Statistics 345: Homework 3

Due on 2014-09-28

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Please complete the problems on a separate sheet of paper with your name at the top. Make sure to show your work and/or provide an explanation for each problem. Partial credit will be given when merited. The total credit is 5.

Problem 1

Let X be a random variable with probability mass function

$$f(x) = \frac{2x+1}{25}, x = 0, 1, 2, 3, 4.$$

(a) Find $P(2 \leq X < 4)$ and $P(X \leq 1)$.

(b) Determine the cumulative distribution function $F(x)$ of X.

Problem 2

Let X be a random variable with cumulative distribution function $F(x) = \begin{cases} 0 & x < 1 \\ 0.7 & 1 \leq x < 4 \\ 0.9 & 4 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$

(a) Determine the probability mass function of X.

(b) Determine probabilities $P(X \leq 5)$ and $P(X > 2)$.

Problem 3

Let X be a random variable with the following probability mass function (pmf):

$$f(x) = \begin{cases} 0.4 & \text{if } x = 0 \\ 0.2 & \text{if } x = 1 \\ 0.1 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Verify that $f(x)$ is a valid pmf (Hint: show $\sum_x f(x) = 1$).

(b) Find the mean of X.

(c) Find the Variance of X.

Problem 4

Compute (by hand) the mean and standard deviation of a random variable X that has the following distributions:

1. A Bernoulli distribution with probability of success $p = 0.1$ (denoted $X \sim \text{Bern}(0.1)$).
2. $X \sim \text{Bern}(0.5)$.
3. $X \sim \text{Bern}(0.9)$.
4. A binomial distribution with $n = 20$ trials and $p = 0.1$ (denoted $X \sim \text{bin}(20,0.1)$).
5. $X \sim \text{bin}(20,0.5)$.
6. $X \sim \text{bin}(20,0.9)$.

Problem 5

A particularly long traffic light on your morning commute is green 30% of the time that your approach it. Assume that each morning represents an independent trial.

(a) Over five mornings, what is the probability that the light is green on exactly one day?

(b) Over 20 mornings, what is the probability that the light is green on exactly four days?

(c) Over 20 mornings, what is the probaiblity that the light is green on more than four days?