PHYS2114 Laboratory 1: Transient and steady-state response of an RLC Circuit

J.J. Whittaker (z5363798)[1,2]
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This experiment explores the transient and steady-state response of a Resistor, Inductor and Capacitor (RLC) circuit to verify response driving values and confirm theoretical relationships. From the transient response of the circuit, it was found that the damping constant, $k = 3507 \pm 82$ and the period of oscillation, $T = 64 \pm 0.5 \times 10^{-6}\ s$. From the steady-state response, it was found that the Q-factor, $Q = 15.0 \pm 1.2$, the bandwidth, $\Delta\nu = 1043 \pm 78\ Hz$ and the resonant frequency, $\nu_0 = 15599 \pm 17\ Hz$.

INTRODUCTION

The Resistor, Inductor and Capacitor (RLC) circuit is an oscillatory circuit that oscillates due to the out of phase storing and release of energy in the magnetic field of the inductor and the electric field of the capacitor. This oscillation can be seen in multiple ways, as a transient response to a perturbation in the circuit, or a steady state (resonant response) from an oscillatory forcing signal.

For the **transient response** like that seen in FIG. 3, the voltage in the circuit would oscillate around $0\ V$ at its *natural frequency* ω_n slowly losing magnitude due to the resistance in the circuit. This can be modelled as;

$$V_c(t) = \frac{q_0}{C} e^{-\left(\frac{R}{2L}\right)t} \cos(\omega_n t), \qquad [1]$$

where;

$$\omega_n = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \qquad [2]$$

If we assume that the circuit is lightly damped, then we can assume;

$$\frac{1}{LC} \gg \frac{R^2}{4L^2} \qquad \text{and thus} \qquad \omega_n = \frac{1}{\sqrt{LC}}. \qquad [3]$$

The function for voltage can be transformed to a function of n, number of peaks;

$$V_n = \frac{q_0}{C} e^{-k(nT - t_0)}, \qquad [4]$$

such the gradient $-kT$ can be extracted through a log transformation, where the damping coefficient k and period T;

$$k = \frac{R}{2L} \qquad \text{and} \qquad T = \frac{2\pi}{\omega_0}. \qquad [5]$$

For the **steady state response** where the battery and switch seen in FIG. 3 is replaced with an oscillatory signal, the signal does not decay and instead is forced at the frequency that the oscillatory signal is applied at. The current through the circuit (proportional to voltage through the circuit) can be modelled as;

$$I(t) = \frac{V_0}{|Z|} e^{i(\omega t - \theta)}, \qquad [6]$$

where the magnitude of the current is determined by the Impedance $|Z|$, and the phase response by θ defined by;

$$|Z| = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]^{\frac{1}{2}} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right). \qquad [7]$$

The maximum magnitude of voltage/current and lowest phase response occurs at the lowest impedance $|Z|$ which occurs at the frequency that minimises $\omega L - \frac{1}{\omega C}$. This is the reactance caused by both the inductor and capacitor, and when its minimised, their reactance's are equal but opposite and cancel each other out. This frequency is called the resonant frequency which occurs at the natural frequency of the circuit;

$$\omega_0 = \frac{1}{\sqrt{LC}}. \qquad [8]$$

We define two other parameters associated with the steady state response of an RLC circuit called the Bandwidth $\Delta\omega$ and the Q-Factor Q. The bandwidth of the circuit is the width of the range of frequencies that are within $\frac{1}{\sqrt{2}}$ of the maximum response magnitude which occurs at resonance. This is defined as;

