This is an abridged version of *Small-area Pubic Opinion Estimation Using Gaussian Process Regression and Post-stratification*, for the convenience of the package team.

1 MCMC Sampler

Each step of the MCMC sampler:

(1) Sample $\omega_{it}^{(t)} \sim \text{PG}(n_{ij}, \mu_{ij}^{(t-1)}) \forall i,j$ where $\text{PG}(\cdot)$ is the Pólya-Gamma density function from the `BayesLogit` package.

- n_{ij} is the group of respondents who have profile $i \in 1, \dots, N$ answering survey items $j \in 1, \dots, J$.
- $\mu_{ij} = \theta_i \beta_j - \alpha_j$

(2) Sample $\tilde{\boldsymbol{\beta}} \sim \text{N}(m_\beta, V_\beta)$, with:

- $V_\beta = (\boldsymbol{\Lambda}_{\tilde{\boldsymbol{\beta}}} + \boldsymbol{X}^\top \boldsymbol{\Omega}_j \boldsymbol{X})^{-1}$
- $m_\beta = V_\beta(\boldsymbol{X}^\top \boldsymbol{\kappa}_j)$
- $\boldsymbol{\Omega}_j = \text{diag}(\{\omega_{ij}^{(t)}\}_{i=1}^N)$
- $\boldsymbol{X}$ has rows $\boldsymbol{x}_i = [\theta_i^{t-1}, -1]$
- $\boldsymbol{\kappa}_j = [\kappa_{1j}, \dots, \kappa_{nj}]^\top$
- $\boldsymbol{\Lambda}_{\tilde{\boldsymbol{\beta}}} = ?$

(3) Sample $\theta_i^{(t)} \sim \text{N}(m_\theta, V_\theta)$, with:

- $V_\theta = (\sigma_\theta^{-2} + \boldsymbol{\beta}^{(t)\top} \boldsymbol{\beta}^{(t)})^{-1}$
- $m_\theta = V_\theta(f_i^{(t-1)}/\sigma_\theta^2 + \boldsymbol{\beta}^{(t)\top} \tilde{\boldsymbol{y}}_i)$
- $\tilde{\boldsymbol{y}} = [\{\kappa_{ij}/\omega_{ij}^{(t)} + \alpha_j^{(t)}\}_{j=1}^J]^\top$

(4) Sample $\mathbf{f}^{(t)} \sim \text{N}(m_f, V_f)$, with:

- $V_f = \mathbf{K}_\rho - \mathbf{K}_\rho(\mathbf{K}_\rho + \boldsymbol{\Sigma}_\theta^{-1})^{-1} \mathbf{K}_\rho$
- $m_f = \mathbf{K}_\rho(\mathbf{K}_\rho + \boldsymbol{\Sigma}_\theta^{-1})^{-1} \boldsymbol{\theta}^{(t)}$
- $\mathbf{K}_\rho = K(\mathbf{Z}|\boldsymbol{\rho})$ is an $N \times N$ covariance generated using a kernel computed on a $\mathbf{Z}_{N \times D}$ matrix of demographic features
- $\boldsymbol{\Sigma}_\theta = \sigma_\theta^2 \mathbf{I}$