Module 1

Alternative Approaches to Valuation and Investment

Portfolio Return and Risk
(The more the merrier…)

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Motivation

Most investors hold assets as part of a diversified portfolio.

Important for us to work out two things:

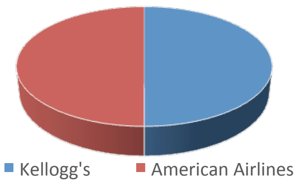
1. What happens to expected returns when you combine assets?
2. What happens to risk?

Our illustration

Consider the following assets, where we have estimated expected returns and risk:

Asset	Risk (σ)	Expected return E(r)
Kellogg's	17.14%	7.41%
American Airlines	38.53%	13.13%

Now let's assume that we form a portfolio consisting of 50% of our wealth invested in Kellogg's shares and 50% in American Airlines shares.



Expected return of a portfolio

The expected return of a portfolio is simply the **weighted average** expected return of the assets in the portfolio – where the weights reflect the proportion of wealth invested in each asset.

$$E(R_{Portfolio}) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3) + ... + w_n E(R_n)$$

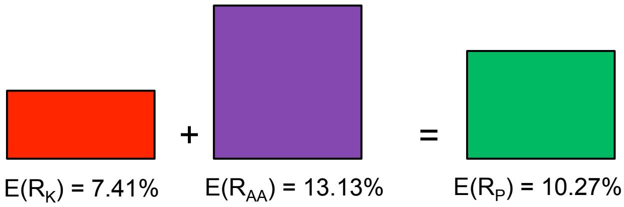
So for our portfolio of two assets:

$$E(R_{Portfolio}) = \underbrace{(0.50)}_{w_{Kellogg's}}\underbrace{(0.0741)}_{E(R)_{Kellogg's}} + \underbrace{(0.50)}_{w_{AmAirlines}}\underbrace{(0.1313)}_{E(R)_{AmAirlines}}$$

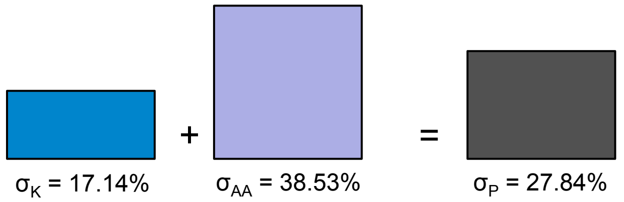
$$E(R_{Portfolio}) = 0.1027 = 10.27\%$$

Averaging return… averaging risk?

So, the expected return for an asset is simply the average of the E(R) of the assets in the portfolio.

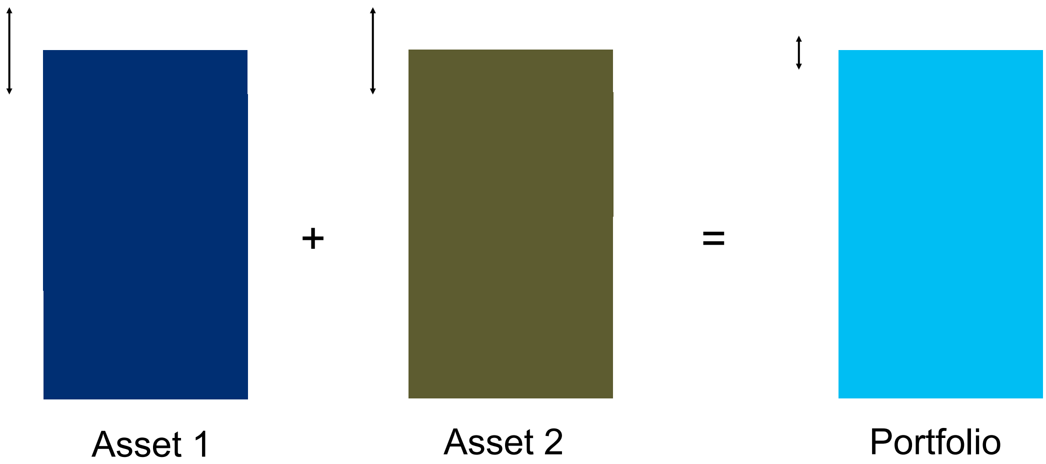


So, what about risk? It's tempting to think that risk would work in the same way right? That is:



Averaging return… averaging risk?

Imagine two assets with the same level of expected return and the same level of risk.



