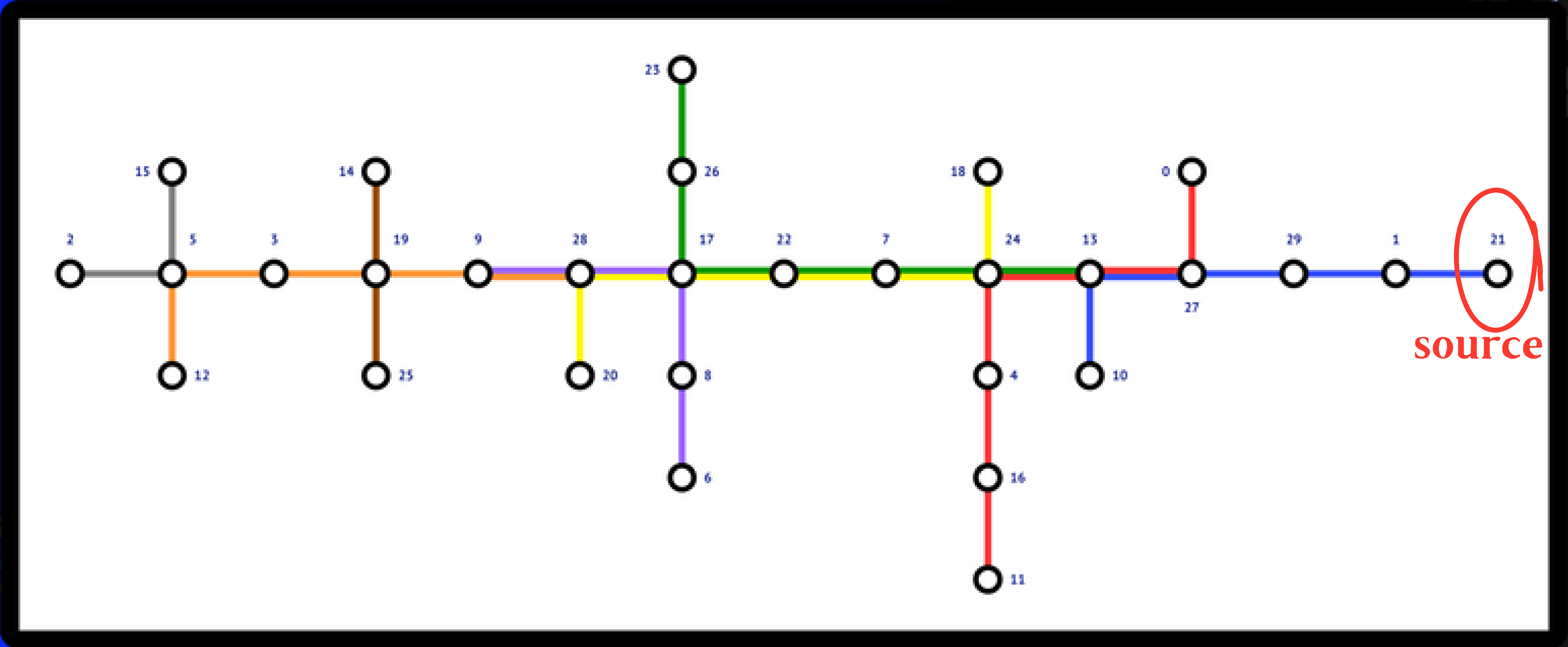
Transfer frequency:

1.Consider how many times one needs to transfer from a source to all other stations. Starting from the lines that the source lies on, we take the union of the lines with the neighboring lines and record the number of new stations in the set. These are the stations with transfer time = 1 from the source.

2. Take the union of the current lines with their neighboring lines again and record newly added stations. These are the stations with transfer times = 2