$$\Delta\omega = \frac{R}{L}. \qquad [9]$$

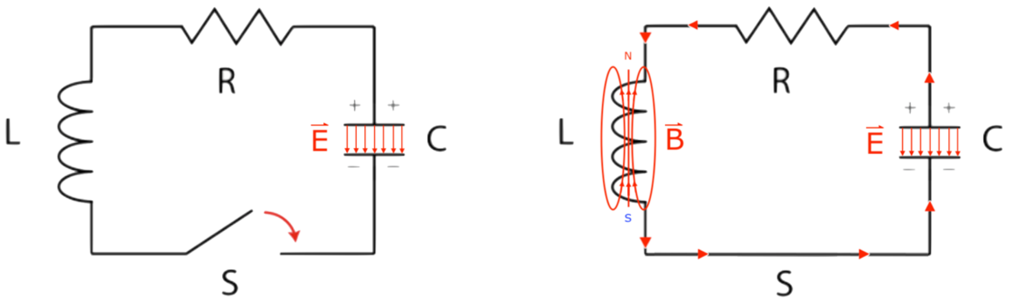
The Q-Factor is a value that quantifies the ratio between how much energy is stored and how much is dissipated per cycle of an RLC circuit. An RLC circuit with a high Q-factor indicates that it has a very steep increase to its resonance response, and it also implies that energy can be stored in the oscillations for a long time. On the other hand, a low Q-factor circuit has a broad resonance and energy in the oscillations are more quickly dissipated. The Q-factor is defined by;

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}. \qquad [10]$$

PRE-WORK

Theory Pre-work

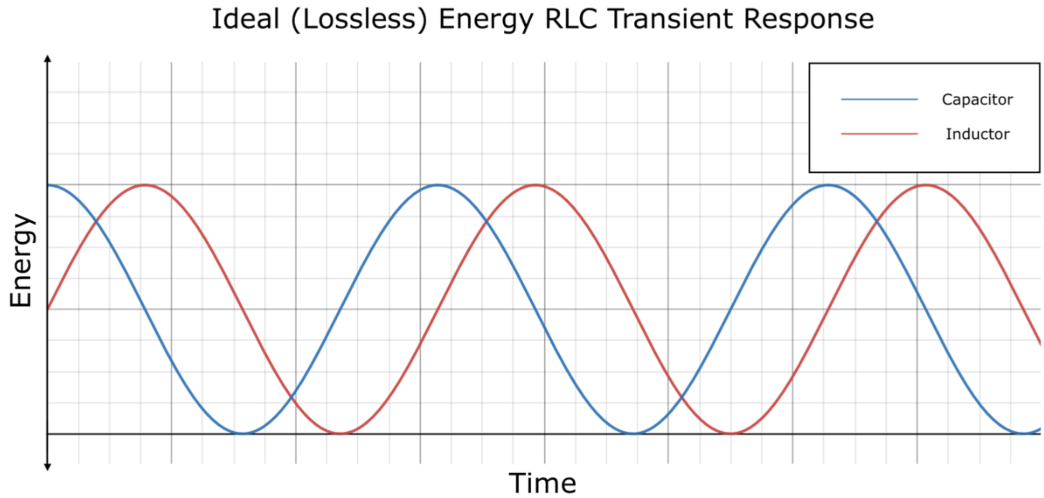
Question. 1 a)



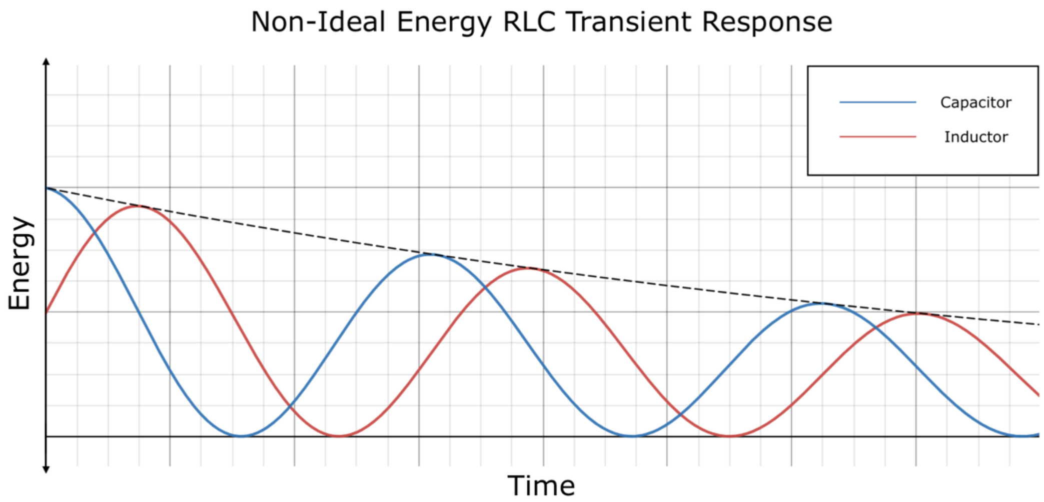
(a) Circuit diagram when switch is open. (b) Circuit diagram when switch is closed ($t = 0$).