Portfolio risk and the correlation coefficient – rho (ρ)

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{1,2}}$$

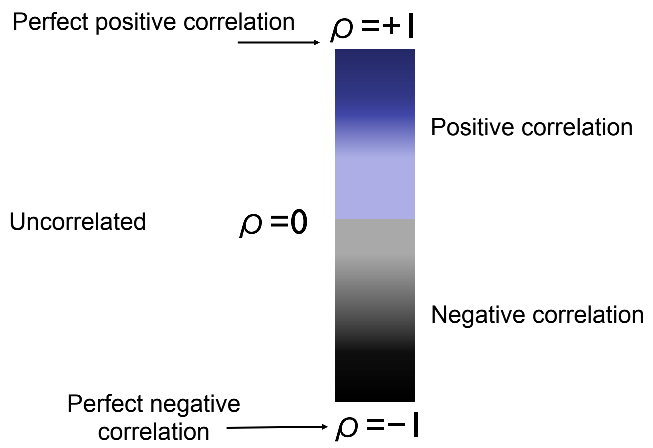
$$where\ \sigma_{1,2} = \sigma_1 \times \sigma_2 \times \rho_{1,2}$$

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}}$$

- The correlation coefficient, ρ, is **crucial** in determining the risk of a portfolio.
- It measures the degree of **correlation** between the returns of the two assets in the portfolio.
- Simple maths tells us that the higher the value of ρ the greater the risk of the portfolio...
  Let's see why...

The correlation coefficient – rho (ρ)

The correlation coefficient can take on a value anywhere between +1 and −1.



The correlation coefficient – rho (ρ)

Perfect Positive Correlation: $\rho_{1,2} = +1$

- A return of X% in Asset 1 will always be matched by a return of Y% in Asset 2 (in the same direction) where the ratio of X to Y is constant.

Perfect Negative Correlation: $\rho_{1,2} = -1$

- A return of X% in one asset will always be matched by a return of Y% (in the opposite direction) in Asset 2 where the ratio of X to Y is constant.

Zero Correlation: $\rho_{1,2} = 0$

- Observing the return for Asset 1 gives us no information about the return for Asset 2.

The risk of our portfolio

The way that the we estimate the **correlation coefficient** between the returns of two variables is very similar to how we measured the **standard deviation** of returns in the previous session:

1. Download each asset's price series (from Yahoo finance or similar) into Excel
2. Calculate returns for each asset on a daily basis
3. Estimate the correlation coefficient using the function "=CORREL(..)"

When I do this using Yahoo finance price series data from 1 January to 31 December 2014 I end up with $\rho_{K,AA}$= 0.1844.

The risk of our portfolio

The risk of our portfolio
with the following characteristics:

w_1	0.50
w_2	0.50
$\sigma_{Kellogg's}$	17.14%
$\sigma_{American\ Airlines}$	38.53%
$\rho_{Kellogg's,American\ Airlines}$	0.1844

$$\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}}$$

$$\sigma_P = \sqrt{(0.5)^2(0.1714)^2 + (0.5)^2(0.3853)^2 + 2(0.5)(0.5)(0.1714)(0.3853)(0.1844)}$$

$$\sigma_P = \sqrt{(0.007344) + (0.037114) + (0.006089)}$$

$$\sigma_P = \sqrt{0.050547} = 0.2248 = 22.48\%$$

The risk of our portfolio

Notice how we have been able to reduce the risk of the portfolio below the weighted average risk of the individual assets!

That is:

$$\sigma_{Average} = w_1\sigma_1 + w_2\sigma_2$$

$$\sigma_{Average} = (0.5)(0.1714) + (0.5)(0.3853)$$

$$\left.\begin{aligned}\sigma_{Average} &= 27.84\% \\ \sigma_{Portfolio} &= 22.48\%\end{aligned}\right\}$$

Difference is known as the **diversification** benefit!

Summary

The expected return of a portfolio is simply the weighted average of the expected returns of the assets in the portfolio.

The risk of a portfolio is determined by three factors:

1. Proportion invested in each asset
2. Risk of individual assets
3. Correlation between returns of assets.

The risk of a portfolio is less than the weighted average risk of the assets in the portfolio – provided the assets in the portfolio are less than perfectly positively correlated – this is the **diversification benefit.**

Source list

Slides 6 and 8:

Tables and figures created by Sean Pinder © The University of Melbourne, using fictional data.

Slides 3, 4, 5, 11 and 12:

Figures created by Sean Pinder © The University of Melbourne, using data obtained from Yahoo! Finance (June 2015, https://au.finance.yahoo.com).