

2.3.1 Adiabatic, steady, throttling of a gas (flow through a valve or other restriction)

Question: Why is the work done: $P_2 \cdot V_2 - P_1 \cdot V_1$?

My explanation: (am I correct?)

Based on figure 2.6, let's first define the left piston, which has an external pressure P_1 , as piston A, and the right piston, which has an external pressure P_2 , as piston B.

As the gas flows through a valve, the upstream surroundings do work on piston A, in the same sense that we compress the piston A, at constant external pressure P_1 , by volume V_1 . This work done on the system is therefore $-P_1 \cdot V_1$.

Similarly, the system does work on the downstream surroundings by piston B, at constant pressure P_2 by volume V_2 . This work done by the system is $P_2 \cdot V_2$. So eventually the net work output is,
$$P_2 \cdot V_2 - P_1 \cdot V_1.$$

2.3.3 Transient filling of a tank

Question: Why is the work done: $-P_0 \cdot V_0$?

My explanation: (am I correct?)

Seeing figure 2.7, we can imagine an invisible piston A, which has a constant external pressure P_0 pushing it toward the tank. (similar to the 2.3.1 example). So the work done on the system is: $-P_0 \cdot V_0$. On the other side, we can imagine an invisible piston B, which has a constant external pressure = 0, because it is vacuum. So the work done by the system is 0 (the free expansion idea). Eventually, the new work output is,

$$-P_0 \cdot V_0$$



Thermodynamics and Propulsion

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- 2.3.4 The First Law in Terms of Enthalpy

2.3 Example Applications of the First Law to motivate the use of a property called ``enthalpy''

[VW, S & B: 5.4-5.5]

2.3.1 Adiabatic, steady, throttling of a gas (flow through a valve or other restriction)

Figure 2.5 shows the configuration of interest. We wish to know the relation between properties upstream of the valve, denoted by ``1'' and those downstream, denoted by ``2''.

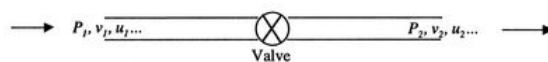


Figure 2.5: Adiabatic flow through a valve, a generic throttling process

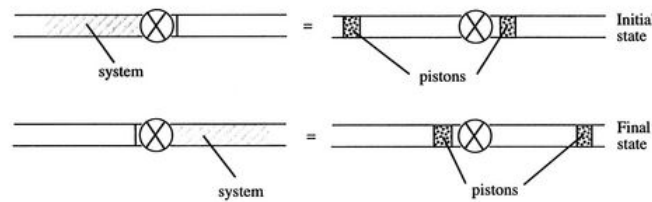


Figure 2.6: Equivalence of actual system and piston model

To analyze this situation, we can define the system (choosing the appropriate system is often a critical element in effective problem solving) as a unit mass of gas in the following two states. Initially the gas is upstream of the valve and just through the valve. In the final state the gas is downstream of the valve plus just before the valve. The figures on the left of Figure 2.6 show the actual configuration just described. In terms of the system behavior, however, we could replace the fluid external to the system by pistons which exert the same pressure that the external fluid exerts, as indicated schematically on the right side of Figure 2.6.

The process is adiabatic, with changes in potential energy and kinetic energy assumed to be negligible. The first law for the system is therefore

$$\Delta U = -W.$$

The work done by the system is

$$W = P_2 V_2 - P_1 V_1.$$

Use of the first law leads to

$$U_2 + P_2 V_2 = U_1 + P_1 V_1.$$

In words, the initial and final states of the system have the same value of the quantity $U + PV$. For the case examined, since we are dealing with a unit mass, the initial and final states of the system have the same value of $u + Pv$.

We define this quantity as the "enthalpy," usually denoted by H ,

$$H = U + PV.$$

In terms of the specific quantities, the enthalpy per unit mass is

$$h = u + Pv = u + P/\rho.$$

It is a function of the state of the system. H has units of Joules, and h has units of Joules per kilogram.

The utility and physical significance of enthalpy will become clearer as we work with more flow problems. For now, you may wish to think of it as follows (Levenspiel, 1996). When you evaluate the energy of an object of volume V , you have to remember that the object had to push the surroundings out of the way to make room for itself. With pressure p on the object, the work required to make a place for itself is pV . This is so with any object or system, and this work may not be negligible. (The force of one atmosphere pressure on one square meter is equivalent to the force of a mass of about 10 tons.) Thus the total energy of a body is its internal energy plus the extra energy it is credited with by having a volume V at pressure p . We call this total energy the enthalpy, H .

