

This is an abridged version of *Small-area Public Opinion Estimation Using Gaussian Process Regression and Post-stratification*, for the convenience of the package team.

1 MCMC Sampler

Each step of the MCMC sampler:

(1) Sample $\omega_{it}^{(t)} \sim \text{PG}(n_{ij}, \mu_{ij}^{(t-1)}) \forall i, j$ where $\text{PG}(\cdot)$ is the Pólya-Gamma density function from the **BayesLogit** package.

- n_{ij} is the group of respondents who have profile $i \in 1, \dots, N$ answering survey items $j \in 1, \dots, J$.
- $\mu_{ij} = \theta_i \beta_j - \alpha_j$

(2) Sample $\tilde{\beta} \sim \text{N}(m_\beta, V_\beta)$, with:

- $V_\beta = (\Lambda_{\tilde{\beta}} + \mathbf{X}^\top \Omega_j \mathbf{X})^{-1}$
- $m_\beta = V_\beta (\mathbf{X}^\top \kappa_j)$
- $\Omega_j = \text{diag}(\{\omega_{ij}^{(t)}\}_{i=1}^N)$
- \mathbf{X} has rows $\mathbf{x}_i = [\theta_i^{t-1}, -1]$
- $\kappa_j = [\kappa_{1j}, \dots, \kappa_{nj}]^\top$
- $\Lambda_{\tilde{\beta}} = ? = \text{diag}(0.1)$

(3) Sample $\theta_i^{(t)} \sim \text{N}(m_\theta, V_\theta)$, with:

- $V_\theta = (\sigma_\theta^{-2} + \beta^{(t)\top} \beta^{(t)})^{-1}$
- $m_\theta = V_\theta (f_i^{(t-1)} / \sigma_\theta^2 + \beta^{(t)\top} \tilde{\mathbf{y}}_i)$
- $\tilde{\mathbf{y}} = [\{\kappa_{ij} / \omega_{ij}^{(t)} + \alpha_j^{(t)}\}_{j=1}^J]^\top$

(4) Sample $\mathbf{f}^{(t)} \sim \text{N}(m_f, V_f)$, with:

- $V_f = \mathbf{K}_\rho - \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \mathbf{K}_\rho$
- $m_f = \mathbf{K}_\rho (\mathbf{K}_\rho + \Sigma_\theta^{-1})^{-1} \boldsymbol{\theta}^{(t)}$
- $\mathbf{K}_\rho = K(\mathbf{Z}|\boldsymbol{\rho})$ is an $N \times N$ covariance generated using a kernel computed on a $\mathbf{Z}_{N \times D}$ matrix of demographic features
- $\Sigma_\theta = \sigma_\theta^2 \mathbf{I} = (1.0) \mathbf{I}$