3. Repeat until the set covers the entire network

Transfer frequency



Transfer 1

Blue ∪ Red ∪ Green
0,24,4,16,11,7,22,17,26,23

Transfer 2

Yellow ∪ Purple
18,8,6,28,20,9

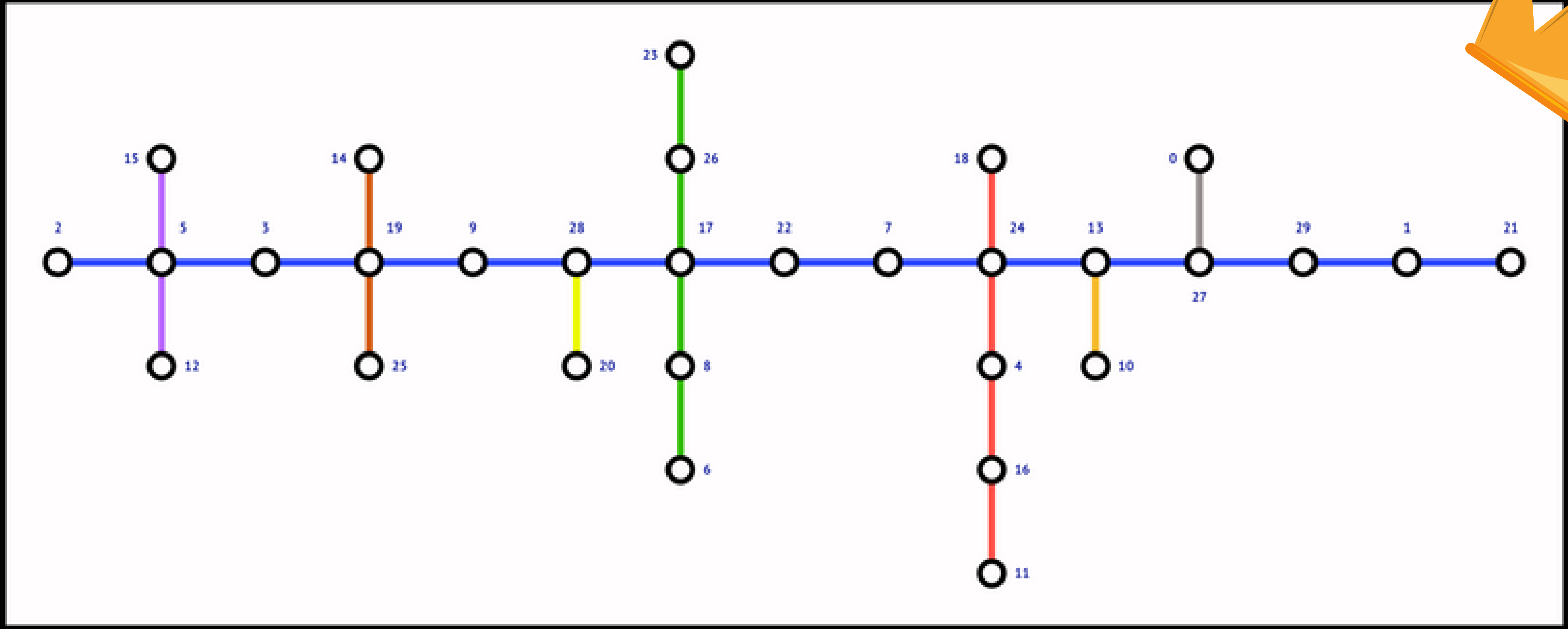
