Langmuir Probes

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1 Introduction

This report on the physics of Langmuir probes is a creative summary of section 3 of "Principles of Plasma Diagnostics" by I. H. Hutchinson [1], section 6 of "Principles of Plasma Discharges and Materials Processing" by M.A. Lieberman and A.J. Lichtenberg [2] and Lecture 4 of "Principles Of Plasma Diagnostics" MIT open course by Prof. J.D. Hare [3].

A Langmuir probe is a plasma diagnostics device that perturbs and measures the subsequent current response to diagnose plasma properties. It applies a voltage sweep and measures the I-V characteristic curve where properties such as electron density n_∞, electron temperature T_e, and plasma potential V_∞ can be calculated. They are generally constructed from a thin high-melting temperature wire of which a specific section is uninsulated to the plasma. This report will only consider that of cylindrical tips.

The goal of this report is to find a physical description of the function and phenomena occurring within a Langmuir probe and find a mathematical description of a plasma in terms of measurable quantities.

2 Physics description

2.1 Flux description

For an isotropic homogeneous plasma the particle flux density Γ, entering a single side of a planar surface (planar condition holds for basic cylindrical probes) is;

$$\Gamma_j = \frac{1}{4} n_j \bar{v}_j, \tag{1}$$

where $\bar{v}$ is species mean velocity, n is species number density and the factor of 1/4 arises statistically. For a simple two species thermal plasma of equal and oppositely charged electrons and ions, the mean ion velocity will be much smaller than the mean electron velocity due to its much greater mass and thus $\bar{v}_i + \bar{v}_e \simeq \bar{v}_e$. The net current entering the

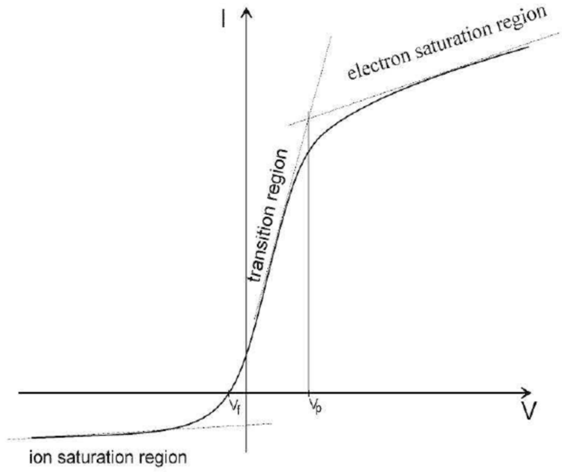

Figure 1: I-V characteristic response of a Langmuir probe in plasma. [4] (not to scale)

surface is;

$$I = -eA(\Gamma_i - \Gamma_e), \tag{2}$$
$$= -\frac{1}{4}eA(n_i\bar{v}_i - n_e\bar{v}_e) \simeq \frac{1}{4}eAn_e\bar{v}_e, \tag{3}$$

where A is the effective collection area of the probe tip, for cylindrical probes, $A_{cyl} = 2\pi rL$, for spherical, $A_{sph} = 4\pi r^2$.

For a probe placed into a plasma (a "floating" probe), incident electrons and ions charge the probe until it reaches an equilibrium. The probe will charge negative until all electrons are repelled and the net current drops to zero. This zero-current potential is called the floating potential denoted by V_f. The plasma potential denoted by V_∞ is defined to be 0 and thus is the datum for all potential values.

If a potential greater than the plasma potential is placed on the probe, electrons will be attracted to the probe, and ions will be repelled. The current would increase until, in principle (approximately in practice), the electron current saturates which occurs at the plasma potential, V_∞. This current is called the electron saturation current, denoted by I_{se}. Similarly, when a potential less than the plasma potential is placed on the probe, electrons start to repel and ions start to attract. As the potential decreases further, all electrons are repelled and the ion current saturates. This current is called the ion saturation current denoted by I_{si}. This current voltage response can be seen in Figure 1.

