Optimizing Marketing Budget Allocation

Project Directive: Find the optimal investment strategy for our $10,000,000 marketing budget among the available advertising mediums.

Available Advertising Mediums: Print, TV, SEO, AdWords, Facebook, LinkedIn, Instagram, Snapchat, Twitter, and Email

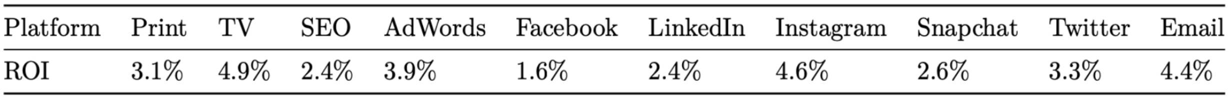
Project Guidelines:

1. The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
2. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
3. For each platform, the amount invested should be no more than $3,000,000.

Initial Recommended Allocation

In order to determine the optimal budget allocation, we partnered with an outside consulting firm to estimate the return on investment (ROI) of each marketing medium under consideration.

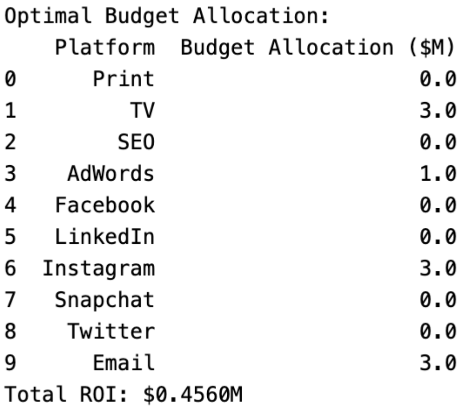
Consulting Firm - ROI Forecast:



Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

Using the above ROI forecast, we were able to build an optimization algorithm that would efficiently allocate our marketing budget, while following the project guidelines. The code for this optimization can be found in the Appendix as Exhibit A.

Optimal Allocation:



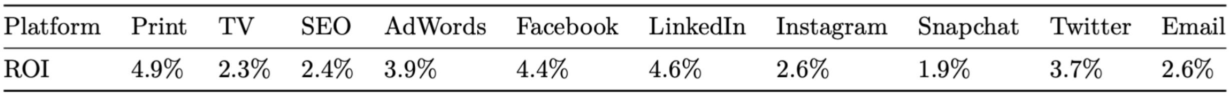
```
Optimal Budget Allocation:
    Platform  Budget Allocation ($M)
0      Print                     0.0
1         TV                     3.0
2        SEO                     0.0
3    AdWords                     1.0
4   Facebook                     0.0
5   LinkedIn                     0.0
6  Instagram                     3.0
7   Snapchat                     0.0
8    Twitter                     0.0
9      Email                     3.0
Total ROI: $0.4560M
```

Initial recommended allocation plan: The maximum total return on investment is $456,000, allocating $3,000,000 to TV, Instagram, and Email and $1,000,000 to AdWords.

Alternative Recommended Allocation

To verify the accuracy of the results of the initial recommendation, we have partnered with another consulting firm to compare their estimates of the ROI forecast for each advertising medium.

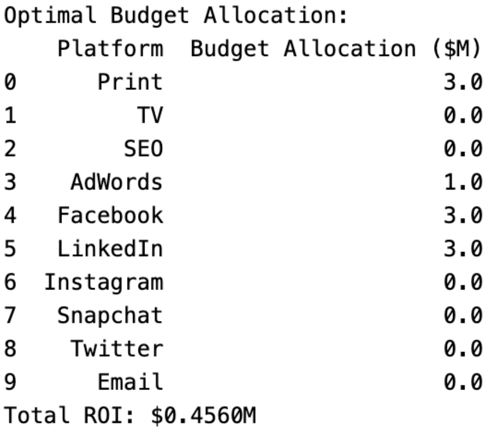
Consulting Firm #2 - ROI Forecast:



Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

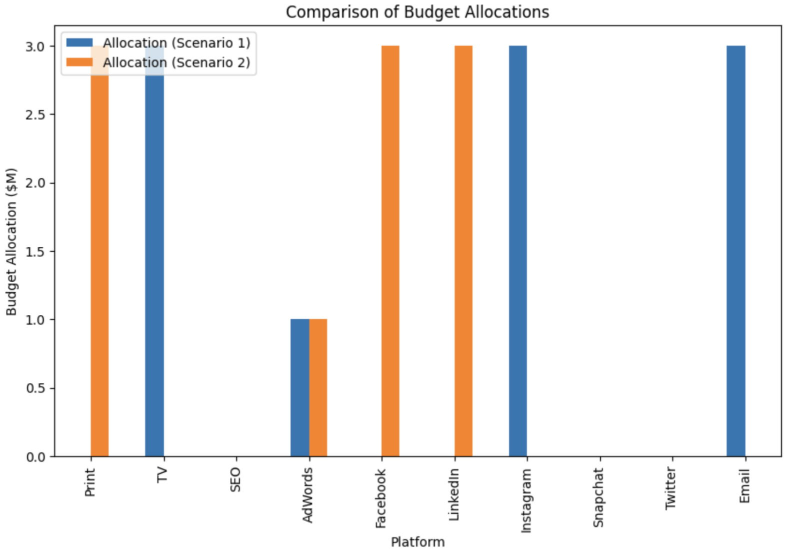
Using the new ROI forecast above, we re-ran our optimization algorithm (Exhibit B in Appendix) to see the impact on our recommended budget allocation, while still following the project guidelines.

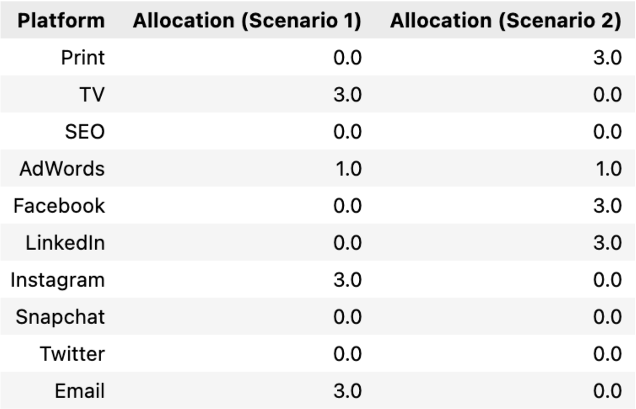
Optimal Allocation:



```
Optimal Budget Allocation:
    Platform  Budget Allocation ($M)
0      Print                     3.0
1         TV                     0.0
2        SEO                     0.0
3    AdWords                     1.0
4   Facebook                     3.0
5   LinkedIn                     3.0
6  Instagram                     0.0
7   Snapchat                     0.0
8    Twitter                     0.0
9      Email                     0.0
Total ROI: $0.4560M
```

Comparison of the Recommended Allocations: The maximum ROI remains consistent among both sets of ROI forecasts at $456,000. However, the distribution of the budget among various platforms has been altered. In this scenario, a budget of $3,000,000 should be assigned to Print, Facebook, and LinkedIn, while AdWords should receive an allocation of $1,000,000.