Transfer 3

Orange
19,3,5,12

Transfer 4

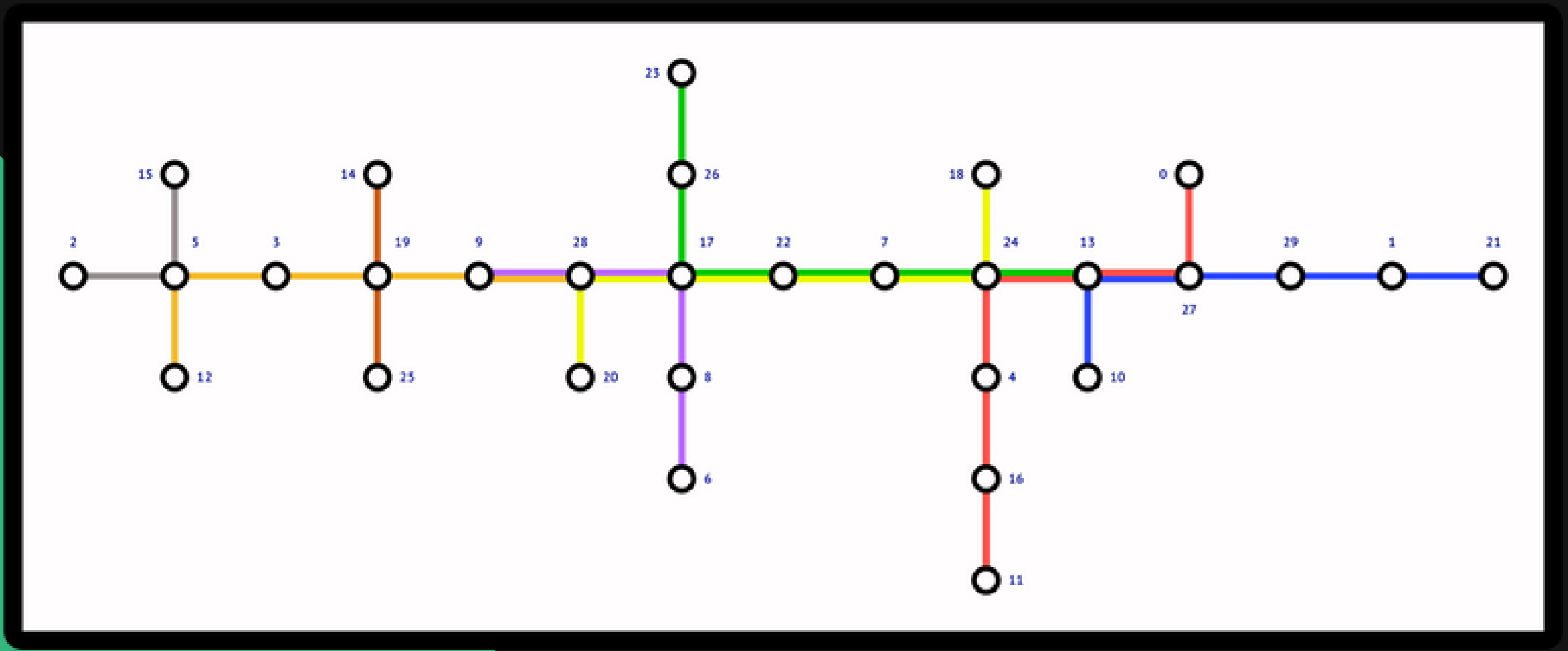
Brown ∪ Grey
14,25,2,15

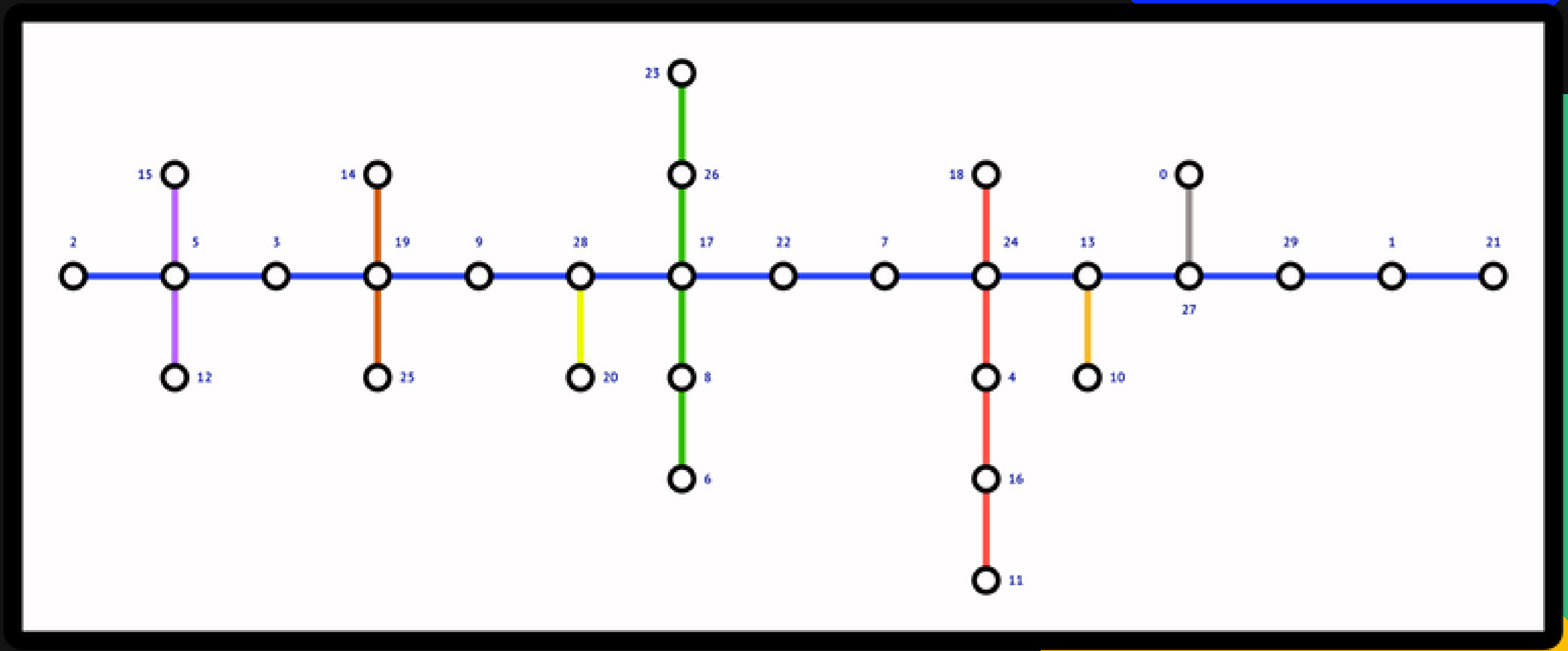
Optimal line assignment with low transfer frequency & no extra overlap