2.2 Debye shielding

Plasma is a quasi-neutral fluid, i.e. $n_i \simeq n_e$. This neutrality holds at length scales greater than the "Debye Length" denoted by λ_D, past which the electric field within the plasma can be considered zero. Plasma has a strong "shielding" effect where in any local break of neutrality, electrons will shift to counteract the electric field. Due to $m_i \gg m_e$, temperature of the ions can be considered zero and are thus stationary. The electrons are approximated by a thermal equilibrium Maxwellian distribution where density is approximated with a Boltzmann factor of form;

$$n_e \simeq n_\infty \exp\left(\frac{eV}{T_e}\right), \tag{4}$$

where n_∞ is free-stream density. Poisson's equations can then be constructed;

$$\nabla^2 V = -\frac{\rho}{\epsilon_0} = \frac{-e}{\epsilon_0}(n_i - n_e). \tag{5}$$

By assuming that $n_i = n_\infty$ due to ions unaffected by potential, Eq. 5 can be written as;

$$\nabla^2 V = \frac{-e}{\epsilon_0} n_\infty \left(1 - \exp\left(\frac{eV}{T_e}\right)\right). \tag{6}$$

We approximate $\exp\left(\frac{eV}{T_e}\right) \simeq 1 + \frac{eV}{T_e}$ by assuming $eV \ll T_e$ and thus can represent Eq. 6 as;

$$\nabla^2 V - \frac{1}{\lambda_D^2} V = 0, \tag{7}$$

where the Debye Length λ_D is defined as;

$$\lambda_D = \left(\frac{\epsilon_0 T_e}{e^2 n_\infty}\right)^{1/2}. \tag{8}$$

The potential response of the plasma is described by;

$$V \propto \exp\left(\frac{\pm x}{\lambda_D}\right), \tag{9}$$

which notably drops off over significantly shorter lengths scales than that of free space's $\propto \frac{1}{r}$ as for a low temperature plasma, $(T \sim 1\ eV,\ n \sim 10^{17}\ m^{-3})$ $\lambda_D \sim 20\ \mu m$.

2.3 Collisional effects

If the concentration condition;

$$n\lambda_D^3 \gg 1, \tag{10}$$

is met, then the mean free path, $\ell \gg \lambda_D$ and thus the probe sheath can be considered collisionless. Macroscopically, the probe's length scale, a, must meet the condition;

$$\ell \gg a, \tag{11}$$

to be collisionless which can be easily satisfied as for a low temperature plasma, ($T \sim 1\ eV,\ n \sim 10^{17}\ m^{-3}$) $\ell \sim 5\ cm$. Derivations of collisional conditions in Eq. 10 and Eq. 11 are found in section 3.1.3 of Hutchinson's [1].

2.4 Sheath physics

When a probe is placed into a plasma, the potential drop is contained to a region around the probe with thickness of order of the Debye length. This region is called the "Sheath" where quasineutrality is not valid and thus $n_i \neq n_e$. As the sheath is thin, the planar assumption of the probe is still valid.

Consider a probe which is biased negative such that it attracts ions and repels electrons, $n_i > n_e$. This is generally the most important case for plasma diagnostics and the case that will be investigated below.

The condition for sheath formation is;

$$V_0 < -\frac{T_e}{2e}, \tag{12}$$

where V_0 is the potential at the probe. The potential at the sheath/plasma boundary, V_s, where quasinetrality breaks is given by;

$$V_s = -\frac{T_e}{2e}. \tag{13}$$

Derivations of sheath voltage conditions in Eq. 12 and Eq. 13 are found in section 3.2.1 of Hutchinson's [1].

The ion flux continuity from the sheath to probe surface can be written as;

$$\Gamma_i = n_i\left(s\right)\overline{v}_i\left(s\right) = n_i\left(0\right)\overline{v}_i\left(0\right). \tag{14}$$

This allows the properties at the sheath to be analysed not at the probe surface. As the sheath boundary, $n_e - n_i \ll n_e$ and thus is still considered to be quasineutral. The ion density therefore can be equated to the electron density found in Eq. 4;

$$n_{si} \simeq n_{se} = n_\infty \exp\left(\frac{eV_s}{T_e}\right), \tag{15}$$

and by substituting the value for V_s from Eq. 13 in to n_{si};

$$n_{si} = n_\infty \exp\left(-\frac{1}{2}\right) \simeq 0.61 n_\infty. \tag{16}$$

