The departure from a reversible process is directly reflected in the entropy change and the decrease in engine effectiveness.

Muddy Points

Why does $\Delta S_{\text{irrev}} = \Delta S_{\text{total}}$? (MP 6.17)

In discussing the terms ``closed system'' and ``isolated system,'' can you assume that you are discussing a cycle or not? (MP 6.18)

Does a cycle process have to have $\Delta S = 0$? (MP 6.19)

In a real heat engine, with friction and losses, why is ΔS still 0 if $TdS = dQ + d\Phi$? (MP 6.20)

4. **Propulsive Power and Entropy Flux**

The final example in this section combines a number of ideas presented in this subject and in Unified in the development of a relation between entropy generation and power needed to propel a vehicle.

Figure 6.15 shows an aerodynamic shape (airfoil) moving through the atmosphere at a constant velocity. A coordinate system fixed to the vehicle has been adopted so that we see the airfoil as fixed and the air far away from the airfoil moving at a velocity c_0 . Streamlines of the flow have been sketched, as has the velocity distribution at station ``0'' far upstream and station ``d'' far downstream.

The airfoil has a wake, which mixes with the surrounding air and grows in the downstream direction. The extent of the wake is also indicated. Because of the lower velocity in the wake the area between the stream surfaces is larger downstream than upstream.

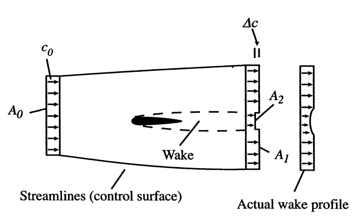

Figure 6.15: Airfoil with wake and control volume for analysis of propulsive power requirement

We use a control volume description and take the control surface to be defined by the two stream surfaces and two planes at station 0 and station d. This is useful in simplifying the analysis because there is no flow across the stream surfaces. The area of the downstream plane control surface is broken into A_1 , which is the area outside the wake and A_2 , which is the area occupied by wake fluid, i.e., fluid that has suffered viscous losses. The control surface is also taken far enough away from the vehicle so that the static pressure can be considered uniform. For fluid which is not in the wake (no viscous forces), the momentum equation is

$$c dc = -dP/\rho.$$

Uniform static pressure therefore implies uniform velocity, so that on A_1 the velocity is equal to the upstream value, c_0 . The downstream velocity profile is actually continuous, as indicated. It is approximated in the analysis as a step change to make the algebra a bit simpler. (The conclusions apply to the more general velocity profile as well and we would just need to use integrals over the wake instead of the algebraic expressions below.)

The equation expressing mass conservation for the control volume is

$$\rho_0 A_0 c_0 = \rho_0 A_1 c_0 + \rho_2 A_2 c_2. \tag{6..12}$$

The vertical face of the control surface is far downstream of the body. By this station, the wake fluid has had much time to mix and the velocity in the wake is close to the free stream value, c_0 . We can thus write,

$$\text{wake velocity } = c_2 = (c_0 - \Delta c); \qquad \Delta c/c_0 \ll 1. \tag{6..13}$$

(We chose our control surface so the condition $\Delta c/c_0 \ll 1$ was upheld.)

The integral momentum equation (control volume form of the momentum equation) can be used to find the drag on the vehicle:

$$\rho_0 A_0 c_0^2 = -\text{Drag} + \rho_0 A_1 c_0^2 + \rho_2 A_2 c_2^2. \tag{6..14}$$

There is no pressure contribution in Eq. (6.14) because the static pressure on the control surface is uniform. Using the form given for the wake velocity and expanding the terms in the momentum equation we obtain

$$\rho_0 A_0 c_0^2 = -\text{Drag} + \rho_0 A_1 c_0^2 + \rho_2 A_2 [c_0^2 - 2c_0 \Delta c + (\Delta c)^2]. \tag{6..15}$$

The last term in the right hand side of the momentum equation, $\rho_2 A_2 (\Delta c)^2$, is small by virtue of the choice of control surface and we can neglect it. Doing this and grouping the terms on the right hand side of Eq. (6.15) in a different manner, we have

$$c_0[\rho_0 A_0 c_0] = c_0[\rho_0 A_1 c_0 + \rho_2 A_2 (c_0 - \Delta c)] + \{-\text{Drag} - \rho_2 A_2 c_0 \Delta c\}.$$

The terms in the square brackets on both sides of this equation are the continuity equation multiplied by c_0 . They thus sum to zero leaving the curly bracketed terms as

$$\text{Drag} = -\rho_2 A_2 c_0 \Delta c. \tag{6..16}$$

The wake mass flow is $\rho_2 A_2 c_2 = \rho_2 A_2 (c_0 - \Delta c)$. All this flow has a velocity defect (compared to the free stream) of Δc , so that the defect in flux of momentum (the mass flow in the wake times the velocity defect) is, to first order in Δc ,

$$\text{Momentum defect in wake } = -\rho_2 A_2 c_0 \Delta c = \text{Drag}.$$

The combined first and second law gives us a means of relating the entropy and velocity:

$$T ds = dh - dP/\rho.$$

The pressure is uniform ($dP = 0$) at the downstream station. There is no net shaft work or heat transfer to the wake so that the mass flux of stagnation enthalpy is constant. We can also approximate that the condition of constant stagnation enthalpy holds locally on all streamlines. Applying both of these to the combined first and second law yields

$$T ds = dh_t - c dc.$$

For the present situation, $dh_t = 0$; $cdc = c_0 \Delta c$, so that

$$T_0 \Delta s = -c_0 \Delta c. \tag{6..17}$$

In Equation (6.17) the upstream temperature is used because differences between wake quantities and upstream quantities are small at the downstream control station. The entropy can be related to the drag as

$$\text{Drag} = \rho_2 A_2 T_0 \Delta s. \tag{6..18}$$

The quantity $\rho_2 A_2 c_0 \Delta s$ is the entropy flux (mass flux times the entropy increase per unit mass; in the general case we would express this by an integral over the locally varying wake velocity and density). The power needed to propel the vehicle is the product of $\text{drag} \times \text{flight speed}$, $\text{Drag} \times c_0$. From Eq. (6.18), this can be related to the entropy flux in the wake to yield a compact expression for the propulsive power needed in terms of the wake entropy flux:

$$\text{Propulsive power needed} = T_0(\rho_2 A_2 c_0 \Delta s) = T_0 \times \text{Entropy flux in wake}. \tag{6..19}$$

This amount of work is dissipated per unit time in connection with sustaining the vehicle motion. Equation (6.19) is another demonstration of the relation between lost work and entropy generation, in this case manifested as power that needs to be supplied because of dissipation in the wake.

Muddy Points

Is it safe to say that entropy is the tendency for a system to go into disorder? (MP 6.21)

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