THE UNIVERSITY OF MELBOURNE

Module 4

Alternative Approaches to Valuation and Investment

Approximation of Real Option Values Using Decision Trees (Options do grow on trees!)

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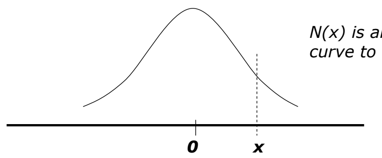
Option valuation: Financial Options

To value options written on shares – we have developed a range of very fancy techniques such as:

Black-Scholes-Merton Option Pricing Model

$$C = P(N(d_1)) - \mathrm{PV}(X)(N(d_2))$$

$$\boxed{d_1 = \frac{\ln\left(P/X\right) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}} \qquad \boxed{d_2 = d_1 - \sigma\sqrt{t}}$$



N(x) is area under the curve to the left of x

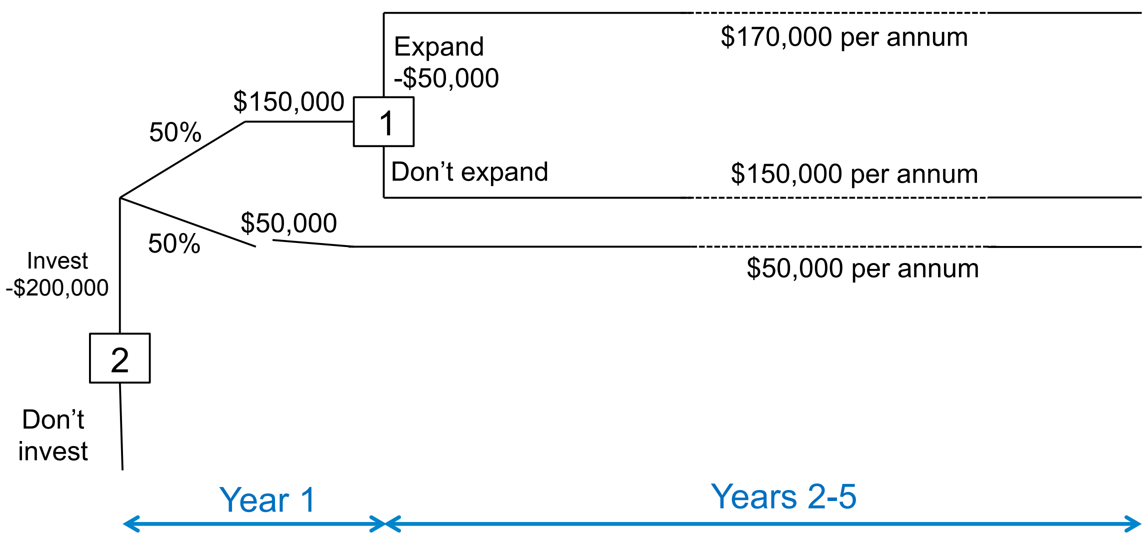
Decision trees: Example

A DCF-based technique that allows you to assess a project that may involve a **sequence** of decisions through time.

Example: You are trying to decide whether to invest $200,000 up-front in a new retail outlet – with a life of 5 years.

- There is a 50% chance that the retail outlet will experience high demand in the 1st year – generating $150,000. Demand will continue to be high for the remaining 4 years.
- If demand is low in year 1, it will remain low – with only $50,000 generated each year.
- If demand is high in the 1st year then the store will have the option to spend an additional $50,000 to expand its range of products – this will increase the expected net cash flows for the next 4 years to $170,000 per annum.
- The discount rate is 10% per annum and assume all cash flows occur at year-end (except for initial investments).