The ions are assumed to be "cold" meaning the free-stream velocity of the ions can be set $\overline{v}_{\infty i} = 0$. This assumption holds for $T_i < T_e$. The velocity of the ions is then assumed to be solely from the kinetic energy gained from the voltage drop to the sheath, $V_s = -\frac{T_e}{2e}$ and thus has form;

$$|v_i| = \left(-\frac{2eV_s}{m_i}\right)^{\frac{1}{2}}, \tag{17}$$

$$= \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}. \tag{18}$$

The ion current at the probe is therefore;

$$I_i = -eA\Gamma_i = -eA_s n_{si}|v_{si}|, \tag{19}$$

$$= -eA_s n_\infty \exp\left(-\frac{1}{2}\right)\left(\frac{T_e}{m_i}\right)^{\frac{1}{2}} \simeq -0.61 n_\infty e A_s \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}, \tag{20}$$

where notably, A_s is area of the sheath not the probe and as the ions are considered cold and thus anisotropic, there is no factor of $1/4$.

Under the assumption that the probe is biased negative enough that most electrons are repelled, the electron properties are given by its thermal equilibrium values where at probe voltage V_0, electron density is given by Eq. 4;

$$n_{0e} = n_\infty \exp\left(\frac{eV_0}{T_e}\right), \tag{21}$$

and mean electron velocity due to being a Maxwellian distribution is;

$$\overline{v}_e = 2\left(\frac{2T_e}{\pi m_e}\right)^{\frac{1}{2}}, \tag{22}$$

The election current at the probe is therefore;

$$I_e = eA\Gamma_e = \frac{1}{4} eA_0 n_{0e} \overline{v}_{0e}, \tag{23}$$

$$= \frac{1}{4} eA_0 n_\infty \exp\left(\frac{eV_0}{T_e}\right) \cdot 2\left(\frac{2T_e}{\pi m_e}\right)^{\frac{1}{2}}, \tag{24}$$

where A_0 is area of the probe.

The net current through the probe is then;

$$I = I_i + I_e, \tag{25}$$

$$= n_\infty e A_0 \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}} \left[\frac{1}{2}\left(\frac{2m_i}{\pi m_e}\right)^{\frac{1}{2}} \exp\left(\frac{eV_0}{T_e}\right) - \frac{A_s}{A_0}\exp\left(-\frac{1}{2}\right)\right]. \tag{26}$$

The area factor for cylindrical probes is;

$$\frac{A_s}{A_0} \approx \left(1 + \frac{x_s}{a}\right), \tag{27}$$

where for spherical probes is the area factor squared and the sheath thickness x_s is approximately constant with a small proportionality to probe potential V_0 (full formula seen in Appendix 1). For $x_s \ll a$, the area factor is ~ 1 and thus can be ignored. For hydrogen, $x_s \sim 4\lambda_D$, thus in a low temperature plasma, ($T \sim 1\ eV,\ n \sim 10^{17}\ m^{-3}$), $x_s \sim 80\ \mu m$. For significantly large probe potentials, sheath thickening does affect current and this can be seen by the magnitude of the ion saturation current continually increasing as the potential decreases in Figure 1.

2.5 Current Response Analysis

For probe potentials $V_0 \ll -T_e/2e$, the electron current terms in Eq. 26 can be ignored as $\exp(eV_0/T_e) \to 0$. When the probe size, a, is sufficiently large such that $\lambda_D \ll a$ then the area ratio A_s/A_0 is approximately unity. The probe current in Eq. 26 then approximately becomes the ion saturation current;

$$I \simeq I_{si} = -n_\infty e A_s \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2}\right) \simeq -0.61 n_\infty e A_s \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}. \tag{28}$$

At the floating potential V_f; $I = 0$, both of which are measurable quantities. The electron temperature T_e can then be found using Eq. 26 in terms of known quantities in form;

$$\frac{eV_f}{T_e} = \frac{1}{2}\left[\ln\left(2\pi\frac{m_e}{m_i}\right) - 1\right]. \tag{29}$$

The problem is that V_f is defined relative to the plasma potential which was defined as 0, thus $V_f = V(I=0) - V_\infty$. The plasma potential V_∞ is measurable at the potential at which the electron current saturates, but in reality, this is not accurately measurable, thus other analysis is performed.

