A confidence interval for a population mean with a known standard deviation is based on the fact that the sample means follow an approximately normal distribution. Suppose that our sample has a mean of $\overline{x} = 10$, and we have constructed the 90% confidence interval (5, 15) where $EBM = 5$.

To get a 90% confidence interval, we must include the central 90% of the probability of the normal distribution. If we include the central 90%, we leave out a total of $\alpha = 10\%$ in both tails, or 5% in each tail, of the normal distribution.
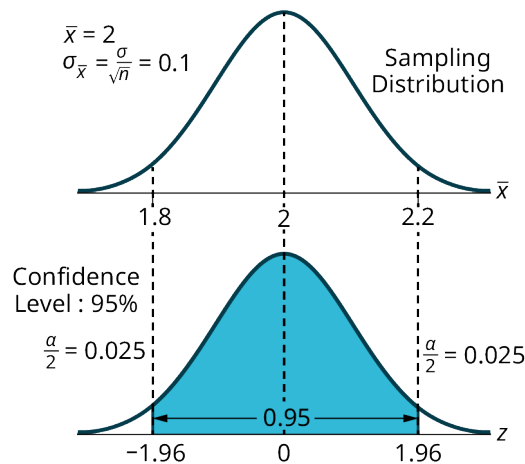

Figure 8.2

$$\mu = \overline{X} \pm Z_\alpha \left(\frac{\sigma}{\sqrt{n}} \right)$$
$$= 2 \pm 1.96(0.1)$$
$$= 2 \pm 0.196$$
$$1.804 \leq \mu \leq 2.196$$

To capture the central 90%, we must go out 1.645 "standard deviations" on either side of the calculated sample mean. The value 1.645 is the z-score from a standard normal probability distribution that puts an area of 0.90 in the center, an area of 0.05 in the far left tail, and an area of 0.05 in the far right tail.

It is important that the "standard deviation" used must be appropriate for the parameter we are estimating, so in this section we need to use the standard deviation that applies to sample means, which is $\frac{\sigma}{\sqrt{n}}$. The fraction $\frac{\sigma}{\sqrt{n}}$, is commonly called the "standard error of the mean" in order to distinguish clearly the standard deviation for a mean from the population standard deviation σ.

In summary, as a result of the central limit theorem:

- $\overline{X}$ is normally distributed, that is, $\overline{X} \sim N\left(\mu_X, \frac{\sigma}{\sqrt{n}} \right)$.

- **When the population standard deviation σ is known, we use a normal distribution to calculate the error bound.**

Calculating the Confidence Interval

To construct a confidence interval estimate for an unknown population mean, we need data from a random sample. The steps to construct and interpret the confidence interval are:

- Calculate the sample mean $\overline{x}$ from the sample data. Remember, in this section we already know the population standard deviation σ.
- Find the z-score that corresponds to the confidence level.
- Calculate the error bound EBM.
- Construct the confidence interval.
- Write a sentence that interprets the estimate in the context of the situation in the problem. (Explain what the confidence interval means, in the words of the problem.)

We will first examine each step in more detail, and then illustrate the process with some examples.

Finding the z-score for the Stated Confidence Level

When we know the population standard deviation σ, we use a standard normal distribution to calculate the error bound