Decision trees



Decision trees: Example

Step One: Assess the most distant decision first

$$NPV_1^{Expand} = -\$50{,}000 + \frac{\$170{,}000}{(1.10)^1} + \frac{\$170{,}000}{(1.10)^2} + \frac{\$170{,}000}{(1.10)^3} + \frac{\$170{,}000}{(1.10)^4}$$

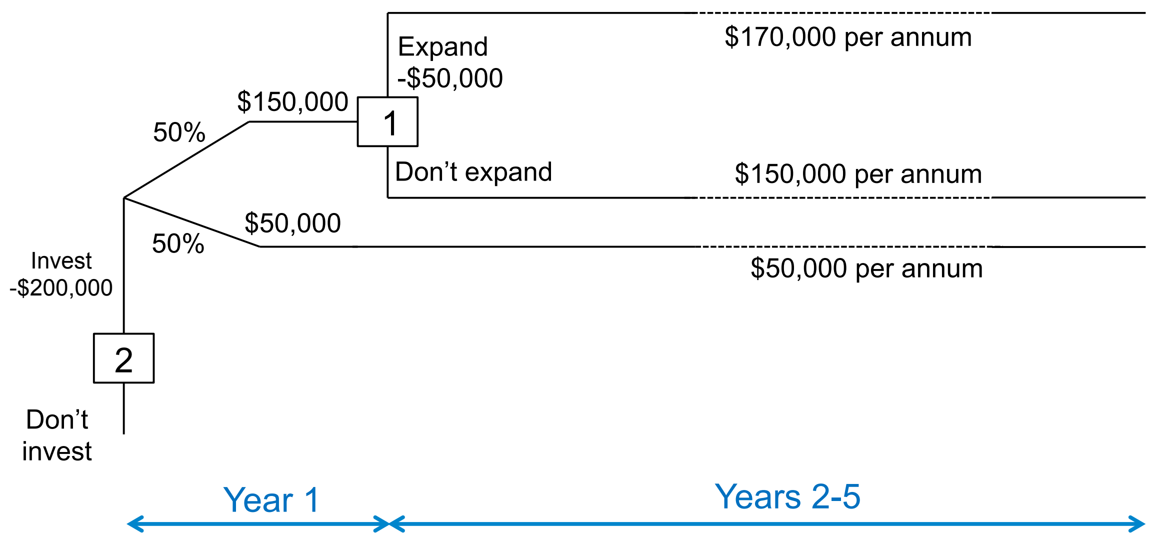
$$NPV_1^{Expand} = \$488{,}877$$

$$NPV_1^{Not\ Expand} = \frac{\$150{,}000}{(1.10)^1} + \frac{\$150{,}000}{(1.10)^2} + \frac{\$150{,}000}{(1.10)^3} + \frac{\$150{,}000}{(1.10)^4}$$

$$NPV_1^{Not\ Expand} = \$475{,}480$$

Therefore – we know that if demand was to be high in the first year of operations – we **would** expand operations.

Decision trees



Decision trees: Example

Step Two: Assess the next most distant decision: Invest or don't invest

$$NPV_0 = -\$200,000 + (0.50)\left(\frac{\$50,000}{(1.10)^1} + \frac{\$50,000}{(1.10)^2} + \frac{\$50,000}{(1.10)^3} + \frac{\$50,000}{(1.10)^4} + \frac{\$50,000}{(1.10)^5}\right)$$

$$+(0.50)\left(\frac{\$150,000}{(1.10)^1} + \frac{\$488,877}{(1.10)^1}\right)$$

$$NPV_0 = -\$200,000 + (0.50)(\$189,539) + (0.50)(\$580,797)$$

$$NPV_0 = -\$200,000 + \$94,770 + \$290,399$$

$$NPV_0 = \$185,169$$

Therefore – we **would** invest in the project.

Using decision trees for real options analysis

Steps

1. Utilize a standard decision tree to estimate the value of the project.
2. Re-structure the decision tree to remove the real option you wish to value. Estimate the value of the modified decision tree.
3. The difference between the value of your estimates in 1. and 2. gives you an approximation of the value of the real option embedded in your project.