Instead, the slope of the current around V_f is found as;

$$\left.\frac{dI}{dV_0}\right|_{V_f} = \frac{e}{T_e}\left(I - I_{si}\right) + \frac{dI_{si}}{d_{V_0}} \simeq \frac{e}{T_e}\left(I - I_{si}\right), \tag{30}$$

where the dI_{si}/d_{V_0} term is from I_{si} dependence on V_0 through dA_s/dV_0, but as the dependence is small, the term is ignored.

This is a better representation of T_e in terms of known quantities as there is no need to estimate the plasma potential. For analysis, $\ln|I - I_{si}|$ should be plotted against V_0 around V_f to find;

$$\left.\frac{d\ln|I - I_{si}|}{dV_0}\right|_{V_f} = \frac{e}{T_e}, \tag{31}$$

thus;

$$T_e = \frac{e}{\left.\frac{d\ln|I-I_{si}|}{dV_0}\right|_{V_f}}. \tag{32}$$

In addition, by plotting I against potentials below V_f, the value for I_{si} can be accurately estimated. The determined value of T_e can then be placed into the formula for I_{si} found in Eq. 28 and solve for n_∞ in terms of know quantities;

$$n_\infty = \frac{I_{si}}{eA_0\left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}\exp\left(-\frac{1}{2}\right)}. \tag{33}$$

2.6 Effect of magnetic fields

The greatest effect a magnetic field has on the plasma is the species gyrate according to their Larmor radius ρ_L. Electrons are more greatly effected due to their reduced mass which will effect the value for I_{se}. If ion Larmor radius satisfies $\rho_{Li} \gg a$, then the electron gyration effect is minimal and the previous analysis is valid.

For a strong magnetic field resulting in $\rho_{Li} \ll a$, the plasma is considered quasi-collisionless and presheath physics needs to be considered. The resultant change in ion current at the probe can be estimated by;

$$\Gamma_{si} = -n_\infty\left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}\frac{\sqrt{2}}{\pi\eta_s^{\frac{1}{2}}} \simeq -0.49n_\infty\left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}, \tag{34}$$

where $\eta = \frac{-eV}{T_e}$ and it was found that $\eta_s = 0.854$. The effective collection area A_0 now becomes the projection of the probe surface in the direction of the magnetic field. The dependence on any spatial variable or magnetic field strength is very low such that if the condition of $\rho_{Li} \ll a$ is met, then Eq. 34 is valid and constant. Comparing this to the fieldless analysis, the only significant change is the factor of $\exp(-1/2) \simeq 0.61$ in I_{si} has been replaced by that of $\sqrt{2}/\left(\pi\eta_s^{1/2}\right) \simeq 0.49$. It is generally accepted that if a plasma is within a strong magnetic field, then the value of 1/2 is used. By placing this new expression for ion flux at the sheath into Eq. 19, ion saturation current in a strong magnetic field is;

$$I_{si} \simeq -\frac{1}{2} n_{\infty} e A_0 \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}. \tag{35}$$

In the presence of a strong magnetic field one must be careful propagating this change in coefficient through equations of consequence. This has not been done any further in this report. Derivations of magnetic field effects on ion saturation current in Eq. 35 is found in section 3.3 of Hutchinson's [1]. There are many other collisional magnetic fields effects described further in literature but most agree on a value of 1/2 for the plasma ion saturation current where any subsequent changes fall within data error $\lesssim 10\%$.

2.7 Non-hydrogenic plasmas

The density n_{∞} is the free-stream electron density and if ions are singly charged, it is the free-stream ion density. For non-hydrogenic plasmas with ion charges of Ze, the formulae are corrected by replacing m_i with m_i/Z and accounting for greater than unity charge for currents. More generally, for a mixture of ion species j, of charge Z_j and mass m_j, m_i is replaced with;

$$\sum \frac{m_j n_j}{n_e}, \tag{36}$$

where the ratio n_j/n_e accounts for charge difference due to quasineitrality condition. Derivations of non-hydrogenic plasma effects are found in section 3.2.5 of Hutchinson's [1].

