

STAT345: Homework 8

Jordan Medlock

November 09, 2014

Problem 1

a

$$\begin{aligned} Z &= \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \\ Z &= \frac{19.8889 - 20}{0.75 / \sqrt{10}} \\ Z &= \frac{-0.1111}{0.2371708245} \\ Z &= \frac{-0.1111}{0.2371708245} \\ Z &= -0.4684387308 \end{aligned}$$

b

$$\begin{aligned} z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} &= 1 \\ 1.96 \left(\frac{2}{\sqrt{n}} \right) &= 1 \\ 1.96(2) &= \sqrt{n} \\ 1.96(2)^2 &= n \\ 15.3664 &= n \\ n &= 16 \end{aligned}$$

Problem 2

a

$$\begin{aligned} \bar{x} &= 65.57725 \\ s &= 4.225464 \\ CI &= \left[\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \right] \\ CI &= \left[65.57725 - 1.645 \frac{4.225464}{\sqrt{16}}, 65.57725 + 1.645 \frac{4.225464}{\sqrt{16}} \right] \\ CI &= [65.57725 - 1.73772207, 65.57725 + 1.73772207] \end{aligned}$$

$$CI = [63.83952793, 67.31497207]$$

We are 90% sure that the population's mean is between 63.83952793 and 67.31497207.

b

$$\bar{x} = 65.57725$$

$$s = 4.225464$$

$$CI = [\bar{x} - z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}]$$

$$CI = [65.57725 - 1.96 \frac{4.225464}{\sqrt{16}}, 65.57725 + 1.96 \frac{4.225464}{\sqrt{16}}]$$

$$CI = [65.57725 - 2.07047736, 65.57725 + 2.07047736]$$

$$CI = [63.50677264, 67.64772736]$$

We are 95% sure that the population's mean is between 63.50677264 and 67.31497207.

Problem 3

a

$$\hat{p} = \frac{x}{n}$$

$$\hat{p} = \frac{823}{1000}$$

$$\hat{p} = 0.823$$

$$CI = [\hat{p} - z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$$

$$CI = [0.823 - 1.96 \sqrt{\frac{0.823(1-0.823)}{1000}}, 0.823 + 1.96 \sqrt{\frac{0.823(1-0.823)}{1000}}]$$

$$CI = [0.823 - 1.96 * 0.01206942418, 0.823 + 1.96 * 0.01206942418]$$

$$CI = [0.7993439286, 0.8466560714]$$

$$CI = [79.9\%, 84.7\%]$$

b

$$n = (\frac{z_{\frac{\alpha}{2}}}{E})^2 \hat{p}(1 - \hat{p})$$

$$n = (\frac{1.96}{0.03})^2 0.823(1 - 0.823)$$

$$n = 622$$