static is used to label the *thermodynamic properties* of the gas (p , T , etc.), and these are no
frame dependent.

Thus in our re-entry vehicle example, looking at the still atmosphere from the vehicle frame
a stagnation temperature *hotter* than the atmospheric (static) temperature. If we look at the
still atmosphere from a stationary frame, the stagnation temperature is *the same as* the stat
temperature.

2.5.3.2 Example

For the case shown below, a jet engine is sitting motionless on the ground prior to take-off. A
entrained into the engine by the compressor. The inlet can be assumed to be frictionless and
adiabatic.

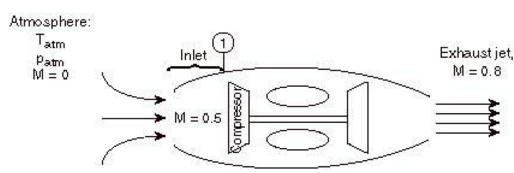

Figure 2.12: A stationary gas turbine drawing air in from the atmosphere

Considering the state of the gas within the inlet, prior to passage into the compressor, as sta
and working in the *reference frame of the motionless airplane*:

1. Is T_{t1} greater than, less than, or equal to T_{atm} ?

   The stagnation temperature of the atmosphere, $T_{\text{t,atm}}$, is equal to T_{atm} since it is moving th
   speed as the reference frame (the motionless airplane). The steady flow energy equation
   that if there is no heat or shaft work (the case for our adiabatic inlet) the stagnation enth
   (and thus stagnation temperature for constant c_p) remains unchanged. Thus $T_{\text{t1}} = T_{\text{t,atm}} = T_{\text{at}}$

2. Is T_1 greater than, less than, or equal to T_{atm} ?

   If $T_{\text{t1}} = T_{\text{atm}}$ then $T_1 < T_{\text{atm}}$ since the flow is moving at station 1 and therefore some of the tota
   energy is composed of kinetic energy (at the expense of internal energy, thus lowering T_1

3. Is p_{t1} greater than, less than, or equal to p_{atm} ?

   Equal, by the same argument as 1.

4. Is p_1 greater than, less than, or equal to p_{atm} ?

   Less than, by the same argument as 2.

2.5.3.3 Steady Flow Energy Equation in terms of Stagnation Enthalpy

The form of the ``Steady Flow Energy Equation'' (SFEE) that we will most commonly use is

Equation 2.11 written in terms of stagnation quantities, and neglecting chemical and potenti energies,

$$\text{Steady Flow Energy Equation:} \qquad \dot{Q}_{\text{cv}} - \dot{W}_{\text{shaft}} = \dot{m}(h_{te} - h_{ti}).$$

The steady flow energy equation finds much use in the analysis of power and propulsion dev and other fluid machinery. Note the prominent role of enthalpy.

Muddy Points

What is the difference between enthalpy and stagnation enthalpy? (MP 2.8)

2.5.4 Example Applications of the First Law of Thermodynamics

[VW, S& B: 6.4]

2.5.4.1 Tank Filling

Using what we have just learned we can attack the tank filling problem solved in Section 2.3 an alternate point of view using the control volume form of the first law. In this problem the s work is zero, and the heat transfer, kinetic energy changes, and potential energy changes ar neglected. In addition there is no exit mass flow.

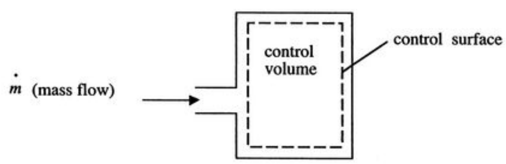

Figure 2.13: A control volume approach to the tank filling problem

The control volume form of the first law is therefore

$$\frac{dU}{dt} = \dot{m}_i h_i.$$

The equation of mass conservation is

$$\frac{dm}{dt} = \dot{m}_i.$$

Combining we have

$$\frac{dU}{dt} = \frac{dm}{dt}h_i.$$

Integrating from the initial time to the final time (the incoming enthalpy is constant) and usir $U = mu$ gives the result $u_{\text{final}} = h_i = h_0$ as before.

2.5.4.2 Flow through a rocket nozzle

A liquid bi-propellant rocket consists of a thrust chamber and nozzle and some means for for the liquid propellants into the chamber where they react, converting chemical energy to ther energy.

