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Infrasound Analysis Proposal

Figure 1. Location of Villarrica, Los Rios Region, Chile

Infrasonic data is obtained using instruments that detect low frequency sounds that are below the limits of human hearing at 20 Hz. This is a common geophysical method. The project I plan on completing for the Math 365 end of semester project is to determine azimuthal direction of incoming signal solely from infrasonic and position data obtained from Villarrica volcano located in central Chile (Figure 1). Figure 2. Illustrates the relative orientation of the instrumentation in UTM coordinates. In order to proceed with the analysis, these will be moved to an arbitrary datum where ch2 is positioned at (0,0). This would mean that ch1 would be at (17,18), and ch3 would be at (-7,15) on the new datum.

The 5 Hz filtered geophysical data will be analyzed using a digital signal processing technique called cross correlation. Cross correlation is a measure of how similar two signals are. Let x = discretized signal 1 and y = discretized signal 2 and length(x) = length(y). These are [n m] = [1 N] dimensional where length(x) = N. Then the cross correlation operation, indicated by the star operator, is as follows:

$$x[n] \star y[n] = \sum_{n=0}^{N-1} x[n]y[n] \qquad \text{Eq. (1)}$$

My program will use the normalized cross correlation operation:

$$x[n] \star y[n] = \frac{\sum_{n=0}^{N-1} x[n]y[n]}{\sqrt{\sum_{n=0}^{N-1} x^2[n] \sum_{n=0}^{N-1} y^2[n]}} \qquad \text{Eq. (2)}$$

where the denominator is the scaling factor to limit output values between -1 and 1. These correlations will be completed using the MatLab built in function xcorr( ).

Lag times can be obtained by finding the maximum indexed cross correlation value. Let lag time sample # 2,1 = max _index between signal channels 2 and 1 and lag time sample # 2,3 = max _index between signal channels 2 and 3. After finding the midpoint of the number of samples, subtract the maximum sample number from the midpoint sample number, and divide it by the number of samples per second. This will yield a lag time between the two signals in the cross correlation.

After the lag times are found, I will then utilize a for loop to test the indexed unit vector of the incoming signal at all possible orientations. If the channel position vector is dotted with this new array, we'll obtain a matrix of [360, 2] values indicating the dot product at each possible orientation of wave propagation. In order to find the best fit angle of orientation, the following formula must be minimized:

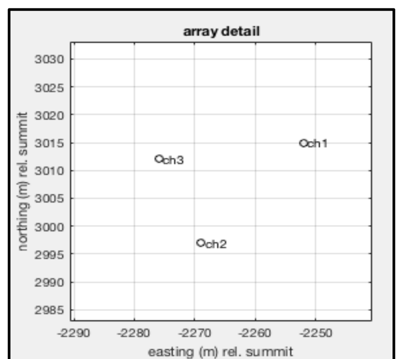

Figure 2. Relative locations of infrasound channels in UTM

$$\epsilon = (\text{dot sample \# 2,1} - \text{lag time sample \# 2,1})^2 + (\text{dot sample \# 2,3} - \text{lag time sample \# 2,3})^2$$

The minimum value of this error function should tell us where the error is exactly zero, or the orientation in which the dot product is the smallest. The index of the minimum value will correspond to the azimuthal direction I was looking for.