The Euler-Maclaurin summation formula

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Whiteside [62, pp. 44, 257]
Newton and Collins [8, pp. 186, 199]
Domingues [12, p. 44]
Todhunter [54, p. 192]
Estrada and Kanwal [16, p. 36]
Bourbaki [8, Chapter VI]
[17, pp. 45, 160, 337, 475, 531]
[52, pp. XL–XLIX]
Stirling [57, p. 274]
Euler correspondence R. 1998, 236; p. 53, 113, 137, 433
Institutiones calculi differentialis, E212
E19, E20, E25, E43, E47, E55, E125, E130, E247, E352, E368, E393, E432, E642, E746

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