

This is an abridged version of *Small-area Public Opinion Estimation Using Gaussian Process Regression and Post-stratification*, for the convenience of the package team.

## 1 MCMC Sampler

Each step of the MCMC sampler:

(1) Sample  $\omega_{it}^{(t)} \sim \text{PG}(n_{ij}, \mu_{ij}^{(t-1)}) \forall i, j$  where  $\text{PG}(\cdot)$  is the Pólya-Gamma density function from the **BayesLogit** package.

- $n_{ij}$  is the group of respondents who have profile  $i \in 1, \dots, N$  answering survey items  $j \in 1, \dots, J$ .
- $\mu_{ij} = \theta_i \beta_j - \alpha_j$

(2) Sample  $\tilde{\beta} \sim \text{N}(m_\beta, V_\beta)$ , with:

- $V_\beta = (\Lambda_{\tilde{\beta}} + \mathbf{X}^\top \Omega_j \mathbf{X})^{-1}$
- $m_\beta = V_\beta (\mathbf{X}^\top \kappa_j)$
- $\Omega_j = \text{diag}(\{\omega_{ij}^{(t)}\}_{i=1}^N)$
- $\mathbf{X}$  has rows  $\mathbf{x}_i = [\theta_i^{t-1}, -1]$
- $\kappa_j = [\kappa_{1j}, \dots, \kappa_{nj}]^\top$
- $\Lambda_{\tilde{\beta}} = \text{diag}(0.1)$

(3) Sample  $\theta_i^{(t)} \sim \text{N}(m_\theta, V_\theta)$ , with:

- $V_\theta = (\sigma_\theta^{-2} + \beta^{(t)\top} \beta^{(t)})^{-1}$
- $m_\theta = V_\theta (f_i^{(t-1)} / \sigma_\theta^2 + \beta^{(t)\top} \tilde{\mathbf{y}}_i)$
- $\tilde{\mathbf{y}} = [\{\kappa_{ij} / \omega_{ij}^{(t)} + \alpha_j^{(t)}\}_{j=1}^J]^\top$

(4) Sample  $\mathbf{f}^{(t)} \sim \text{N}(m_f, V_f)$ , with:

- $V_f = \mathbf{K}_\rho - \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \mathbf{K}_\rho$
- $m_f = \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \boldsymbol{\theta}^{(t)}$
- $\mathbf{K}_\rho = K(\mathbf{Z}|\boldsymbol{\rho})$  is an  $N \times N$  covariance generated using a kernel computed on a  $\mathbf{Z}_{N \times D}$  matrix of demographic features
- $\Sigma_\theta = \sigma_\theta^2 \mathbf{I} = (1.0) \mathbf{I}$



## 2 Parameter Names

|              |   |
|--------------|---|
| w_it         | $\omega_{it}^{(t)}$                           |
| w            | $\{\omega_{ij}^{(t)}\}_{i=1}^N$               |
| w_j          | jth column of $\{\omega_{ij}^{(t)}\}_{i=1}^N$ |
| beta_tilde   | $\tilde{\beta}$                               |
| beta_tilde_j | $\tilde{\beta}$ for response item $j$         |
| V_beta       | $V_{\beta}$                                   |
| m_beta       | $m_{\beta}$                                   |
| Omega_j      | $\Omega_j$                                    |
| X            | $\mathbf{X}$                                  |
| k            | $\kappa_j$                                    |
| Lambda       | $\Lambda_{\tilde{\beta}}$                     |
| V_theta      | $V_{\theta})$                                 |
| sigma2_theta | $\sigma_{\theta}^2$                           |
| m_theta      | $m_{\theta}$                                  |
| f_prior      | $f_i^{(t-1)}$                                 |
| y_tilde      | $\tilde{\mathbf{y}}$                          |
| f_t          | $\mathbf{f}^{(t)}$                            |
| V_f          | $V_f$   |
| m_f          | $m_f$   |
| Kappa_rho    | $\mathbf{K}_{\rho}$                           |
| Sigma_theta  | $\Sigma_{\theta}$                             |