Platform	Allocation (Scenario 1)	Allocation (Scenario 2)
Print	0.0	3.0
TV	3.0	0.0
SEO	0.0	0.0
AdWords	1.0	1.0
Facebook	0.0	3.0
LinkedIn	0.0	3.0
Instagram	3.0	0.0
Snapchat	0.0	0.0
Twitter	0.0	0.0
Email	3.0	0.0

Scenario Analysis

We wanted to test the accuracies of each ROI dataset and view the relationships between each ROI data and budget allocation recommendation. Therefore, we tested the 1st Firm's ROI data with the second Budget Allocation Plan and vice versa (Exhibit C in Appendix).

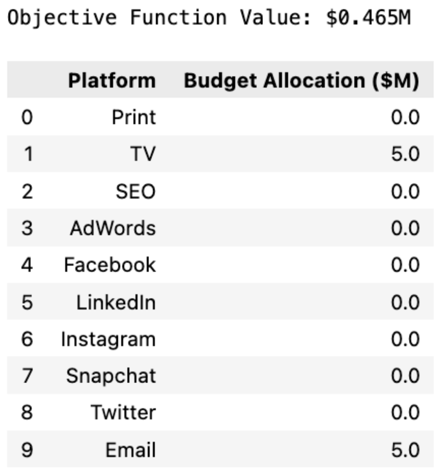
ROI data: 1st Firm ***Budget Allocation: 2nd Recommendation***

Under these conditions our total ROI would be $252,000, which is $204,000 lower than our maximum ROI of $456,000.

ROI data: 2nd Firm ***Budget Allocation: 1st Recommendation***

Under these conditions our total ROI would be $264,000, which is $192,000 lower than our maximum ROI of $456,000.

Additionally, we decided to verify whether the constraint that no more than $3,000,000 be invested for each platform had any impact on our recommended allocation or total project ROI (Exhibit D in Appendix).

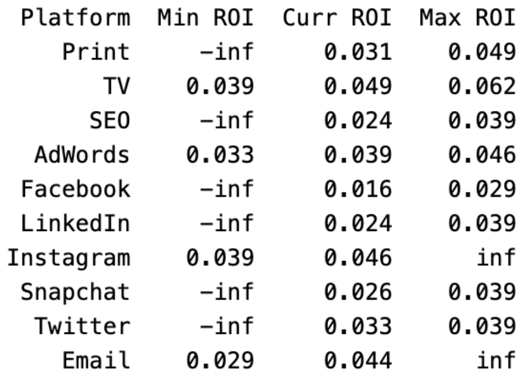


```
Objective Function Value: $0.465M
```

	Platform	Budget Allocation ($M)
0	Print	0.0
1	TV	5.0
2	SEO	0.0
3	AdWords	0.0
4	Facebook	0.0
5	LinkedIn	0.0
6	Instagram	0.0
7	Snapchat	0.0
8	Twitter	0.0
9	Email	5.0

If we were to eliminate this $3,000,000 maximum investment constraint, the maximum ROI value would increase by $9,000 to $465,000. However, it's important to consider that this approach focuses on just two channels, which may not be the most effective strategy due to the potential for diminishing returns.

Using the first firm's ROI data as our starting point, we calculated how much each advertising medium's ROI could increase or decrease but still result in the same optimal allocation we found in our initial recommendation (Exhibit E in Appendix).



```
 Platform  Min ROI  Curr ROI  Max ROI
    Print     -inf     0.031    0.049
       TV    0.039     0.049    0.062
      SEO     -inf     0.024    0.039
  AdWords    0.033     0.039    0.046
 Facebook     -inf     0.016    0.029
 LinkedIn     -inf     0.024    0.039
Instagram    0.039     0.046      inf
 Snapchat     -inf     0.026    0.039
  Twitter     -inf     0.033    0.039
    Email    0.029     0.044      inf
```

Min ROI (Minimum ROI): This column identifies the minimum ROI value that each platform's ROI can decrease to while still maintaining the same optimal budget allocation. A value of "-inf" indicates that the ROI for that platform can decrease to negative infinity without affecting the allocation.

Curr ROI (Current ROI): This column identifies the current ROI values for each platform based on the initial ROI data.

Max ROI (Maximum ROI): This column identifies the maximum ROI value that each platform's ROI can increase to while still maintaining the same optimal budget allocation. A value of "inf" indicates that the ROI for that platform can increase to positive infinity without affecting the allocation.

For example, with the "Print" platform:

- The minimum value that the ROI can decrease to is negative infinity, which means that the Print platform's ROI can decrease significantly without affecting the budget allocation.
- The current ROI for the Print platform is 3.1%.
- The maximum value that the ROI can increase to is 4.9%, meaning that if the Print platform's ROI improves to greater than 4.9%, the budget allocation will no longer apply and needs to be changed.

In contrast, for the "Instagram" platform:

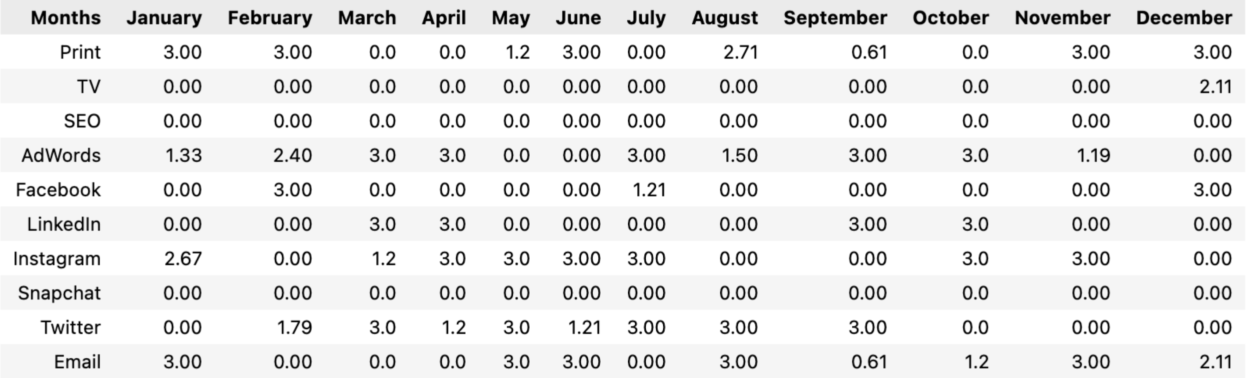
- The minimum value that the ROI can decrease to is 3.9%, which means that the ROI of Instagram would need to decrease to lower than 3.9% to impact the budget allocation.
- The current ROI for Instagram is 4.6%.
- The maximum value of ROI has no bounds and can increase to positive infinity, indicating that even a substantial increase in Instagram's ROI won't change the budget allocation.

These results allow us to understand the flexibility and sensitivity of the budget allocation to changes in ROI for each advertising platform. Platforms with wider ranges of ROI values (both minimum and maximum) have more flexibility in terms of ROI changes that will not affect the budget allocation. Conversely, platforms with narrower ranges are more sensitive to ROI fluctuations.