FIG. 1: RLC Circuit diagrams showing electric and magnetic fields.

Question 1. b)



(a) Energy relationship of an ideal RLC circuit.



(b) Energy relationship of a non-ideal RLC circuit.

FIG. 2: Energy transfer relationship between energy stored in the capacitor and inductor of ideal and non-ideal RLC circuit.

Question 2.

$$L = \frac{N\Phi_0}{I}, \quad [11]$$

$$\Phi_0 = BA. \quad [12]$$

Substitute into [2] into [1];

$$\therefore\ L = \frac{NBA}{I}. \quad [13]$$

$$\oint Bds = \mu_0 I_{enc}, \quad [14]$$

$$n = \frac{N}{l} = \text{desnity of loops.} \quad [15]$$

Multiply [4] by [5];

$$\therefore n \oint Bds = B = \mu_0 I_{enc} n = \frac{\mu_0 IN}{l}. \quad [16]$$

Substitute [6] into [3];

$$\therefore L = \frac{N^2 \mu_0 IA}{lI} = \frac{N^2 \mu_0 A}{l}. \quad [17]$$

$$A = \pi r^2 = \pi \left(\frac{D}{2}\right)^2 = \frac{\pi}{4} D^2. \quad [18]$$

Substitute [8] into [7];

$$\therefore L = \frac{N^2 \mu_0 \pi}{4l} D^2. \quad [19]$$

As it is an ideal close packed solenoid, we can assume that;

$$N = \frac{l}{d} \quad \Rightarrow \quad N^2 = \frac{l^2}{d^2}. \quad [20]$$

Substitute [10] into [9];

$$L = \frac{l^2 \mu_0 \pi}{4l} \frac{D^2}{d^2} = \frac{\mu_0 \pi l}{4} \left(\frac{D}{d}\right)^2. \quad [21]$$

Question 3.

Calculate the inductance of the solenoid, assuming it is ideal. Assume that $l = 110\, mm$, $D = 82.5\, mm$ and $d = 0.5\, mm$.

$$L = \frac{\mu_0 \pi l}{4} \left(\frac{D}{d}\right)^2, \quad [22]$$

$$L = \frac{\pi\, (4\, \pi \times 10^{-7})(0.11)}{4} \left(\frac{82.5}{0.5}\right)^2, \quad [23]$$

$$= 2.955\, mH. \quad [24]$$

Question 4.

For the circuit shown in FIG. 3, assume $L = 2.56\, mH$, the net resistance of the circuit is $12\,\Omega$ and $C = 47\, nF$.

Calculate:

a) the **Resonant Frequency**, ω_0 in Hz:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \qquad [25]$$

$$= \frac{1}{\sqrt{(2.56 \times 10^{-3})(47 \times 10^{-9})}}, \qquad [26]$$

$$= 91170\, rad\, s^{-1}, \qquad [27]$$

$$= 14510\, Hz. \qquad [28]$$

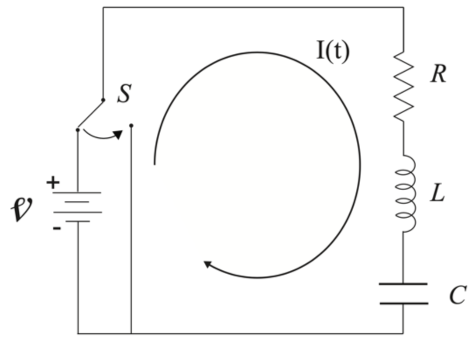


FIG. 3: A series circuit consisting of a resistor, an inductor, and a capacitor, subject to an instantaneous voltage change when the switch is opened.

b) the **Q-factor**, Q:

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}, \qquad [29]$$

$$= \frac{1}{12}\sqrt{\frac{2.56 \times 10^{-3}}{47 \times 10^{-9}}}, \qquad [30]$$

$$= 19.45. \qquad [31]$$

c) the **Bandwidth**, $\Delta\omega$:

$$\Delta\omega = 2|\delta\omega|, \qquad [32]$$

$$= \frac{R}{L}, \qquad [33]$$

$$= \frac{12}{2.56 \times 10^{-3}}, \qquad [34]$$

$$= 4688\, rad\, s^{-1}, \qquad [35]$$

$$= 746.0\, Hz. \qquad [36]$$

Experimental Pre-work

1. Transient Response.

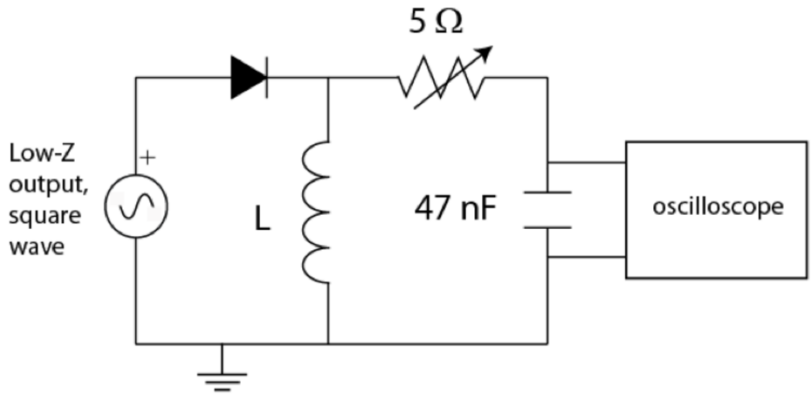


FIG. 4: Experimental arrangement for measuring the transient response of the RLC circuit.

Question 1.

With the aid of the experimental diagram provided in Fig. 4 describe how the capacitor is charged and discharged in order to replicate the response of the circuit depicted in Fig. 3.

The capacitor is charged by the voltage applied by the square wave. When the square signal switches, it stops charging the capacitor and the capacitor discharges as if the source did not exist. As the voltage source is connected in parallel to the inductor, there is no change in voltage and thus no energy stored in the inductor in the charging phase. This replicates the discharge response of the circuit found in FIG. 3.

Question 2.

How does the combination of a square wave passing through a diode act in a similar way to the battery and switch?

When the current is passing through in one direction, it passes through the diode and charges the capacitor like a DC current, like a battery. But when the square wave switches direction, it can't pass through the diode, so no current is passed through, just like if the switch was open.

Question 3.

We can either measure the oscillations in the current (magnetic field) or the voltage (electric field). Why do we connect the oscilloscope across the capacitor?

Measuring the voltage across the capacitor is a direct observation of the voltage behaviour as the voltage across the capacitor is directly related to the electric field within the capacitor plates. It is also much easier to measure the voltage across something as the oscilloscope can be placed in parallel to the circuit where to measure current, the oscilloscope must be placed in series.

Question 4.

An oscilloscope measures periodic signals. Why is this helpful in recording the decay trace.

The decay response doesn't just decay exponentially but oscillates positive to negative as its peak voltage decays exponentially. The oscilloscope can measure this oscillation making it easy to record the decay trace.

Question 5.

We will be using a 160 Hz AC signal to pump the damped RLC circuit. What sets this frequency? Does it have anything to do with the resonance frequency of the circuit? Does it have to do with the damping constant?

The 160 Hz AC signal is a specifically arbitrary number, but it has to avoid some certain values in order for the decay response to be valid. It can't be close to the resonant frequency otherwise the circuit would not decay but the circuit would be driven instead. The frequency has to be low enough so that the circuit has time to completely decay which is determined by the damping constant. If the damping constant is low, then the time for it to completely decay would increase, and the frequency would have to decrease and vice versa.

