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Professor Paul Humke
Editor in Chief
Real Analysis Exchange

Dear Professor Humke,

I am submitting my paper, “Sums, series, and products in Diophantine approximation”, to Real Analysis Exchange.

I investigated the behavior of the partial products of $\prod_{k=1}^{\infty} (1-x^k)$ and $\prod_{k=1}^{\infty} (1+x^k)$. In this paper I deal with these partial products as trigonometric polynomials, organize what is known about them, and prove several new results about the L^p norms of these functions, their Fourier coefficients, and their analytic behavior.

These products are intimately related to $\prod_{k=1}^n \sin(k\theta)$ and $\prod_{k=1}^n \cos(k\theta)$. The behavior of the Dirichlet kernel, which amounts to the sum $\sum_{k=1}^n \cos(k\theta)$, is well understood and is of substantial interest, and it’s natural to look at the behavior of the products of these terms.

The main original results of the paper are Theorem 3, Theorem 5, and Theorem 7. Aside from the original results, I hope readers will find the paper useful as a survey of the topic. (In particular, Theorem 2 can be shown by modifying a proof of Wright so I didn’t mention it as one of my original results, but it has not been stated in the literature before and Theorem 2’s statement is only clear once you have read Wright’s proof, not just seen the statement of Wright’s result.) I have paid special attention to exposition, motivation, and readability in this paper.

Jeffrey Shallit and Hervé Queffélec

Sincerely,

Jordan Bell