Reinvesting Returns

While conducting our analysis, we received news that we would be able to reinvest half of the return back into our marketing budget. For example, if the marketing obtains a 4% return in January, the budget of February will be \$10M + \$10M × 4% × 50% = \$10.2M. The monthly ROI for next year can be found in the Appendix as Exhibit F. To consider this, we kept the three project guidelines in place for each month and created a new optimization algorithm to find the optimal allocation for each month (Exhibit G in Appendix).

Monthly Optimal Budget Allocation:

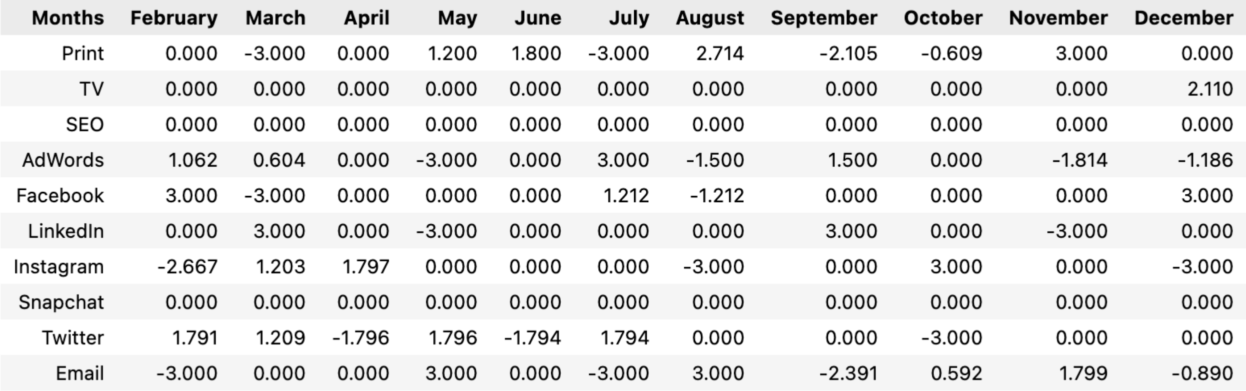


Months	January	February	March	April	May	June	July	August	September	October	November	December
Print	3.00	3.00	0.0	0.0	1.2	3.00	0.00	2.71	0.61	0.0	3.00	3.00
TV	0.00	0.00	0.0	0.0	0.0	0.00	0.00	0.00	0.00	0.0	0.00	2.11
SEO	0.00	0.00	0.0	0.0	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00
AdWords	1.33	2.40	3.0	3.0	0.0	0.00	3.00	1.50	3.00	3.0	1.19	0.00
Facebook	0.00	3.00	0.0	0.0	0.0	0.00	1.21	0.00	0.00	0.0	0.00	3.00
LinkedIn	0.00	0.00	3.0	3.0	0.0	0.00	0.00	0.00	3.00	3.0	0.00	0.00
Instagram	2.67	0.00	1.2	3.0	3.0	3.00	3.00	0.00	0.00	3.0	3.00	0.00
Snapchat	0.00	0.00	0.0	0.0	0.0	0.00	0.00	0.00	0.00	0.0	0.00	0.00
Twitter	0.00	1.79	3.0	1.2	3.0	1.21	3.00	3.00	3.00	0.0	0.00	0.00
Email	3.00	0.00	0.0	0.0	3.0	3.00	0.00	3.00	0.61	1.2	3.00	2.11

Looking at the table, we can analyze how the company's budget allocation or ROI varies across different advertising platforms over the course of the year. This information is essential for making decisions regarding where to allocate resources for marketing efforts. The specific values in the table provide insights into the platforms that are performing well (high ROI) and those that might need adjustments or reallocations of the budget. Additionally, this data can be used in optimization or budget allocation models to determine the optimal allocation of the marketing budget for each month based on certain constraints and objectives.

A stable budget is defined as a monthly allocation such that for each platform, the monthly change in spending is no more than \$1,000,000. We wanted to analyze if our recommended allocation qualifies as a stable budget (Exhibit H in Appendix).

Month-on-Month Budget Adjustments:



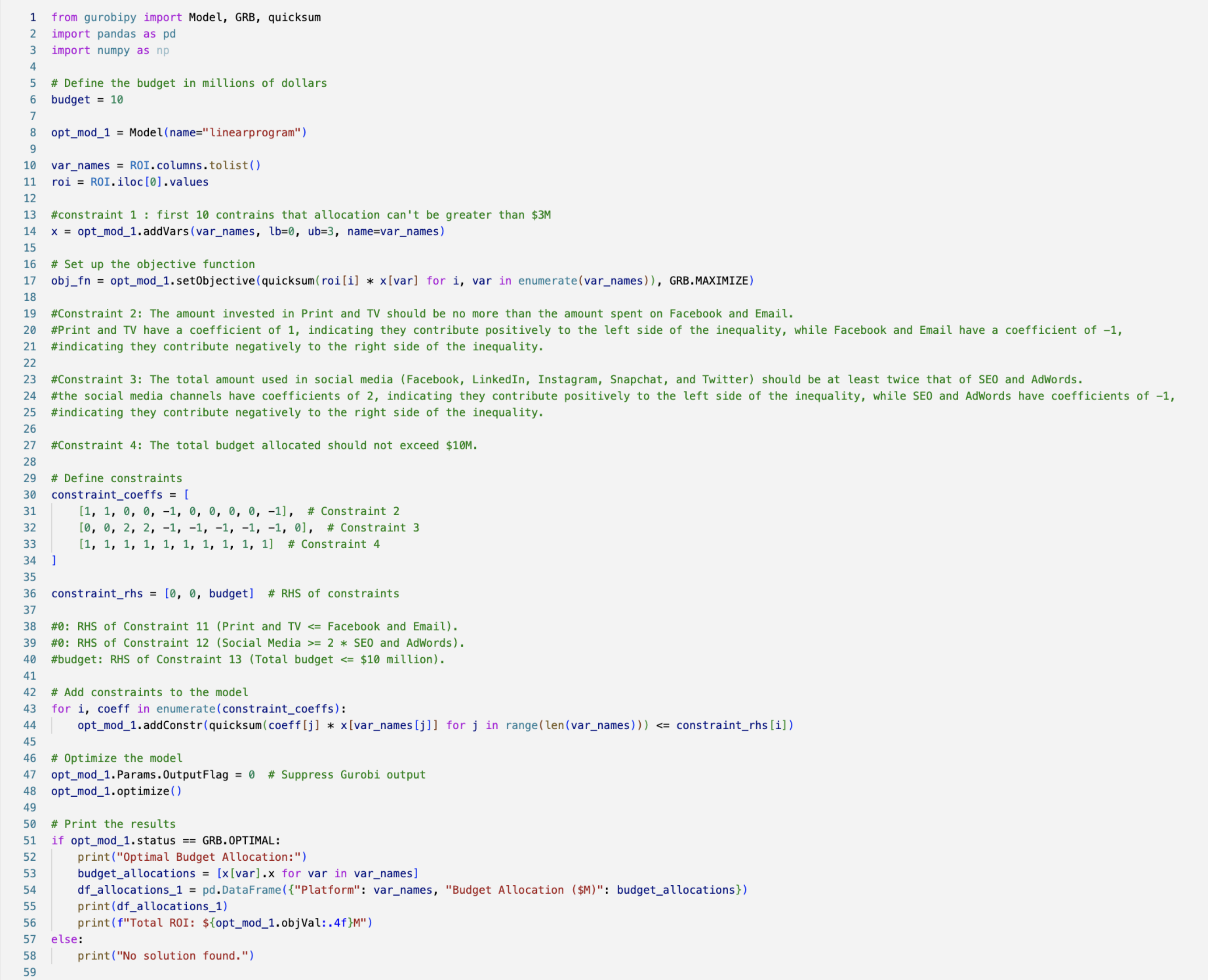
Months	February	March	April	May	June	July	August	September	October	November	December
Print	0.000	-3.000	0.000	1.200	1.800	-3.000	2.714	-2.105	-0.609	3.000	0.000
TV	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	2.110
SEO	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
AdWords	1.062	0.604	0.000	-3.000	0.000	3.000	-1.500	1.500	0.000	-1.814	-1.186
Facebook	3.000	-3.000	0.000	0.000	0.000	1.212	-1.212	0.000	0.000	0.000	3.000
LinkedIn	0.000	3.000	0.000	-3.000	0.000	0.000	0.000	3.000	0.000	-3.000	0.000
Instagram	-2.667	1.203	1.797	0.000	0.000	0.000	-3.000	0.000	3.000	0.000	-3.000
Snapchat	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Twitter	1.791	1.209	-1.796	1.796	-1.794	1.794	0.000	0.000	-3.000	0.000	0.000
Email	-3.000	0.000	0.000	3.000	0.000	-3.000	3.000	-2.391	0.592	1.799	-0.890

