Gradient Descent Multivariable.

for each sample.

$$\theta_j := \theta_j - \alpha \frac{1}{m} \cdot \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) x_j^{(i)}$$

$$X = \begin{pmatrix} - x - \\ - x_i - \\ - x_m - \end{pmatrix}$$

in our implementation,

$$\theta_j = \theta_j - \alpha \frac{1}{m} \cdot \sum_{i=1}^{m} \left(x^{(i)} \cdot \theta - y^{(i)} \right) \cdot x_j^{(i)}$$

$$= \theta_j - \frac{\alpha}{m} \sum_{i=1}^{m} \left(x^{(i)}\theta - y^{(i)} \right) x_j^{(i)}$$

$$\underline{\theta} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_j \\ \vdots \\ \theta_n \end{pmatrix}$$

scalar.

$$= \theta_j - \frac{\alpha}{m} \sum \begin{pmatrix} - x^{(1)}\theta - y^{(1)} - \\ \vdots \\ x^{(m)}\theta - y^{(m)} \end{pmatrix} \cdot \left(x_j^{(1)} \cdots x_j^{(m)} \right)$$

$$= \theta_j - \frac{\alpha}{m} \cdot \mathrm{sum} \begin{bmatrix} (x^{(1)} \cdot \theta - y^{(1)}) \cdot x_j^{(1)} \\ \vdots \\ (x^{(m)}\theta - y^{(m)}) \cdot x_j^{(m)} \end{bmatrix}$$

θ : $(n+1) \times 1$.

$$X = \begin{array}{c} \begin{array}{ccc} x_1 & \cdots & x_n \end{array} \\ \begin{pmatrix} \vdots & \cdots & \\ \vdots & & \end{pmatrix} \end{array} \; m$$

$(m+1) \times (n+1)$

$$= \theta_j - \frac{\alpha}{m} \cdot \left[\begin{pmatrix} | \\ X\theta - y \\ | \end{pmatrix} \underbrace{\left(x_j^{(1)} \cdots x_j^{(m)} \right)}_{(1 \times m)} \right]$$

column vector $(m \times 1)$

same feature (j) in diff. samples. (a column before)

$$= \theta_j - \frac{\alpha}{m} \cdot \left(\underbrace{(X\theta - y)}_{(m \times 1)} \cdot X(:, j) \right)$$

$$\underline{\theta} = \begin{pmatrix} \theta_0 \\ \vdots \\ \theta_n \end{pmatrix} - \frac{\alpha}{m} \cdot \underbrace{(X\underline{\theta} - \underline{y})}_{m \times 1} \cdot \overbrace{\underbrace{\begin{pmatrix} X(:, 1) \\ \vdots \\ X(:, j) \\ X(:, n) \end{pmatrix}}_{n \times m}}^{X^T} = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} - \frac{\alpha}{m} \cdot \underbrace{\begin{pmatrix} X(:, 1) \\ \vdots \\ X(:, n) \end{pmatrix}}_{n \times m} \cdot \underbrace{(X\underline{\theta} - \underline{y})}_{m \times 1}$$

↑↓ dimension!

Question 3 Cash Flow Analysis.

Part A) We want to compare the present value of

① the machine cost

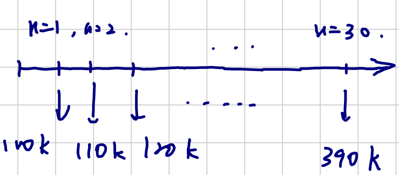
② annuity of labour cost.

$$PV_{labour} = 15000 \cdot (P/A, 12\%, 8)$$

$$= 15000 \cdot 6.1944 = 92\,916.$$

The maximum the company can pay for the machine is the present worth of the annuity cost from labour, \$92,916.

Part B) Option 1. PV growing annuity



PU = PV (basc annuity) + PV (arithmic gradient annuity)

$$= 100k \cdot (P/A, 8\%, 30) + 10k \cdot (P/G, 8\%, 30)$$

$$= 100k \cdot (11.2578) + 10k \cdot (103.4558)$$

$$= 2160.338\ K$$

$$= 2160,338$$

option 2 : PV = 2500,000. ∴ option 2 (\$2500000 today) is a better option.

Part c).

① Get FV. of \$80,000 at 52 month.

② Get FV of annuity payment at 52 month., for 51 periods

③. subtract ① − ②.

$n = 52$ (months) =

$r = \frac{12\%}{12} = 1\%$

Final payment = $\$80{,}000 \cdot (F/P, 1\%, 52) - \$2000\,(F/A, 1\%, 51) \cdot 1.01$

$= \$80{,}000\,(1.6777) - 2000 \cdot 66.1078 \cdot 1.01$

$= 678.244$

∴ Final payment is \$678.24.

Question 4. Comparison method.