

This is an abridged version of *Small-area Pubic Opinion Estimation Using Gaussian Process Regression and Post-stratification*, for the convenience of the package team.

## 1 MCMC Sampler

Each step of the MCMC sampler:

- (1) Sample  $\omega_{it}^{(t)} \sim \text{PG}(n_{ij}, \mu_{ij}^{(t-1)}) \forall i, j$  where  $\text{PG}(\cdot)$  is the Pólya-Gamma density function from the **BayesLogit** package.
  - $n_{ij}$  is the group of respondents who have profile  $i \in 1, \dots, N$  answering survey items  $j \in 1, \dots, J$ .
  - $\mu_{ij} = \theta_i \beta_j - \alpha_j$
- (2) Sample  $\tilde{\beta} \sim \text{N}(m_\beta, V_\beta)$ , with:
  - $V_\beta = (\Lambda_{\tilde{\beta}} + \mathbf{X}^\top \Omega_j \mathbf{X})^{-1}$
  - $m_\beta = V_\beta (\mathbf{X}^\top \kappa_j)$
  - $\Omega_j = \text{diag}(\{\omega_{ij}^{(t)}\}_{i=1}^N)$
  - $\mathbf{X}$  has rows  $\mathbf{x}_i = [\theta_i^{t-1}, -1]$
  - $\kappa_j = [\kappa_{1j}, \dots, \kappa_{nj}]^\top$
  - $\Lambda_{\tilde{\beta}} = \text{diag}(0.1)$
- (3) Sample  $\theta_i^{(t)} \sim \text{N}(m_\theta, V_\theta)$ , with:
  - $V_\theta = (\sigma_\theta^{-2} + \beta^{(t)\top} \beta^{(t)})^{-1}$
  - $m_\theta = V_\theta (f_i^{(t-1)} / \sigma_\theta^2 + \beta^{(t)\top} \tilde{\mathbf{y}}_i)$
  - $\tilde{\mathbf{y}} = [\{\kappa_{ij} / \omega_{ij}^{(t)} + \alpha_j^{(t)}\}_{j=1}^J]^\top$
- (4) Sample  $\mathbf{f}^{(t)} \sim \text{N}(m_f, V_f)$ , with:
  - $V_f = \mathbf{K}_\rho - \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \mathbf{K}_\rho$
  - $m_f = \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \boldsymbol{\theta}^{(t)}$
  - $\mathbf{K}_\rho = K(\mathbf{Z}|\boldsymbol{\rho})$  is an  $N \times N$  covariance generated using a kernel computed on a  $\mathbf{Z}_{N \times D}$  matrix of demographic features
  - $\Sigma_\theta = \sigma_\theta^2 \mathbf{I} = (1.0) \mathbf{I}$

## 2 Parameter Names

w	$\omega_{it}^{(t)}$
beta_tilde	$\tilde{\beta}$
V_beta	$V_{\beta}$
m_beta	$m_{\beta}$
Omega	$\Omega_j$
X	$\mathbf{X}$
k	$\kappa_j$
Lambda	$\Lambda_{\tilde{\beta}}$
V_theta	$V_{\theta})$
sigma2_theta	$\sigma_{\theta}^2$
m_theta	$m_{\theta}$
f_prior	$f_i^{(t-1)}$
y_tilde	$\tilde{\mathbf{y}}$
f_t	$\mathbf{f}^{(t)}$
V_f	$V_f$
m_f	$m_f$
Kappa_rho	$\mathbf{K}_{\rho}$
Sigma_theta	$\Sigma_{\theta}$