This is an abridged version of Small-area Pubic Opinion Estimation Using Gaussian Process Regression and Post-stratification, for the convenience of the package team.

1 MCMC Sampler

Each step of the MCMC sampler:

- (1) Sample $\omega_{it}^{(t)} \sim \mathrm{PG}(n_{ij}, \mu_{ij}^{(t-1)}) \forall i, j \text{ where } \mathrm{PG}(\cdot)$ is the Pólya-Gamma density function from the BayesLogit package.
 - n_{ij} is the group of respondents who have profile $i \in 1, ..., N$ answering survey items $j \in 1, ..., J$.
 - $\mu_{ij} = \theta_i \beta_j \alpha_j$
- (2) Sample $\tilde{\boldsymbol{\beta}} \sim N(m_{\beta}, V_{\beta})$, with:

•
$$V_{\beta} = (\boldsymbol{\Lambda}_{\tilde{\boldsymbol{\beta}}} + \boldsymbol{X}^{\top} \boldsymbol{\Omega}_{j} \boldsymbol{X})^{-1}$$

•
$$m_{\beta} = V_{\beta}(\boldsymbol{X}^{\top}\boldsymbol{\kappa}_{j})$$

•
$$\Omega_j = \operatorname{diag}(\{\omega_{ij}^{(t)}\}_{i=1}^N)$$

•
$$X$$
 has rows $x_i = [\theta_i^{t-1}, -1]$

•
$$\boldsymbol{\kappa}_j = [\kappa_{1j}, \dots, \kappa_{nj}]^{\top}$$

•
$$\Lambda_{\tilde{\beta}} = \text{diag}(0.1)$$

(3) Sample $\theta_i^{(t)} \sim N(m_\theta, V_\theta)$, with:

•
$$V_{\theta} = (\sigma_{\theta}^{-2} + \boldsymbol{\beta}^{(t)\top} \boldsymbol{\beta}^{(t)})^{-1}$$

•
$$m_{\theta} = V_{\theta}(f_i^{(t-1)}/\sigma_{\theta}^2 + \boldsymbol{\beta}^{(t)\top} \tilde{\boldsymbol{y}}_i)$$

•
$$\tilde{\boldsymbol{y}} = \left[\left\{ \kappa_{ij} / \omega_{ij}^{(t)} + \alpha_j^{(t)} \right\}_{j=1}^J \right]^\top$$

(4) Sample $\mathbf{f}^{(t)} \sim N(m_f, V_f)$, with:

•
$$V_f = \mathbf{K}_{\rho} - \mathbf{K}_{\rho} (\mathbf{K}_{\rho} + \boldsymbol{\Sigma}_{\theta}^{-1})^{-1} \mathbf{K}_{\rho}$$

$$\bullet \ m_f = \mathbf{K}_{\rho} (\mathbf{K}_{\rho} + \boldsymbol{\Sigma}_{\theta}^{-1})^{-1} \boldsymbol{\theta}^{(t)}$$

• $\mathbf{K}_{\rho} = K(\mathbf{Z}|\boldsymbol{\rho})$ is an $N \times N$ covariance generated using a kernel computed on a $\mathbf{Z}_{N \times D}$ matrix of demographic features

•
$$\Sigma_{\theta} = \sigma_{\theta}^2 \mathbf{I} = (1.0)\mathbf{I}$$

Gibbs-Sampler: Glossary of Terms

Moy et al.

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2 Parameter Names

wit	$\omega_{it}^{(t)}$
W	$\{\omega_{ij}^{(t)}\}_{i=1}^N$
w_j	jth column of $\{\omega_{ij}^{(t)}\}_{i=1}^{N}$
beta_tilde	$ ilde{oldsymbol{eta}}$
beta_tilde_j	$\tilde{\boldsymbol{\beta}}$ for response item j
V_beta	V_{eta}
m_beta	m_{eta}
Omega_j	Ω_j
X	X
k	$oldsymbol{\kappa}_j$
Lambda	$\Lambda_{ ilde{oldsymbol{eta}}}$
V_theta	$V_{ heta}$
sigma2_theta	$\sigma_{ heta}^2$
m_theta	$m_{ heta}$
f_prior	$f_i^{(t-1)}$
y_tilde	$ ilde{m{y}}$
$\mathtt{f}_{\mathtt{-}}\mathtt{t}$	$\mathbf{f}^{(t)}$
$\mathtt{V}_{-}\mathtt{f}$	V_f
m_f	m_f
Kappa_rho	$\mathbf{K}_{ ho}$
Sigma_theta	$oldsymbol{\Sigma}_{ heta}$