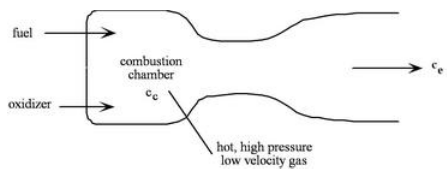

Figure 2.14: Flow through a rocket nozzle

Once the rocket is operating we can assume that all of the *flow processes are steady*, so it is appropriate to use the steady flow energy equation. Also, for now we will assume that the ga behaves as a perfect gas with constant specific heats, though in general this is a poor approximation. There is *no external work*, and we assume that the flow is *adiabatic*. We defir control volume as going between location c , in the chamber, and location e , at the exit, and write the First Law as

$$q_{\text{c-e}} - w_{\text{s,c-e}} = h_{te} - h_{tc} \qquad \text{which becomes} \qquad h_{te} = h_{tc}$$

or

$$c_p T_c + \frac{c_c^2}{2} = c_p T_e + \frac{c_e^2}{2},$$

Therefore

$$c_e = \sqrt{2c_p(T_c - T_e)}.$$

If we assume quasi-static, adiabatic expansion then

$$\frac{T_e}{T_c} = \left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}$$

so

$$c_e = \sqrt{2c_pT_c\left[1-\left(\frac{p_e}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]}.$$

T_c and p_c , the conditions in the combustion chamber, are set by propellants, and p_e is the ext static pressure.

2.5.4.3 Power to drive a gas turbine compressor

Consider for example the PW4084 pictured in Figure 2.15. The engine is designed to produce 84,000 lbs of thrust at takeoff. The engine is a two-spool design. The fan and low pressure compressor are driven by the low pressure turbine. The high pressure compressor is driven b high pressure turbine. We wish to find the total shaft work required to drive the compression system.

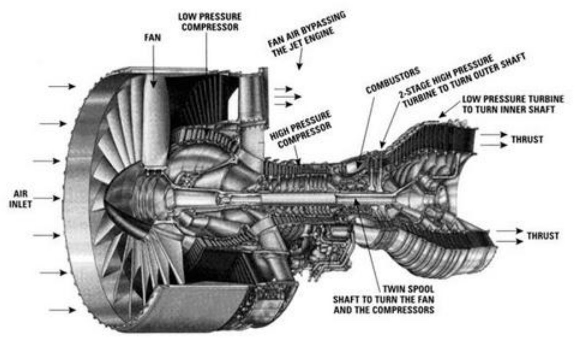

Figure 2.15: The Pratt and Whitney 4084 (drawing courtesy of Pratt and Whitney)

$$\begin{aligned}
\pi_f &= \text{total pressure ratio across the fan}\\
\pi_c &= \text{total pressure ratio across the fan + compressor}\\
\dot m_f &= 610\ \text{kg/s}\\
\dot m_{\text{core}} &= 120\ \text{kg/s}\\
T_{\text{inlet}} &= 300\ \text{K}.
\end{aligned}$$

We define our control volume to encompass the compression system, from the front of the fa the back of the fan and high pressure compressor, with the shaft cutting through the back si the control volume. Heat transfer from the gas streams is negligible, so we write the First La (steady flow energy equation) as:

$$\cancelto{0}{\dot Q} - \dot W_s = \dot m(h_{t2} - h_{t1}).$$

For this problem we must consider two streams, the fan stream, f , and the core stream, c :

$$-\dot{W}_s = \dot{m}_f \Delta h_{t,f} + \dot{m}_c \Delta h_{t,c}$$
$$= \dot{m}_f c_p \Delta T_{t,f} + \dot{m}_c c_p \Delta T_{t,c}$$

We obtain the temperature change by assuming that the compression process is quasi-static adiabatic,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

then

$$\left(\frac{T_{t2}}{T_{t1}}\right)_{\text{fan}} = \pi_f^{\frac{\gamma-1}{\gamma}} = 1.1 \Rightarrow \Delta T_{t,\text{fan}} = 30 \text{ K}$$

$$\left(\frac{T_{t2}}{T_{t1}}\right)_{\text{core}} = \pi_{\text{core}}^{\frac{\gamma-1}{\gamma}} = 3.0 \Rightarrow \Delta T_{t,\text{core}} = 600 \text{ K}$$

Substituting these values into the expression for the first law above, along with estimates of obtain

$$-\dot{W}_s = 610 \text{ kg/s} \times 30 \text{ K} \times 1008 \text{ J/kg-K} + 120 \text{ kg/s} \times 600 \text{ K} \times 1008 \text{ J/kg-K}$$
$$= -91 \times 10^6 \text{ J/s}$$
$$= -91 \text{ Megawatts} \qquad \text{negative sign implies work done on the fluid}$$

Note that $1 \text{ Hp} = 745 \text{ watts}$. If a car engine has $\approx 110 \text{ Hp} = 8.2 \times 10^4 \text{ watts}$, then the power needed to driv compressor is equivalent to 1,110 automobile engines. All of this power is generated by the pressure and high pressure turbines.

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