Module 1

Alternative Approaches to Valuation and Investment

Defining the Diversification Benefit (It's All About the Eggs and Baskets…)

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Motivation

In our last session we defined the expected return and risk of a two asset portfolio as:

$$E(R_{Portfolio}) = w_1 E(R_1) + w_2 E(R_2)$$

and

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}}$$

In this session we are going to show – graphically – the diversification benefit in action.

We will also consider what happens when we add more than 2 assets into a portfolio.

The risk of our portfolio

Consider risk and return for different weights:

w_{AmAir}	$w_{Kellogg's}$	$E(R_P)$	σ_P	σ_{Avg}
1.0	0	13.13%	38.53%	38.53%
0.9	0.1	12.56%	35.03%	36.39%
0.8	0.2	11.99%	31.64%	34.25%
0.7	0.3	11.41%	28.37%	32.11%
0.6	0.4	10.84%	25.30%	29.97%
0.5	0.5	10.27%	22.48%	27.84%
0.4	0.6	9.70%	20.04%	25.70%
0.3	0.7	9.13%	18.13%	23.56%
0.1	0.9	7.98%	16.57%	19.28%
0.05	0.95	7.70%	16.75%	18.21%
0	1.0	7.41%	17.14%	17.14%

No diversification benefit! (1.0 / 0)

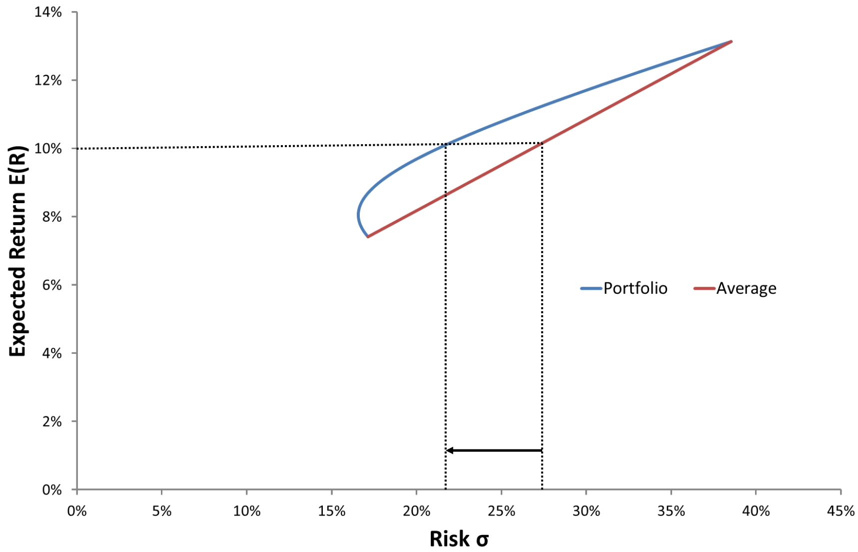
Diversification benefit! (0.9 / 0.1 through 0.05 / 0.95)

Inefficient portfolios (0.05 / 0.95 and 0 / 1.0)

No diversification benefit! (0 / 1.0)

The risk of our portfolio

We can illustrate the relationship between risk and return as follows:



What determines the diversification benefit?

The key element in determining the diversification benefit is the Correlation Coefficient (ρ).

The lower the value of ρ [as in further away from +1] the smaller the portfolio's risk at any given level of expected return.

What sort of factors might impact upon the value of ρ between assets?

- Industry
- Country
- Strategy:
  e.g. leverage levels, hedging decisions etc.

Correlation coefficient between different assets

Consider the following matrix of correlation coefficients estimated using daily returns from U.S. listed shares from 1st Jan to 31st Dec 2014:

	Kellogg's	Exxon Mobil	Walmart	Kraft	Microsoft	S&P500	American Airlines	Trip Advisor	Facebook
Kellogg's	1.0000								
Exxon Mobil	0.3865	1.0000							
Walmart	0.3510	0.2107	1.0000						
Kraft	0.6487	0.4612	0.4507	1.0000					
Microsoft	0.2962	0.3590	0.3691	0.3845	1.0000				
S&P500	0.5112	0.6612	0.4696	0.6057	0.5735	1.0000			
American Airlines	0.1844	0.1089	0.2362	0.2499	0.2195	0.4677	1.0000		
Trip Advisor	0.1949	0.3018	0.1957	0.2691	0.3544	0.5800	0.3885	1.0000	
Facebook	0.1601	0.1990	0.1686	0.1945	0.2915	0.5557	0.3558	0.5048	1.0000

What happens when there are more than two assets in a portfolio?