- The observed budget allocation fluctuations, exceeding the $1 million threshold in some months, highlight the need for a more consistent and stable marketing strategy. Abrupt changes in marketing spending across various platforms can disrupt ongoing campaigns and affect customer engagement.
- The root cause of this instability lies in our current approach, where we optimize budget allocation by maximizing the ROI for each specific month based on that month's ROI. While this method allows for adaptability to varying market conditions, it also leads to pronounced budget shifts.
- To address this issue, we propose a refined strategy. Initially, we allocate budgets based on a single, predetermined ROI, as outlined in Part 1. Subsequently, we leverage the monthly ROIs to calculate the expected ROI for each specific month. To promote stability, we allocate half of this expected ROI increment to the following month. This approach aims to create a smoother transition between budget allocations, resulting in a more predictable and reliable marketing strategy.

Ultimately, achieving budget stability is crucial for maintaining a consistent brand presence and ensuring that marketing efforts yield reliable, long-term results.

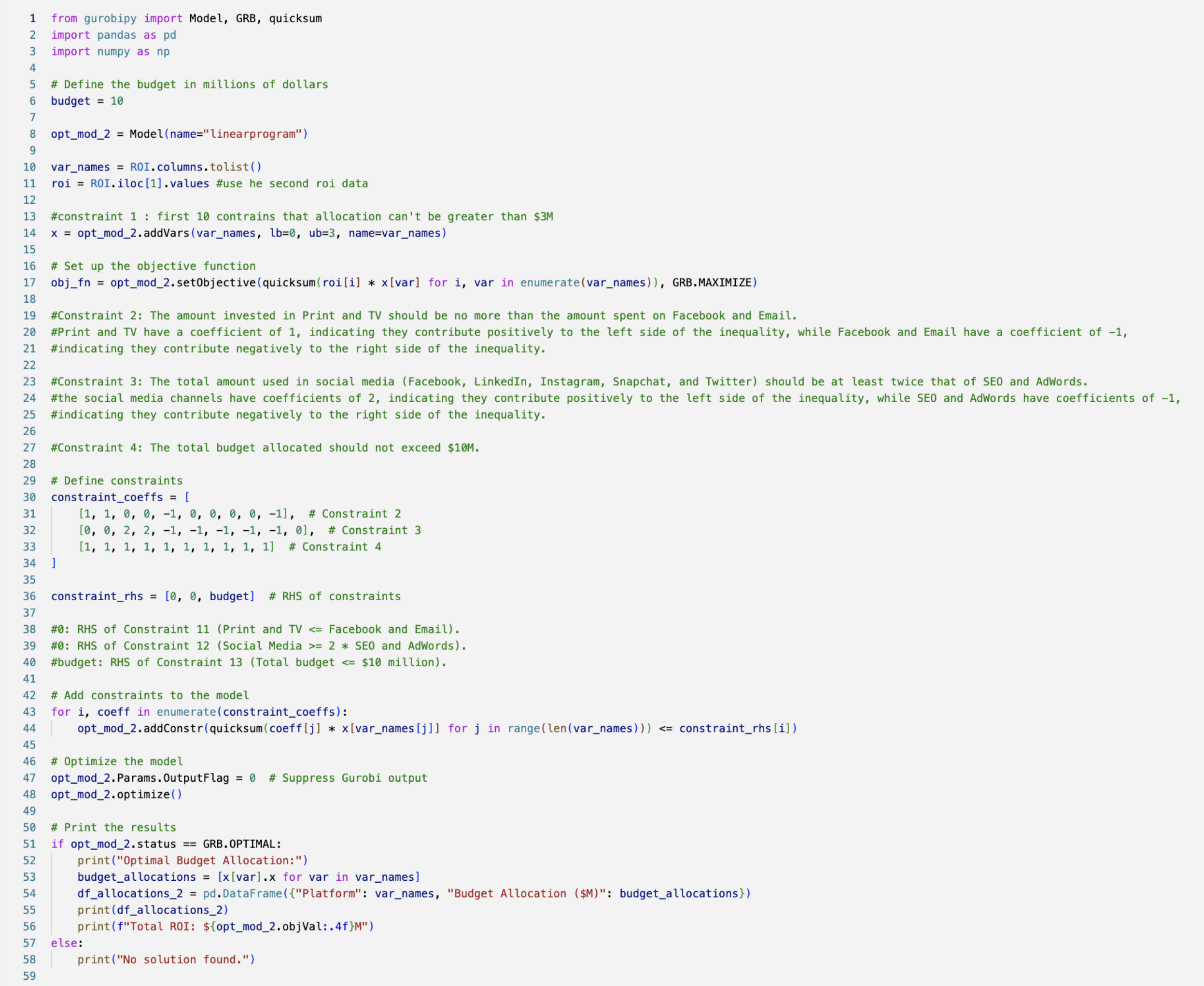
Appendix

Exhibit A



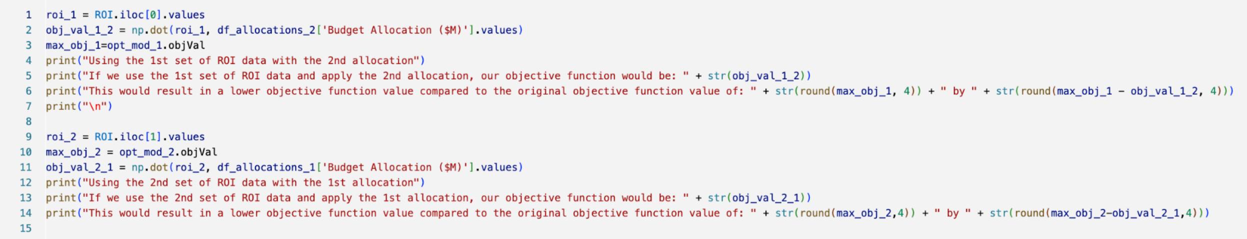
```python
from gurobipy import Model, GRB, quicksum
import pandas as pd
import numpy as np

# Define the budget in millions of dollars
budget = 10

opt_mod_1 = Model(name="linearprogram")

var_names = ROI.columns.tolist()
roi = ROI.iloc[0].values

#constraint 1 : first 10 contrains that allocation can't be greater than $3M
x = opt_mod_1.addVars(var_names, lb=0, ub=3, name=var_names)

# Set up the objective function
obj_fn = opt_mod_1.setObjective(quicksum(roi[i] * x[var] for i, var in enumerate(var_names)), GRB.MAXIMIZE)

#Constraint 2: The amount invested in Print and TV should be no more than the amount spent on Facebook and Email.
#Print and TV have a coefficient of 1, indicating they contribute positively to the left side of the inequality, while Facebook and Email have a coefficient of -1,
#indicating they contribute negatively to the right side of the inequality.

#Constraint 3: The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice that of SEO and AdWords.
#the social media channels have coefficients of 2, indicating they contribute positively to the left side of the inequality, while SEO and AdWords have coefficients of -1,
#indicating they contribute negatively to the right side of the inequality.

#Constraint 4: The total budget allocated should not exceed $10M.

# Define constraints
constraint_coeffs = [
    [1, 1, 0, 0, -1, 0, 0, 0, 0, -1],  # Constraint 2
    [0, 0, 2, 2, -1, -1, -1, -1, -1, 0],  # Constraint 3
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]  # Constraint 4
]

constraint_rhs = [0, 0, budget]  # RHS of constraints

#0: RHS of Constraint 11 (Print and TV <= Facebook and Email).
#0: RHS of Constraint 12 (Social Media >= 2 * SEO and AdWords).
#budget: RHS of Constraint 13 (Total budget <= $10 million).

# Add constraints to the model
for i, coeff in enumerate(constraint_coeffs):