2. Resonance Response.

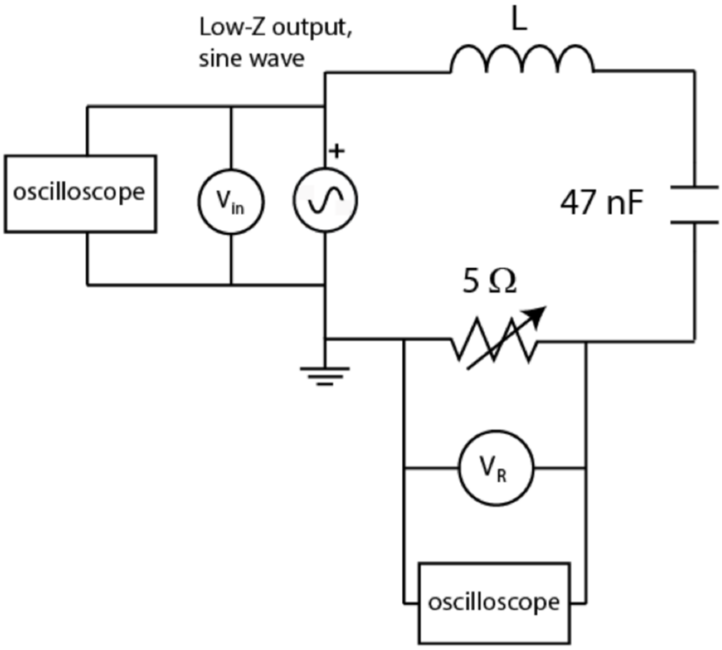


FIG. 5: A driven RLC oscillator circuit. The two oscilloscopes in the diagram represent individual channels on a single oscilloscope.

Question 1.

Assume that the circuit above is an ideal circuit. If you were to output a 1 V r.m.s. sine wave at the resonance frequency of the circuit, what would be the voltage drop across the inductor and capacitor? What would be the voltage drop across the resistor? Express your answer as a ratio $V_R = V_{in}$.

At resonance, the reactance of the capacitor and inductor have equal but opposite magnitudes and thus the impedance of the circuit is purely resistive. If we assume that the inductor and capacitor are ideal and have no resistance, then all the voltage drop is done over the resistor and thus $V_R = V_{in}$ and thus $\frac{V_R}{V_{in}} = 1$. In reality, there will be a net voltage drop over the inductor and capacitor as they do have some resistance.

Question 2.

As you start applying off-resonant sine waves, how does the ratio change?

The ratio will start to get smaller as the reactance of the capacitor and inductor stop cancelling each other out increasing the voltage drop across the capacitor and inductor. This makes the voltage drop across the whole circuit be spread more across the 3 components, thus the voltage drop across the resistor reduces.

Question 3.

Which components are the main energy loss components on and off resonance?

On resonance, the only type of energy loss is resistive which can be from all 3 components as all 3 have a certain resistance but mainly due to the resistor. Off resonance, the reactance's of the Inductor and Capacitor stop cancelling each other out, and the main energy losses come from the Inductor and Capacitor.

Question 4.

List all the components which contribute to the resistance of the circuit. Which resistances are negligible, and which will we need to be included in your analysis?

The resistor, the capacitor, the inductor, and the wires that conduct the electricity. The resistor and the inductors resistance will be considered, but the resistance of the capacitor and the wires will not. The resistance of the wire is negligible, but the resistance of the capacitor is not considered due to it being hard to quantify, but it will affect the experimental value.

Question 5.

How would you use the circuit to measure a resonance curve and the phase response of the circuit. Take into account the fact that the signal generator outputs a non- constant amplitude as a function of frequency.

The voltage ratio is the highest at resonant frequency and thus the voltage ratio can be plotted and at its maximum magnitude, it is at resonant frequency. The V r.m.s is changing through frequencies, but as the ratio of voltages is being considered, this change is constant through both voltages and cancels out. The phase response can be directly measured as it is not affected by the magnitude change.

METHOD

Experiment 1: RLC Transient Response

1. Setup the RLC Circuit in the configuration shown in FIG. 4.
2. Set the signal generator to a 160 Hz square wave with an amplitude of 1V.
3. Set the variable resistor to 5 Ω.
4. Put the oscilloscope across the capacitor and turn on the oscilloscope.
5. Turn on the signal generator.
6. Press "Auto-scale" on the oscilloscope and observe the display.
7. Use the cursor function on the oscilloscope and measure the voltage, position in time, and order of all the discernible peaks of the decay trace.
8. Record measurements and turn off the signal generator.