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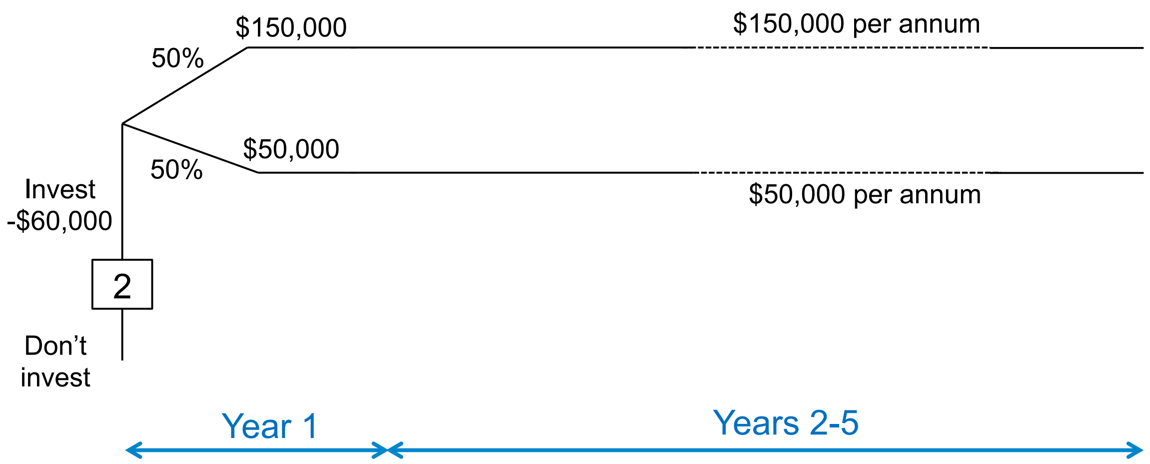
Decision trees



Decision trees: Example

Step Two: Assess the next most distant decision: Invest or don't invest

$$NPV_0^{No-option} = -\$200{,}000 + (0.50)\left(\frac{\$150{,}000}{(1.10)^1} + \frac{\$150{,}000}{(1.10)^2} + \frac{\$150{,}000}{(1.10)^3} + \frac{\$150{,}000}{(1.10)^4} + \frac{\$150{,}000}{(1.10)^5}\right)$$

$$+(0.50)\left(\frac{\$50{,}000}{(1.10)^1} + \frac{\$50{,}000}{(1.10)^2} + \frac{\$50{,}000}{(1.10)^3} + \frac{\$50{,}000}{(1.10)^4} + \frac{\$50{,}000}{(1.10)^5}\right)$$

$$NPV_0 = -\$200{,}000 + (0.50)(\$568{,}618) + (0.50)(\$189{,}539)$$

$$NPV_0 = -\$200{,}000 + \$284{,}309 + \$94{,}770$$

$$NPV_0 = \$179{,}079$$

Using decision trees for real options analysis

Steps

3. The difference between the value of your estimates in 1. and 2. gives you an approximation of the value of the real option embedded in your project.

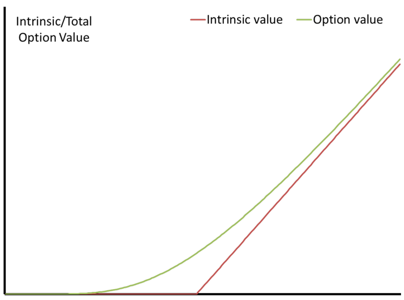
$$Option\ value = NPV_0^{With\ option} - NPV_0^{No-option}$$

$$Option\ value = \$185{,}169 - \$179{,}079 = \$6{,}090$$

Decision trees are just an approximation

Decision trees only provide an approximation of the value of a real option as – by their very nature – they allow only for a discrete range of outcomes.

If the decision tree does not provide a single point where a particular option would be exercised – then the estimate of the option's value would be zero.



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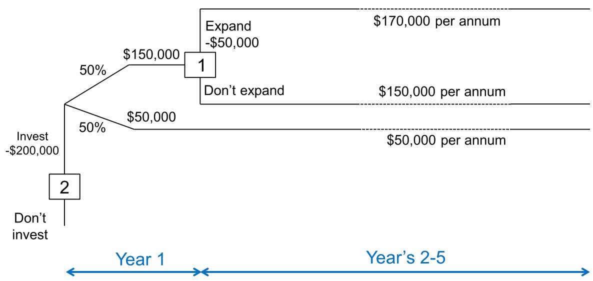

Summary

We have considered:

- How to use decision trees to evaluate project's that involve a sequence of decisions to be made over time
- How to adapt decision-trees to allow for the valuation of real options.

Does anyone use Real Options Analysis?

Source list

All diagrams (slides 2, 4, 6, 9 and 13) and examples produced by Sean Pinder. © The University of Melbourne.