    opt_mod_1.addConstr(quicksum(coeff[j] * x[var_names[j]] for j in range(len(var_names))) <= constraint_rhs[i])

# Optimize the model
opt_mod_1.Params.OutputFlag = 0  # Suppress Gurobi output
opt_mod_1.optimize()

# Print the results
if opt_mod_1.status == GRB.OPTIMAL:
    print("Optimal Budget Allocation:")
    budget_allocations = [x[var].x for var in var_names]
    df_allocations_1 = pd.DataFrame({"Platform": var_names, "Budget Allocation ($M)": budget_allocations})
    print(df_allocations_1)
    print(f"Total ROI: ${opt_mod_1.objVal:.4f}M")
else:
    print("No solution found.")

```

Exhibit B



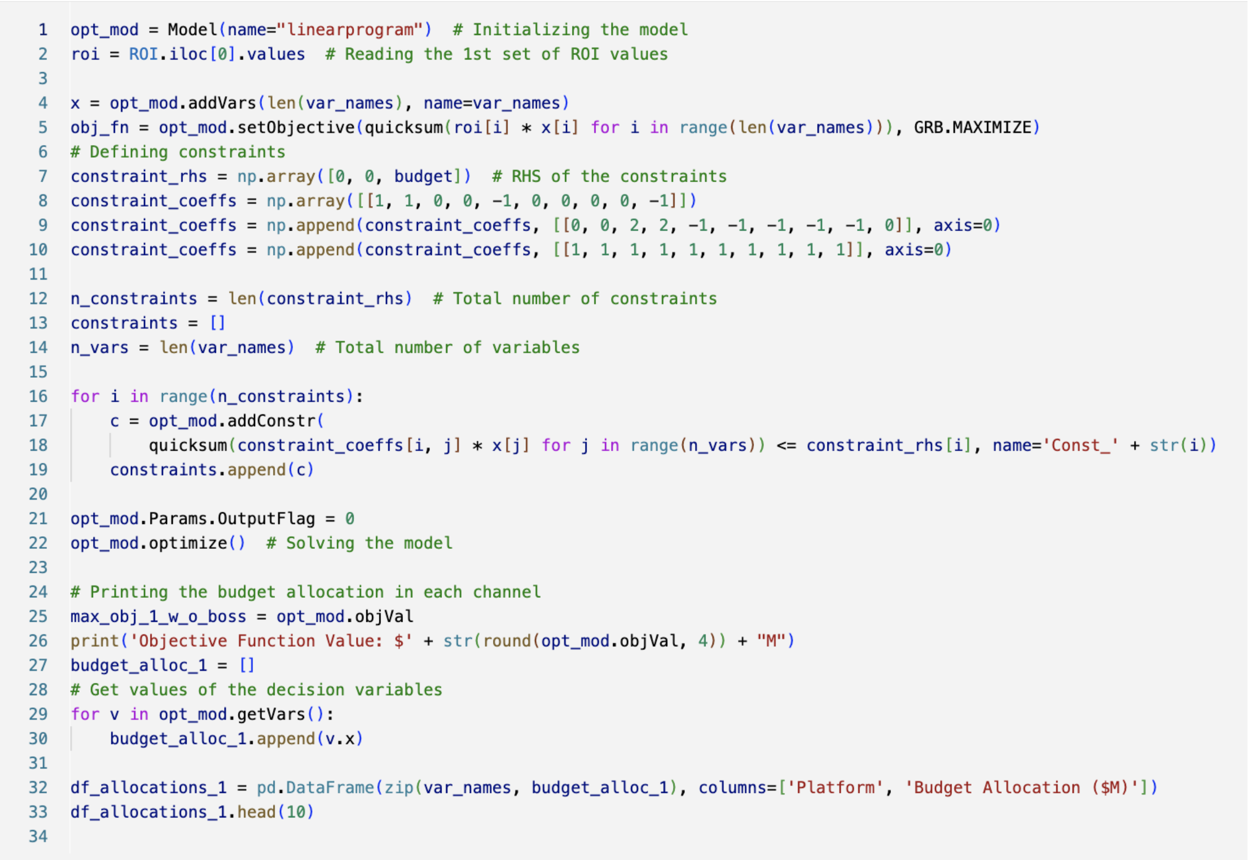
```python
from gurobipy import Model, GRB, quicksum
import pandas as pd
import numpy as np

# Define the budget in millions of dollars
budget = 10

opt_mod_2 = Model(name="linearprogram")

var_names = ROI.columns.tolist()
roi = ROI.iloc[1].values #use he second roi data

#constraint 1 : first 10 contrains that allocation can't be greater than $3M
x = opt_mod_2.addVars(var_names, lb=0, ub=3, name=var_names)

# Set up the objective function
obj_fn = opt_mod_2.setObjective(quicksum(roi[i] * x[var] for i, var in enumerate(var_names)), GRB.MAXIMIZE)

#Constraint 2: The amount invested in Print and TV should be no more than the amount spent on Facebook and Email.
#Print and TV have a coefficient of 1, indicating they contribute positively to the left side of the inequality, while Facebook and Email have a coefficient of -1,
#indicating they contribute negatively to the right side of the inequality.

#Constraint 3: The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice that of SEO and AdWords.
#the social media channels have coefficients of 2, indicating they contribute positively to the left side of the inequality, while SEO and AdWords have coefficients of -1,
#indicating they contribute negatively to the right side of the inequality.

#Constraint 4: The total budget allocated should not exceed $10M.

# Define constraints
constraint_coeffs = [
    [1, 1, 0, 0, -1, 0, 0, 0, 0, -1],  # Constraint 2
    [0, 0, 2, 2, -1, -1, -1, -1, -1, 0],  # Constraint 3
    [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]  # Constraint 4
]

constraint_rhs = [0, 0, budget]  # RHS of constraints

#0: RHS of Constraint 11 (Print and TV <= Facebook and Email).
#0: RHS of Constraint 12 (Social Media >= 2 * SEO and AdWords).
#budget: RHS of Constraint 13 (Total budget <= $10 million).

# Add constraints to the model
for i, coeff in enumerate(constraint_coeffs):
    opt_mod_2.addConstr(quicksum(coeff[j] * x[var_names[j]] for j in range(len(var_names))) <= constraint_rhs[i])

# Optimize the model
opt_mod_2.Params.OutputFlag = 0  # Suppress Gurobi output
opt_mod_2.optimize()

# Print the results
if opt_mod_2.status == GRB.OPTIMAL:
    print("Optimal Budget Allocation:")
    budget_allocations = [x[var].x for var in var_names]
    df_allocations_2 = pd.DataFrame({"Platform": var_names, "Budget Allocation ($M)": budget_allocations})
    print(df_allocations_2)
    print(f"Total ROI: ${opt_mod_2.objVal:.4f}M")
else:
    print("No solution found.")
```