Recall that the risk of a two asset portfolio is:

$$\sigma_P = \sqrt{\underbrace{w_1^2\sigma_1^2 + w_2^2\sigma_2^2}_{\text{2 stand-alone risk measures}} + \underbrace{2w_1w_2\sigma_{1,2}}_{\text{2 covariance risk measures}}}$$

For a three-asset portfolio we have:

$$\sigma_P = \sqrt{\underbrace{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2}_{\text{3 stand-alone risk measures}} + \underbrace{2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_2w_3\sigma_{2,3}}_{\text{6 covariance risk measures}}}$$

Multi-asset demonstration

Estimate the expected return and risk of the following portfolio:

	Kellogg's	Facebook	Microsoft
Weight	40%	30%	30%
E(R)	7.41%	14.25%	8.73%
σ	17.14%	40.80%	19.00%
ρ	As per the previous slide		

$$E(R_{Portfolio}) = w_1E(R_1) + w_2E(R_2) + w_3E(R_3)$$

$$E(R_{Portfolio}) = (0.4)(7.41) + (0.3)(14.25) + (0.3)(8.73)$$

$$E(R_{Portfolio}) = 0.09858 = 9.858\% \textit{ per annum}$$

Multi-asset demonstration

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\sigma_{1,2} + 2w_1w_3\sigma_{1,3} + 2w_2w_3\sigma_{2,3}}$$

$$\sigma_P = \sqrt{\begin{array}{l}(0.4)^2(17.14)^2 + (0.3)^2(40.8)^2 + (0.3)^2(19)^2 + 2(0.4)(0.3)(17.14)(40.8)(0.1601) \\ +2(0.3)(0.3)(40.8)(19)(0.2915) + 2(0.3)(0.3)(17.14)(19)(0.2962)\end{array}}$$

$$\sigma_P = \sqrt{320.01} = 17.89\%\ per\ annum$$

compared with

$$\sigma_{Avg} = (0.4)(17.14) + (0.3)(40.8) + (0.3)(19)$$

$$\sigma_{Avg} = 24.80\%\ per\ annum$$

$$\sigma_{i,j} = \sigma_i \times \sigma_j \times \rho_{i,j}$$

Multi-asset portfolios

As you keep on adding more and more assets to the portfolio – the number of covariance risk factors start to massively outnumber the stand-alone risk factors.

Number of assets	Number of σ_i terms	Number of $\sigma_{i,j}$ terms
2	2	2
3	3	6
4	4	12
5	5	20
10	10	90
50	50	2450

Tip: The number of $\sigma_{i,j}$ terms in an n-asset portfolio is n^2-n.

Something to ponder…

Imagine you have an investment portfolio consisting of 200 of the largest listed companies in the U.S. and are considering adding another asset to the portfolio.

Do you really need to estimate hundreds of measures of σ and thousands of measures of ρ?

Probably not – you could treat the existing portfolio of 200 stocks as a single asset and then simply estimate:

$\sigma_{\text{Existing Port}}$, $\sigma_{\text{New Asset}}$, $\rho_{\text{Existing Port, New Asset}}$

Summary

In this session, we:

- Demonstrated the diversification benefit graphically
- Highlighted the factors that influenced the value of ρ and hence the benefits of diversification
- Worked out how to estimate the risk of a portfolio consisting of more than two assets
- Highlighted how stand-alone risk became increasingly less-important relative to covariance risk as a portfolio grew in size.

Module Summary

1. Measuring stand-alone risk
2. Alternative attitudes towards risk
3. Combining assets into portfolios
4. Defining the diversification benefit

Where to next?

Defining and Measuring Risk → **Linking Risk with Expected Return** → **Using Financial Statements to Estimate Cost of Capital** → **Real Options Analysis**

Source list

Slide 3, 4, 6, 8, 10:

Tables and figures created by Sean Pinder © The University of Melbourne, using data obtained from Yahoo! Finance (June 2015, https://au.finance.yahoo.com).