2.8 Types of probes

The goal has been achieved in finding plasma properties, i.e. n_{∞}, T_e, in terms of measurable quantities from the I-V characteristic of a Langmuir probe.

2.8.1 Single Probe

A single probe has an identical I-V characteristic to that analysed previously. It consists of a singular high-temperature probe conductor tip with a coaxial ceramic insulation and depending on the expected heat flux of plasma, a high-temperature coaxial shield. This coaxial construction can be seen in Figure. 2.

The voltage is sweeped around the floating potential with a somewhat arbitrary delta of;

$$\Delta V = \pm 5 T_e, \tag{37}$$

to ensure ion saturation current is reached where T_e is in energy units. In most cases the electron saturation current is not needed for analysis but if needed, the plasma should be probed positive to the plasma potential which generally can occur in the region of;

$$V_{\infty} \sim (8-12) T_e \tag{38}$$

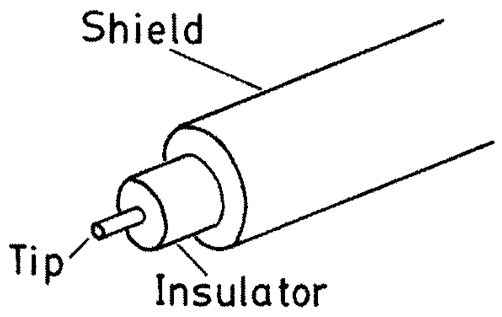

Figure 2: General coaxial construction of the tip of a Langmuir Probe. [1]

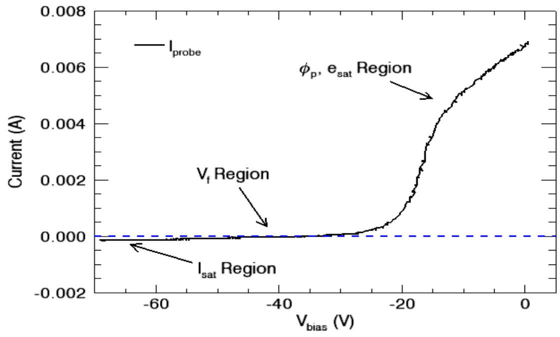

Figure 3: Actual I-V Characteristic of a single probe. [5]

The I_{si} and V_f regions need to be graphed and using Eq. 28 or 35 depending on magnetic field strength and Eq. 32, plasma density and temperature values calculated.

The floating potential is an important parameter as it is the midpoint of the voltage sweep. A crude estimate of the floating potential is;

$$V_f \sim (2-3)T_e. \quad [3] \tag{39}$$

To experimentally determine the floating potential V_f, the probe can be placed into the plasma with 0 voltage bias and the subsequent voltage measured. If a temperature and plasma potential estimate is already known, the floating potential can be estimated using a rearrangement of Eq. 29;

$$V_f - V_\infty = \frac{T_e}{2e}\left[\ln\left(2\pi\frac{m_e}{m_i}\right) - 1\right], \tag{40}$$

which for hydrogen is approximately;

$$V_f \simeq V_\infty - 3.34T_e. \tag{41}$$

It is easy to see in Figure. 3 that the electron saturation current and plasma potential are not easily defined values whereas the ion saturation current is relatively constant and measurable. If the plasma potential is a desired parameter and one knows V_f and T_e, then V_∞ can be found using Eq. 41.

The downside to a single probe is that the floating potential needs to be "guessed" and there is a possibility that the potential applied draws well in the electron saturation region and it damages the probe. The voltage sweep also takes a certain Δt and thus for highly transient plasma phenomena the single probe is inadequate.

