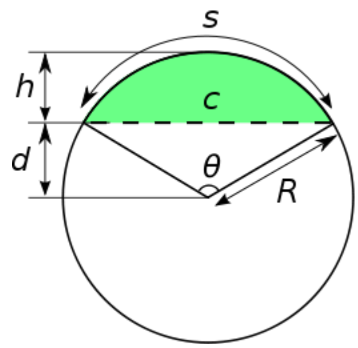
Derivation of formulas:

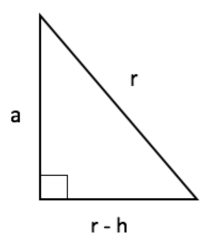
We need to know the area of the shaded region (pretend this picture is gravitationally correct):



Note that the area of the shaded region is equal to the area of the sector minus the area of the unshaded triangle made by θ:

$A_{shaded} = A_{sector} - A_{triangle} = \frac{3}{8} * \pi * r^2$ (1)

Splitting the obtuse triangle in half, you have a right triangle with the sides:



Using the Pythagorean theorem, we find that: $a = \sqrt{-h * (h - 2r)}$

It follows that the area of the obtuse triangle is:

$A_{triangle} = \frac{1}{2}bH = \frac{1}{2} * 2 * a * (r - h)$
$A_{triangle} = (r - h)\sqrt{-h * (h - 2r)}$

The area of the sector is:
$A_{sector} = \frac{1}{2}r^2 * \theta$
$A_{sector} = \frac{1}{2}r^2(2 \arccos(1 - \frac{h}{r}))$
$A_{sector} = r^2 \arccos(1 - \frac{h}{r})$

Substituting these results back into formula (1), we have the area of the shaded region:
$A_{shaded} = [r^2 \arccos(1 - \frac{h}{r})] - [(r - h)\sqrt{-h(h - 2r)}] = \frac{3}{8}\pi r^2$

Now we can apply Newton's Method to h in the recursive formula:
$h_{i+1} = h_i + \frac{f(h)}{f'(h)}$
where $f(h) = A_{shaded} = [r^2 \arccos(1 - \frac{h}{r})] - [(r - h)\sqrt{-h(h - 2r)}] - \frac{3}{8}\pi r^2 = 0$