Experiment 2: RLC Resonance Response

1. Setup the RLC Circuit in the configuration shown in FIG. 5.
2. Set the signal generator to a 10 kHz sine wave with an amplitude of 1V.
3. Set the variable resistor to 5 Ω.
4. Put channel 1 of the oscilloscope across the resistor, put channel 2 of the oscilloscope across the signal generator and turn on the oscilloscope.
5. Place one of the multimeters across the resistor and place the other across the signal generator and turn them both to input Voltage.
6. Turn on the signal generator.
7. Press "Auto-scale" on the oscilloscope and observe the 2-function display.
8. Using the cursor on the oscilloscope, record the peak voltage of both functions and record the phase difference between both functions.
9. 'Sweep' the frequency range until the 2 functions become in phase.
10. Using broad increments in frequency, repeat step 7 for each broad increment from 10 kHz up to and past the frequency that the functions become in phase (resonant frequency).
11. Now using finer increments of frequency, repeat step 7 for each fine increment around the frequency that the functions become in phase (resonant frequency).
12. Record measurements and turn off the signal generator.

RESULTS & ANALYSIS

The measured values of the resistance of the resistor and inductor, inductance, and capacitance are;

$$R_R = 5 \pm 0.5\,\Omega, \quad R_L = 6.5 \pm 0.05\,\Omega, \quad L = 2.18 \pm 0.005\,mH, \quad C = 48.4\ \pm 0.05\,nF. \qquad [37]$$

1. Transient Response

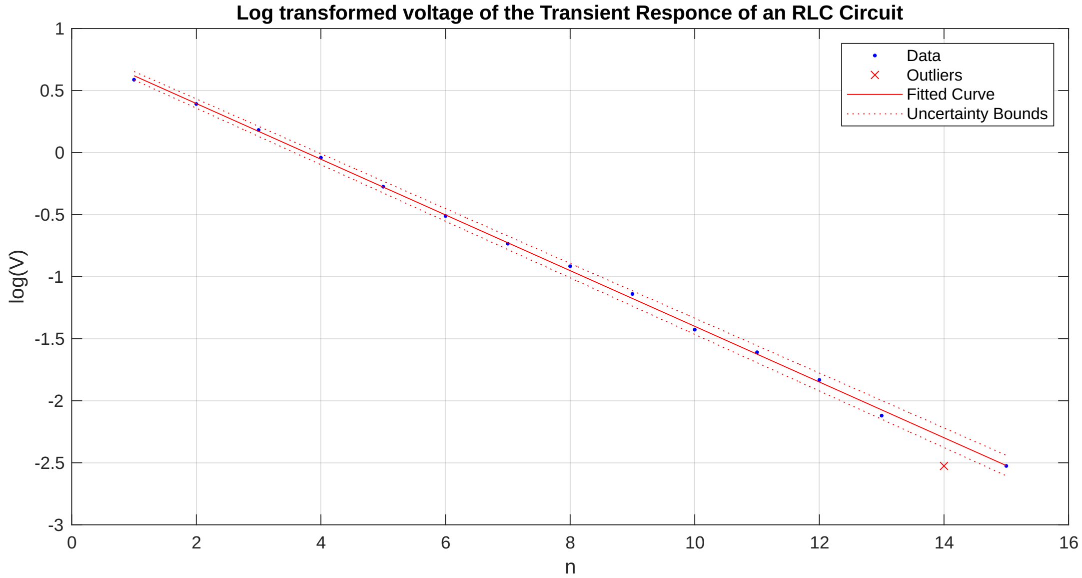


FIG. 6: A graph of the log transform of the decay voltage of the peaks of the transient response of the RLC circuit seen in FIG. 4.

