standard deviation is known to be three meals. Construct an approximate 95% confidence interval for the true mean number of meals students eat out each week.

1. Calculate the sample mean.
2. Let $\sigma = 3$ and n = the number of students surveyed.
3. Construct the interval $\left(\overline{x} - 2 \cdot \frac{\sigma}{\sqrt{n}}, \overline{x} + 2 \cdot \frac{\sigma}{\sqrt{n}}\right)$.

We say we are approximately 95% confident that the true mean number of meals that students eat out in a week is between __________ and ___________.

8.1 A Single Population Mean using the Normal Distribution

A confidence interval for a population mean, when the population standard deviation is known, is based on the conclusion of the Central Limit Theorem that the sampling distribution of the sample means follow an approximately normal distribution. Suppose that our sample has a mean of $\overline{x} = 10$ and we have constructed the 90% confidence interval (5, 15) where *EBM* = 5.

Calculating the Confidence Interval

To construct a confidence interval for a single unknown population mean μ, **where the population standard deviation is known**, we need $\overline{x}$ as an estimate for μ and we need the margin of error. Here, the margin of error (*EBM*) is called the **error bound for a population mean** (abbreviated ***EBM***). The sample mean $\overline{x}$ is the **point estimate** of the unknown population mean μ.

The confidence interval estimate will have the form:

(point estimate - error bound, point estimate + error bound) or, in symbols,$(\overline{x} - EBM, \overline{x} + EBM)$

The margin of error (*EBM*) depends on the **confidence level** (abbreviated ***CL***). The confidence level is often considered the probability that the calculated confidence interval estimate will contain the true population parameter. However, it is more accurate to state that the confidence level is the percent of confidence intervals that contain the true population parameter when repeated samples are taken. Most often, it is the choice of the person constructing the confidence interval to choose a confidence level of 90% or higher because that person wants to be reasonably certain of their conclusions.

There is another probability called alpha (α). α is related to the confidence level, *CL*. α is the probability that the interval does not contain the unknown population parameter.
Mathematically, $\alpha + CL = 1$.

EXAMPLE 8.1

Suppose we have collected data from a sample. We know the sample mean but we do not know the mean for the entire population.
The sample mean is seven, and the error bound for the mean is 2.5.

$\overline{x} = 7$ and *EBM* = 2.5

The confidence interval is (7 – 2.5, 7 + 2.5), and calculating the values gives (4.5, 9.5).

If the confidence level (*CL*) is 95%, then we say that, "We estimate with 95% confidence that the true value of the population mean is between 4.5 and 9.5."

TRY IT 8.1

Suppose we have data from a sample. The sample mean is 15, and the error bound for the mean is 3.2.

What is the confidence interval estimate for the population mean?