Exhibit C



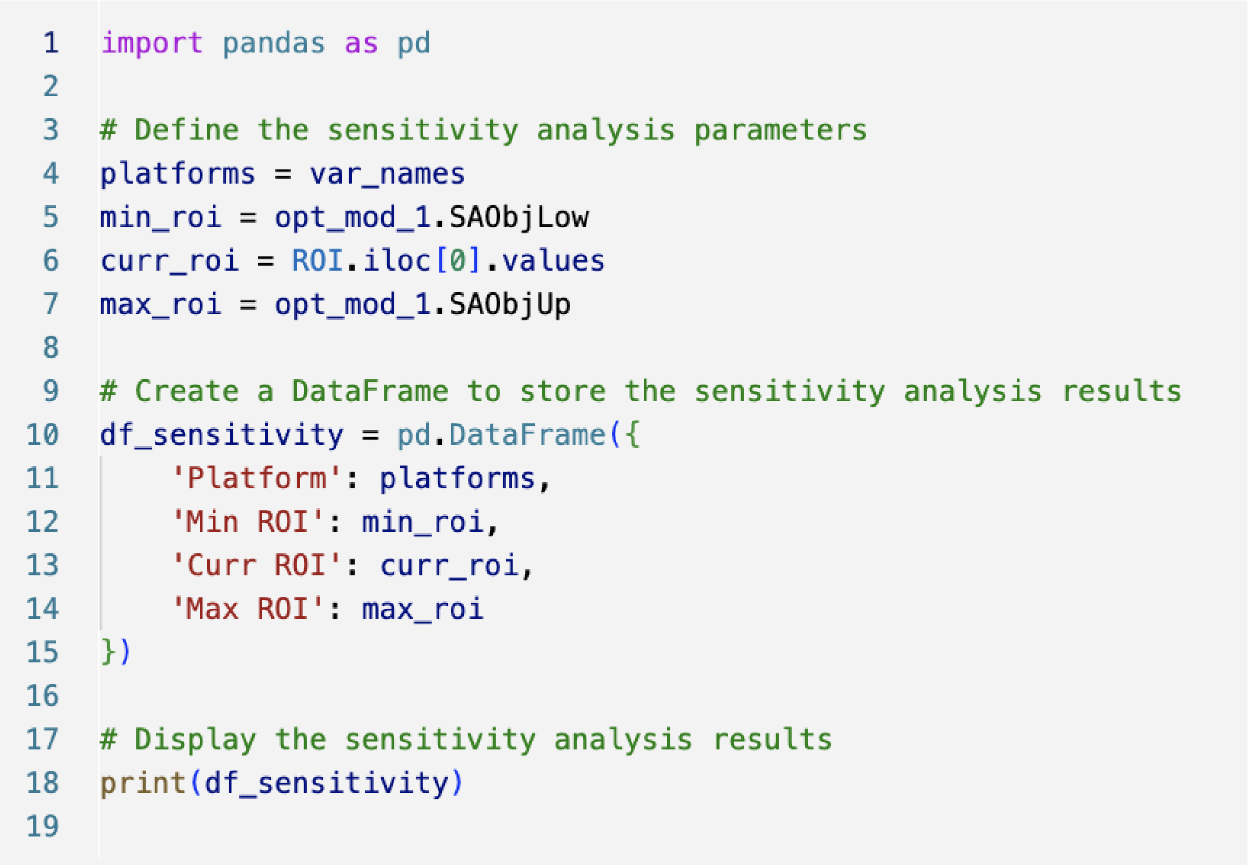
```python
roi_1 = ROI.iloc[0].values
obj_val_1_2 = np.dot(roi_1, df_allocations_2['Budget Allocation ($M)'].values)
max_obj_1=opt_mod_1.objVal
print("Using the 1st set of ROI data with the 2nd allocation")
print("If we use the 1st set of ROI data and apply the 2nd allocation, our objective function would be: " + str(obj_val_1_2))
print("This would result in a lower objective function value compared to the original objective function value of: " + str(round(max_obj_1, 4)) + " by " + str(round(max_obj_1 - obj_val_1_2, 4)))
print("\n")

roi_2 = ROI.iloc[1].values
max_obj_2 = opt_mod_2.objVal
obj_val_2_1 = np.dot(roi_2, df_allocations_1['Budget Allocation ($M)'].values)
print("Using the 2nd set of ROI data with the 1st allocation")
print("If we use the 2nd set of ROI data and apply the 1st allocation, our objective function would be: " + str(obj_val_2_1))
print("This would result in a lower objective function value compared to the original objective function value of: " + str(round(max_obj_2,4)) + " by " + str(round(max_obj_2-obj_val_2_1,4)))
```