Muddy Points

When is enthalpy the same in initial and final states? (MP 2.3)

2.3.2 Quasi-Static Expansion of a Gas

Consider a quasi-static process of constant pressure expansion. We can write the first law in terms of the states before and after the expansion as

$$Q = (U_2 - U_1) + W,$$

and writing the work in terms of system properties,

$$= (U_2 - U_1) + p(V_2 - V_1) \quad \text{since } p_1 = p_2 = p.$$

By grouping terms we can write the heat input in terms of the enthalpy change of the system:

$$\begin{aligned} Q &= (U_2 + pV_2) - (U_1 + pV_1) \\ &= H_2 - H_1. \end{aligned}$$

2.3.3 Transient filling of a tank

Another example of a flow process, this time for an unsteady flow, is the transient process of filling a tank, initially evacuated, from a surrounding atmosphere, which is at a pressure P_0 and a temperature T_0 . The configuration is shown in Figure 2.7.

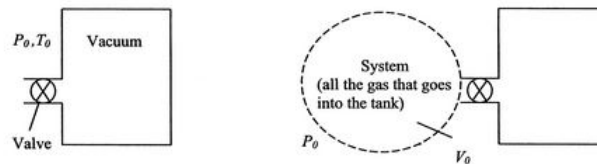


Figure 2.7: A transient problem — filling of a tank from the atmosphere

At a given time, the valve at the tank inlet is opened and the outside air rushes in. The inflow stops when the pressure inside is equal to the pressure outside. The tank is insulated, so there is no heat transfer to the atmosphere. What is the final temperature of the gas in the tank?

This time we take the system to be all the gas that enters the tank. The initial state has the system completely outside the tank, and the final state has the system completely inside the tank. The kinetic energy initially and in the final state is negligible, as is the change in potential energy, so the first law again takes the form

$$\Delta U = -W.$$



Work is done *on* the system, of magnitude $P_0 V_0$, where V_0 is the initial volume of the system, so

$$\Delta U = P_0 V_0.$$

In terms of quantities per unit mass ($\Delta U = m\Delta u$, $V_0 = mv_0$, where m is the mass of the system),

$$\Delta u = u_{\text{final}} - u_i = P_0 v_0.$$

The final value of the internal energy is

$$\begin{aligned}
 u_{\text{final}} &= u_i + P_0 v_0 \\
 &= h_i = h_0.
 \end{aligned}$$

For a perfect gas with constant specific heats (see the next section, Section 2.4),

$$u = c_v T, \quad h = c_p T,$$

$$c_v T_{\text{final}} = c_p T_0,$$

$$T_{\text{final}} = \frac{c_p}{c_v} T_0 = \gamma T_0.$$

The final temperature is thus roughly 200°F hotter than the outside air!

It may be helpful to recap what we used to solve this problem. There were basically four steps:

1. Definition of the system
2. Use of the first law
3. Equating the work to a $\int P dV$ term
4. Assuming the fluid to be a perfect gas with constant specific heats.

A message that can be taken from both of these examples (as well as from a large number of other more complex situations, is that the quantity $h = u + Pv$ occurs naturally in problems of fluid flow. Because the combination appears so frequently, it is not only defined but also tabulated as a function of temperature and pressure for a number of working fluids.

Muddy Points

In the filling of a tank, why (physically) is the final temperature in the tank higher than the initial temperature? (MP 2.4)

2.3.4 The First Law in Terms of Enthalpy

We start with the first law in differential form and substitute $p dV$ for dW by assuming a quasi-static or reversible process:

$$dU = \delta Q - \delta W \quad (\text{true for any process, neglecting } \Delta KE \text{ and } \Delta PE)$$

$$dU = \delta Q - p dV \quad (\text{true for any quasi-static process, no } \Delta KE \text{ or } \Delta PE)$$

The definition of enthalpy,

$$H = U + pV,$$

can be differentiated (applying the chain rule to the pV term) to produce

$$dH = dU + p dV + V dp.$$

Substituting the dU above for the dU in the First Law, we obtain

$$dH = \delta Q - \delta W + p dV + V dp \quad (\text{valid for any process})$$

or

$$dH = \delta Q + V dp \quad (\text{valid for any quasi-static process}).$$

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