To find the **damping coefficient *k***, the log transformed voltage function was fitted to a linear function in MatLab seen in FIG. 6 where the function was estimated to be;

$$\log V = (-0.2245 \pm 0.0035)\, n + (0.845 \pm 0.030) \qquad [38]$$

From the equation for peak voltage V, as a function of number of peaks n;

$$V_n = \frac{q_0}{C} e^{-k(nT - t_0)}, \qquad [39]$$

the gradient of the log transform of V gives;

$$-kT = -0.2245 \pm 0.0035. \qquad [40]$$

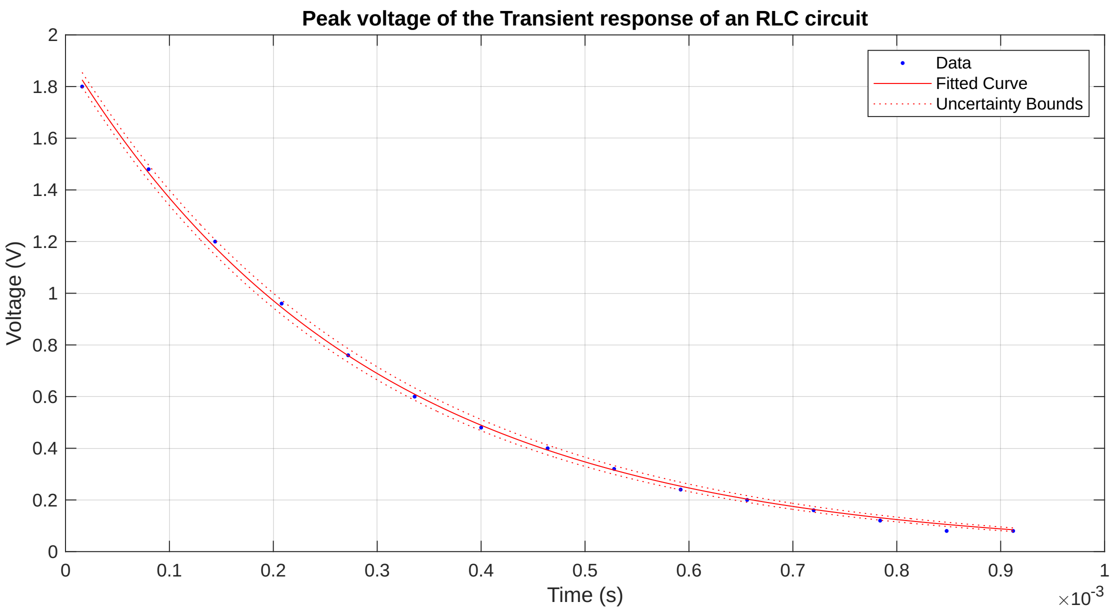


FIG. 7: A graph of the peak voltages of the Transient response of an RLC circuit

Seen in FIG. 7, the **period *T*** of the decay frequency is $T = 64.0 \pm 0.5 \times 10^{-6}\ s$ as the period was measured to the nearest μs. This value can be compared to the theoretical value;

$$T_{theo} = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{1}{\sqrt{LC}}} = \frac{2\pi}{\frac{1}{\sqrt{(2.18 \pm 0.005 \times 10^{-3})(48.4 \pm 0.05 \times 10^{-9})}}} = 64.54 \pm 0.15 \times 10^{-6}\ s. \quad [41]$$

From the value for T, we can estimate the damping constant k by using the gradient found in the linear regression analysis;

$$-kT = -0.2245 \pm 0.0035 \quad [42]$$

$$k = \frac{0.2245 \pm 0.0035}{T} = \frac{0.2245 \pm 0.0035}{(64 \pm 0.5) \times 10^{-6}} \quad [43]$$

$$= 3507 \pm 82. \quad [44]$$

This can be compared to the theoretical value for k which;

$$k_{theo} = \frac{R_{theo}}{2L} = \frac{R_R + R_L}{2L} = \frac{5 \pm 0.5 + 6.5 \pm 0.05}{2(2.18 \pm 0.005 \times 10^{-3})} = 2640 \pm 130. \quad [45]$$

The actual value for k found is much higher than that of its theoretical which is due to there being other sources of resistance in the circuit not the resistor nor the inductor. The actual resistance of the circuit can be estimated by;

$$R_{circuit} = 2kL = 2(3507 \pm 82)(2.18 \pm 0.005) \times 10^{-3} = 15.29 \pm 0.39\ \Omega. \quad [46]$$