Exhibit D



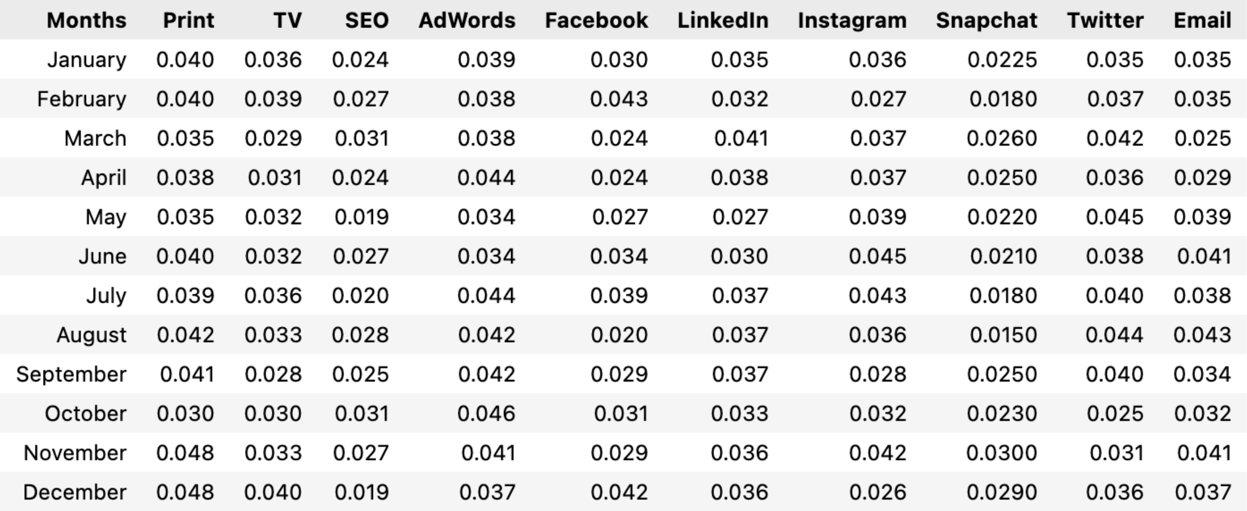
```
opt_mod = Model(name="linearprogram")  # Initializing the model
roi = ROI.iloc[0].values  # Reading the 1st set of ROI values

x = opt_mod.addVars(len(var_names), name=var_names)
obj_fn = opt_mod.setObjective(quicksum(roi[i] * x[i] for i in range(len(var_names))), GRB.MAXIMIZE)
# Defining constraints
constraint_rhs = np.array([0, 0, budget])  # RHS of the constraints
constraint_coeffs = np.array([[1, 1, 0, 0, -1, 0, 0, 0, 0, -1]])
constraint_coeffs = np.append(constraint_coeffs, [[0, 0, 2, 2, -1, -1, -1, -1, -1, 0]], axis=0)
constraint_coeffs = np.append(constraint_coeffs, [[1, 1, 1, 1, 1, 1, 1, 1, 1, 1]], axis=0)

n_constraints = len(constraint_rhs)  # Total number of constraints
constraints = []
n_vars = len(var_names)  # Total number of variables

for i in range(n_constraints):
    c = opt_mod.addConstr(
        quicksum(constraint_coeffs[i, j] * x[j] for j in range(n_vars)) <= constraint_rhs[i], name='Const_' + str(i))
    constraints.append(c)

opt_mod.Params.OutputFlag = 0
opt_mod.optimize()  # Solving the model

# Printing the budget allocation in each channel
max_obj_1_w_o_boss = opt_mod.objVal
print('Objective Function Value: $' + str(round(opt_mod.objVal, 4)) + "M")
budget_alloc_1 = []
# Get values of the decision variables
for v in opt_mod.getVars():
    budget_alloc_1.append(v.x)

df_allocations_1 = pd.DataFrame(zip(var_names, budget_alloc_1), columns=['Platform', 'Budget Allocation ($M)'])
df_allocations_1.head(10)

```

Exhibit E



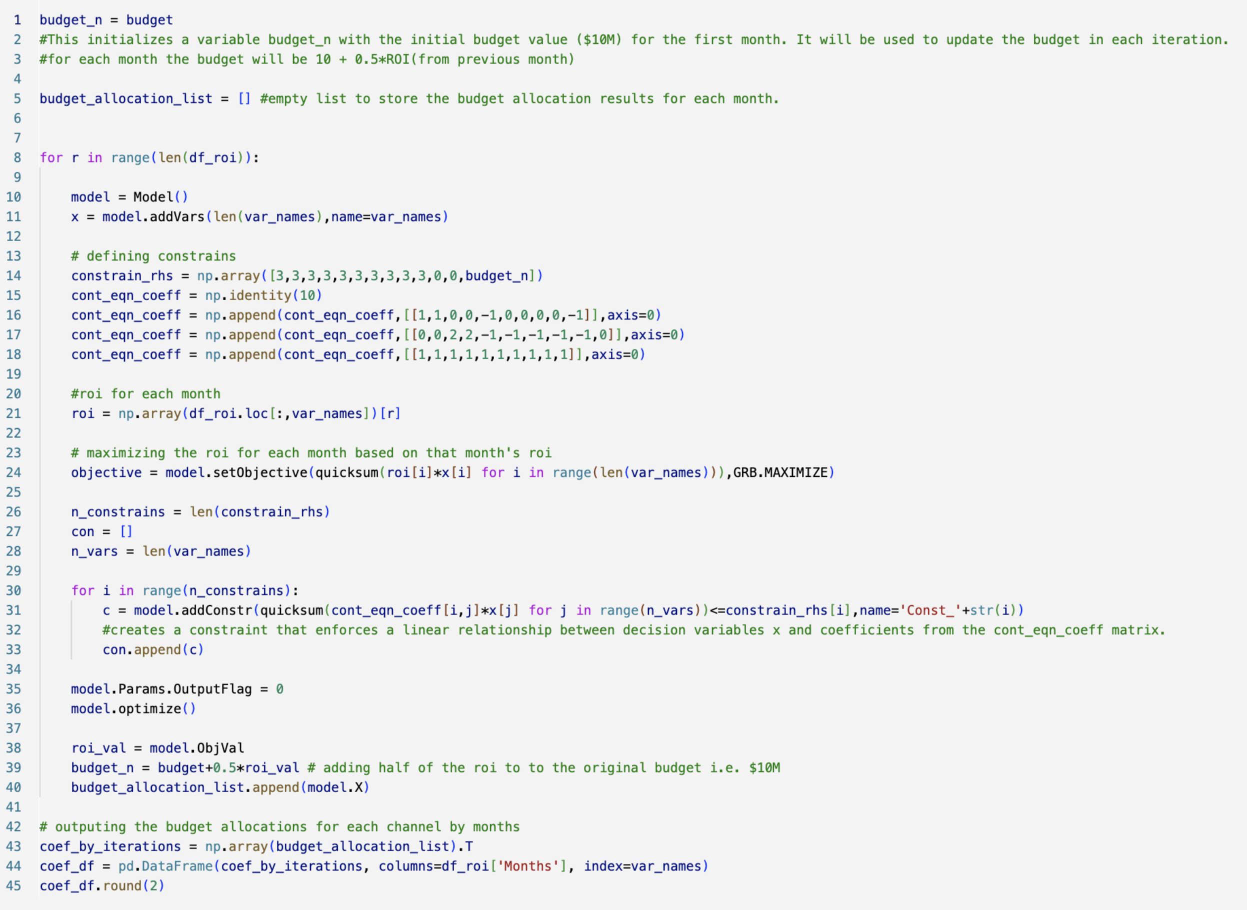
```
import pandas as pd

# Define the sensitivity analysis parameters
platforms = var_names
min_roi = opt_mod_1.SAObjLow
curr_roi = ROI.iloc[0].values
max_roi = opt_mod_1.SAObjUp

# Create a DataFrame to store the sensitivity analysis results
df_sensitivity = pd.DataFrame({
    'Platform': platforms,
    'Min ROI': min_roi,
    'Curr ROI': curr_roi,
    'Max ROI': max_roi
})

# Display the sensitivity analysis results
print(df_sensitivity)

