$\log \sin x$

Jordan Bell

April 6, 2015

That

$$\int_0^{2\pi} \log|1 - e^{i\theta}| d\theta = 0.$$

is used in the proof of Jensen's formula.

Euler [3, §§35–37] E393, [4, §21], [5] E499

Class number formula

Koyama and Kurokawa [8]

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complex analysis: Rudin, Real and Complex, p. 307

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sect. 192 of Introductio in analysin, chap. 11

cotangent series: E128

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[6, p. 8]

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