Average transfer frequency: 0.94 times

Variance of length vs. overlap :





overlap: 21.3% overall length
standard deviation: 2.63

overlap: 0% overall length
standard deviation: 8.366

There seems to be a trade-off between the variance of length and overlaps between lines as well

4 -2

Minimize the variance of length without inducing too many overlaps between lines

Distances between any pair of terminal stations are computed and recorded in W

The binary variable Xij represents the pair of terminal stations (i, j) which forms a line

The sum of the difference between every chosen pair (line) is encoded as the target function

The weighted overlapping distance is also considered as the penalty.

Objective:

$$H_c = \sum_{k<s} (\vec{W}\vec{X}_k - \vec{W}\vec{X}_s)^2 - \frac{n(n-2)}{2} \sum_{k} (\vec{W}\vec{X}_k)^2$$

$$\vec{W} = [W_{12}, W_{23}, \ldots, W_{n-1,n}] \quad \vec{X} = [X_{12}, X_{23}, \ldots, X_{n-1,n}]$$

Penalty:

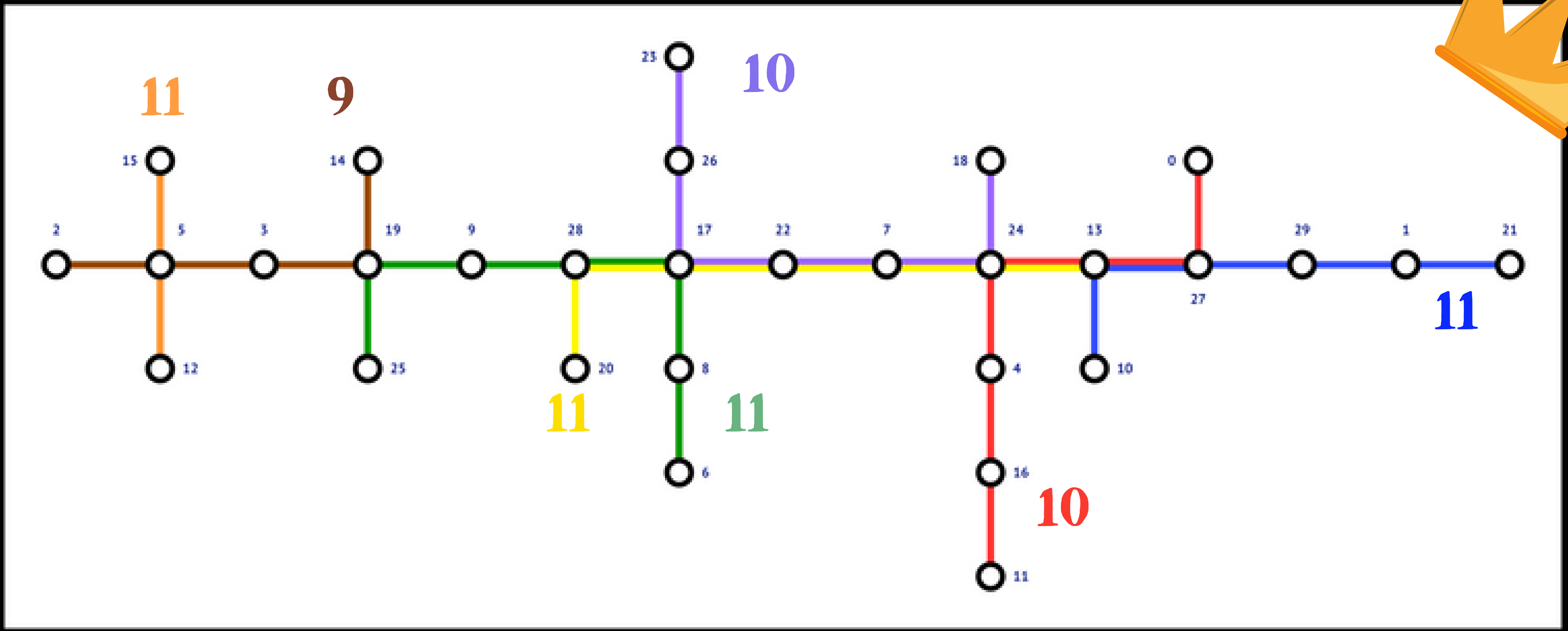
$$\alpha \sum_{k} \vec{W}\vec{X}_k$$

Constraint:

1.Each terminal station can only be chosen once:

$$\sum_{j} X_{ij} = 1$$

Optimal line assignment with low variance of length & low overlap



Average transfer frequency: 1.35 times
overlap = 19% overall length
standard deviation: 0.728

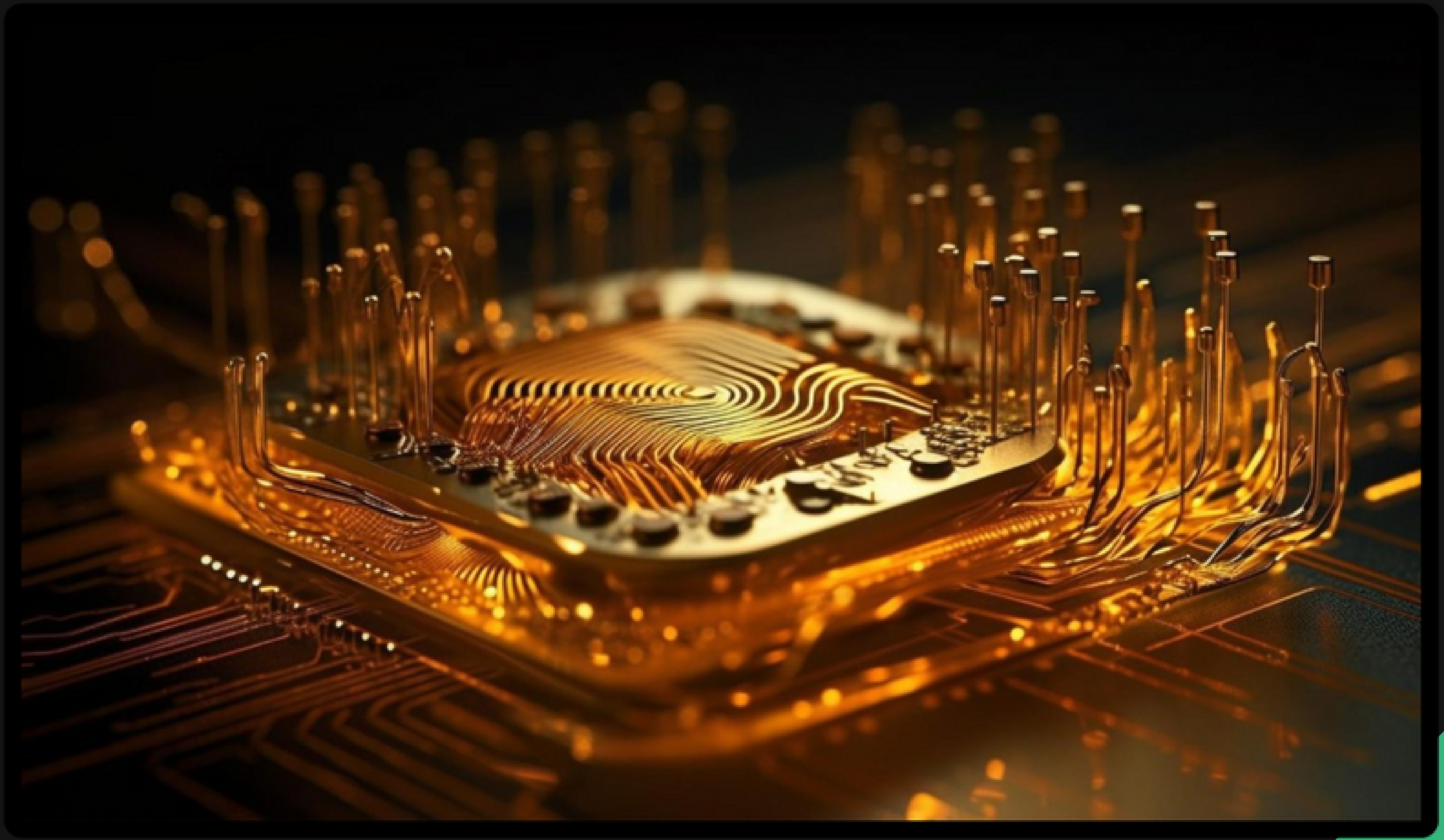
A summary of what we achieved with quantum annealing ...

Grouping nodes of a fully connected graph to minimize the sum of the overall length

Extracting connected subgraphs with minimal overall length from a fully connected graph

Designing quantum shortest path algorithm

Choosing combinations of lines with a low variance of overall length



THANK YOU FOR YOUR PATIENCE

May quantum be with you!