```

Exhibit F



Months	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	0.040	0.036	0.024	0.039	0.030	0.035	0.036	0.0225	0.035	0.035
February	0.040	0.039	0.027	0.038	0.043	0.032	0.027	0.0180	0.037	0.035
March	0.035	0.029	0.031	0.038	0.024	0.041	0.037	0.0260	0.042	0.025
April	0.038	0.031	0.024	0.044	0.024	0.038	0.037	0.0250	0.036	0.029
May	0.035	0.032	0.019	0.034	0.027	0.027	0.039	0.0220	0.045	0.039
June	0.040	0.032	0.027	0.034	0.034	0.030	0.045	0.0210	0.038	0.041
July	0.039	0.036	0.020	0.044	0.039	0.037	0.043	0.0180	0.040	0.038
August	0.042	0.033	0.028	0.042	0.020	0.037	0.036	0.0150	0.044	0.043
September	0.041	0.028	0.025	0.042	0.029	0.037	0.028	0.0250	0.040	0.034
October	0.030	0.030	0.031	0.046	0.031	0.033	0.032	0.0230	0.025	0.032
November	0.048	0.033	0.027	0.041	0.029	0.036	0.042	0.0300	0.031	0.041
December	0.048	0.040	0.019	0.037	0.042	0.036	0.026	0.0290	0.036	0.037

Exhibit G



```
budget_n = budget
#This initializes a variable budget_n with the initial budget value ($10M) for the first month. It will be used to update the budget in each iteration.
#for each month the budget will be 10 + 0.5*ROI(from previous month)

budget_allocation_list = [] #empty list to store the budget allocation results for each month.


for r in range(len(df_roi)):

    model = Model()
    x = model.addVars(len(var_names),name=var_names)

    # defining constrains
    constrain_rhs = np.array([3,3,3,3,3,3,3,3,3,3,0,0,budget_n])
    cont_eqn_coeff = np.identity(10)
    cont_eqn_coeff = np.append(cont_eqn_coeff,[[1,1,0,0,-1,0,0,0,0,-1]],axis=0)
    cont_eqn_coeff = np.append(cont_eqn_coeff,[[0,0,2,2,-1,-1,-1,-1,-1,0]],axis=0)
    cont_eqn_coeff = np.append(cont_eqn_coeff,[[1,1,1,1,1,1,1,1,1,1]],axis=0)

    #roi for each month
    roi = np.array(df_roi.loc[:,var_names])[r]

    # maximizing the roi for each month based on that month's roi
    objective = model.setObjective(quicksum(roi[i]*x[i] for i in range(len(var_names))),GRB.MAXIMIZE)

    n_constrains = len(constrain_rhs)
    con = []
    n_vars = len(var_names)

    for i in range(n_constrains):
        c = model.addConstr(quicksum(cont_eqn_coeff[i,j]*x[j] for j in range(n_vars))<=constrain_rhs[i],name='Const_'+str(i))
        #creates a constraint that enforces a linear relationship between decision variables x and coefficients from the cont_eqn_coeff matrix.
        con.append(c)

    model.Params.OutputFlag = 0
    model.optimize()

    roi_val = model.ObjVal
    budget_n = budget+0.5*roi_val # adding half of the roi to to the original budget i.e. $10M
    budget_allocation_list.append(model.X)

# outputing the budget allocations for each channel by months
coef_by_iterations = np.array(budget_allocation_list).T
coef_df = pd.DataFrame(coef_by_iterations, columns=df_roi['Months'], index=var_names)
coef_df.round(2)
```

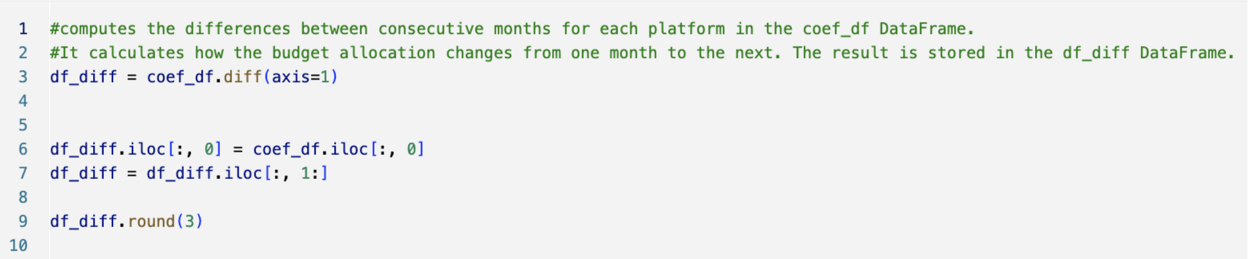
For reference:

- Initializing constrain_rhs: constrain_rhs is an array that defines the right-hand side values of the constraints. It contains 13 values, corresponding to the constraints for the 10 advertising platforms and the 3 additional constraints specified in your problem statement. The first 10 values (3 for each platform) are set to 3, which enforces that each platform's budget allocation should not exceed $3 million, as per constraint c. The last three values are initialized as follows: The second-to-last value (0) corresponds to the constraint that the total amount used in social media should be at least twice the combined budget for SEO and AdWords (b).

The last value (budget_n) is used to dynamically specify the budget for the next month, where budget_n is updated based on the previous month's ROI.

- Initializing cont_eqn_coeff as an identity matrix: cont_eqn_coeff is a coefficient matrix representing the linear coefficients of the decision variables (budget allocations). It is initialized as a 10x10 identity matrix, where the main diagonal elements are 1, and all other elements are 0. This corresponds to the first 10 constraints ensuring that each platform's budget allocation is less than or equal to $3 million. Appending additional constraint rows to cont_eqn_coeff:
- Three additional rows are appended to the cont_eqn_coeff matrix, representing the constraints related to the problem statement.
- The second-to-last row [1, 1, 0, 0, -1, 0, 0, 0, 0, -1] corresponds to the constraint that the amount invested in print and TV should be no more than the amount spent on Facebook and Email (a).
- The third-to-last row [0, 0, 2, 2, -1, -1, -1, -1, -1, 0] corresponds to the constraint that the total amount used in social media should be at least twice the combined budget for SEO and AdWords (b).
- The last row [1, 1, 1, 1, 1, 1, 1, 1, 1, 1] corresponds to the constraint that, for each platform, the amount invested should be no more than $3 million (c).

Exhibit H



```python
#computes the differences between consecutive months for each platform in the coef_df DataFrame.
#It calculates how the budget allocation changes from one month to the next. The result is stored in the df_diff DataFrame.
df_diff = coef_df.diff(axis=1)


df_diff.iloc[:, 0] = coef_df.iloc[:, 0]
df_diff = df_diff.iloc[:, 1:]

df_diff.round(3)

```