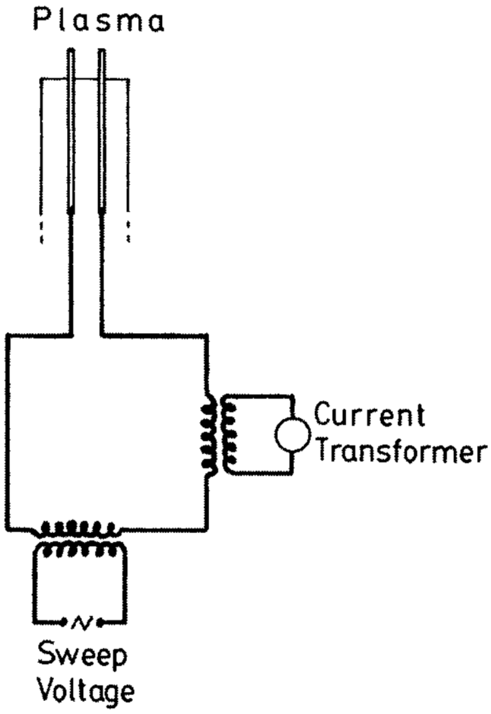

Figure 4: General circuit diagram and construction of a double probe. [1]

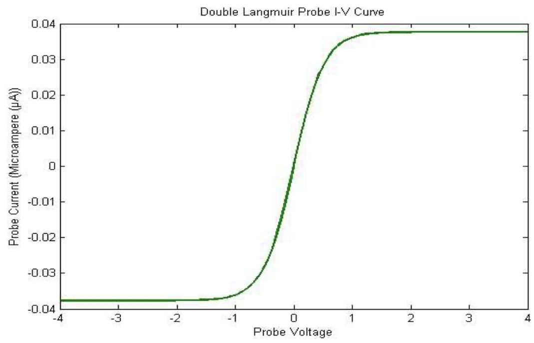

Figure 5: Actual I-V Characteristic of a double probe. [6]

2.8.2 Double Probe

A double probe consists of two floating probes biased together with a voltage. The general circuit diagram is seen in Figure. 4. The benefits of using a double probe is that it bounds the current drawn for a large positive potential to the negative of the ion saturation current. The I-V characteristic adopts a curve with form;

$$I = I_{si} \tanh\left(\frac{eV_0}{2T_e}\right), \tag{42}$$

which can be seen in Figure. 5. Similarly, to a single probe, the voltage must be sweeped, curve graphed, and the values for I_{si} and the slope of the curve at V_f estimated. The formula for the slope of the I-V characteristic at the floating potential is now;

$$\left.\frac{dI}{dV_0}\right|_{V_f} = \frac{I_{si}}{2T_e}, \tag{43}$$

where the ion saturation current is found similarly to the that of the single probe. The single probe's current problem has been fixed in the double probe but the finite time of voltage sweep is still a problem and thus is inadequate for highly transient plasma phenomena.

2.8.3 Triple Probe

A triple probe has a very similar construction to the double probe with the addition of a single floating probe. By applying a certain constant bias voltage to the double probe and allowing the floating probe to reach the floating potential, 3 points of the I-V characteristic are measured, V_f, I_{si} and $-I_{si}$. 3 points is enough to exactly define an exponential thus there is no need for a voltage sweep to estimate the curve. This allows the probe to measure faster phenomena than either a single or double probe alone. The constant V_{bias} needs to be specifically chosen so that ion saturation current is met and the underlying exponential assumption must be accurate or the fitted curve will produce inaccurate results.

References

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Appendix

1. x_s

$$\frac{x_s}{\lambda_D} = \frac{2}{3}\left[\frac{2}{\exp(-1)}\right]^{\frac{1}{4}}\left[\left(\frac{-eV_0}{T}\right)^{\frac{1}{2}} - \frac{1}{\sqrt{2}}\right]^{\frac{1}{2}}\left[\left(\frac{-eV_0}{T}\right)^{\frac{1}{2}} + \sqrt{2}\right]^{\frac{1}{2}}. \tag{44}$$

The coefficient $\frac{2}{3}(2/\exp(-1))^{1/4} \simeq 1.02$ and if $V_0 \sim V_f$ then $eV_0/T_e \sim \frac{1}{2}\ln(m_e/m_i)$ which for hydrogen is ~ 3.75. x_s remains relatively constant unless in significantly large probe potentials.