2. Resonance Response

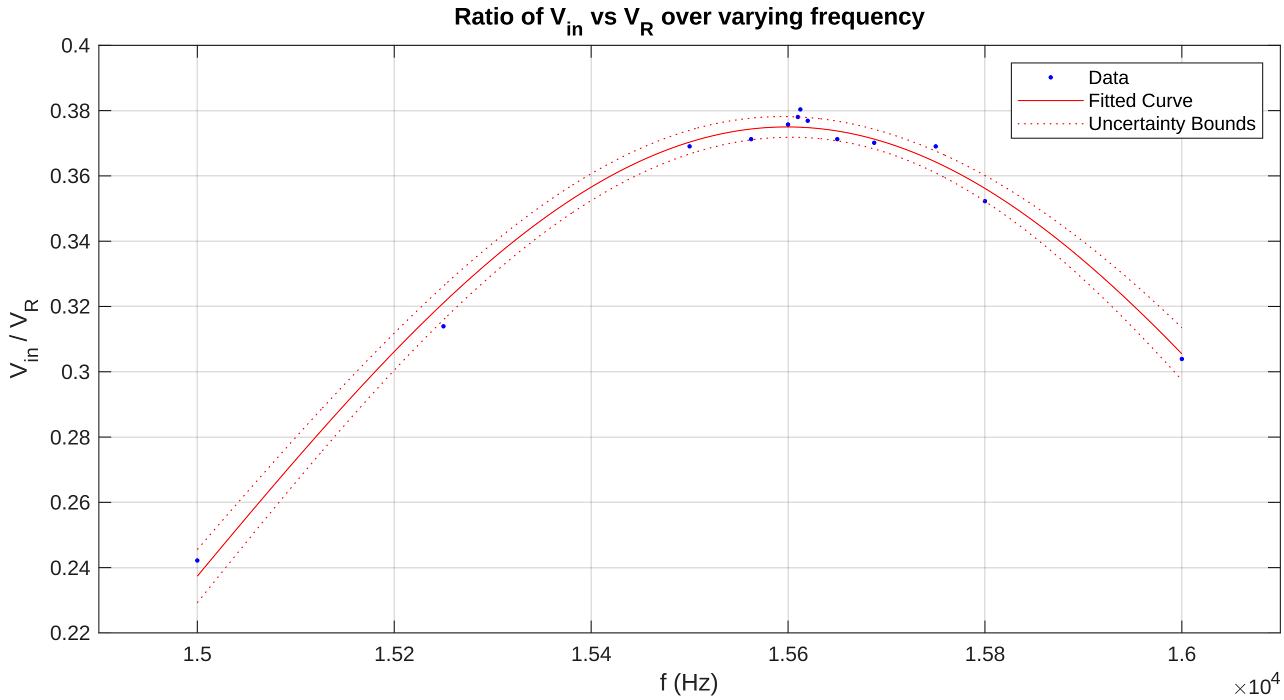


FIG. 8: A graph of the ratio of relative voltage drop across circuit to frequency displaying the frequency of least voltage drop, resonant frequency.

To find **resonant frequency $\boldsymbol{\nu_0}$**, as discussed in the experimental pre-work, the maximum of the Voltage ratio frequency relationship in FIG. 8 must be found. The relationship was fitted to a gaussian function in MatLab which was estimated to be;

$$\frac{V_{in}}{V_R} = (0.3750 \pm 0.0032)\, e^{-\left(\frac{\nu-(1.5599\pm0.0017)\times10^4}{(886\pm37)}\right)^2}, \tag{47}$$

which has a maximum when $\nu = (1.5599 \pm 0.0017) \times 10^4\ Hz$ thus;

$$\nu_0 = 15599 \pm 17\ Hz, \tag{48}$$

corresponding to a;

$$\frac{V_{in}}{V_R} = 0.3750 \pm 0.0032. \tag{49}$$

The theoretical value for resonant frequency is;

$$\nu_{0_{theo}} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(2.18 \pm 0.005)\times10^{-3}(48.4 \pm 0.05)\times10^{-9}}} \tag{50}$$

$$= 15494 \pm 26\ Hz, \tag{51}$$

which is close but lower than the calculated experimental value of the resonant frequency of the circuit.

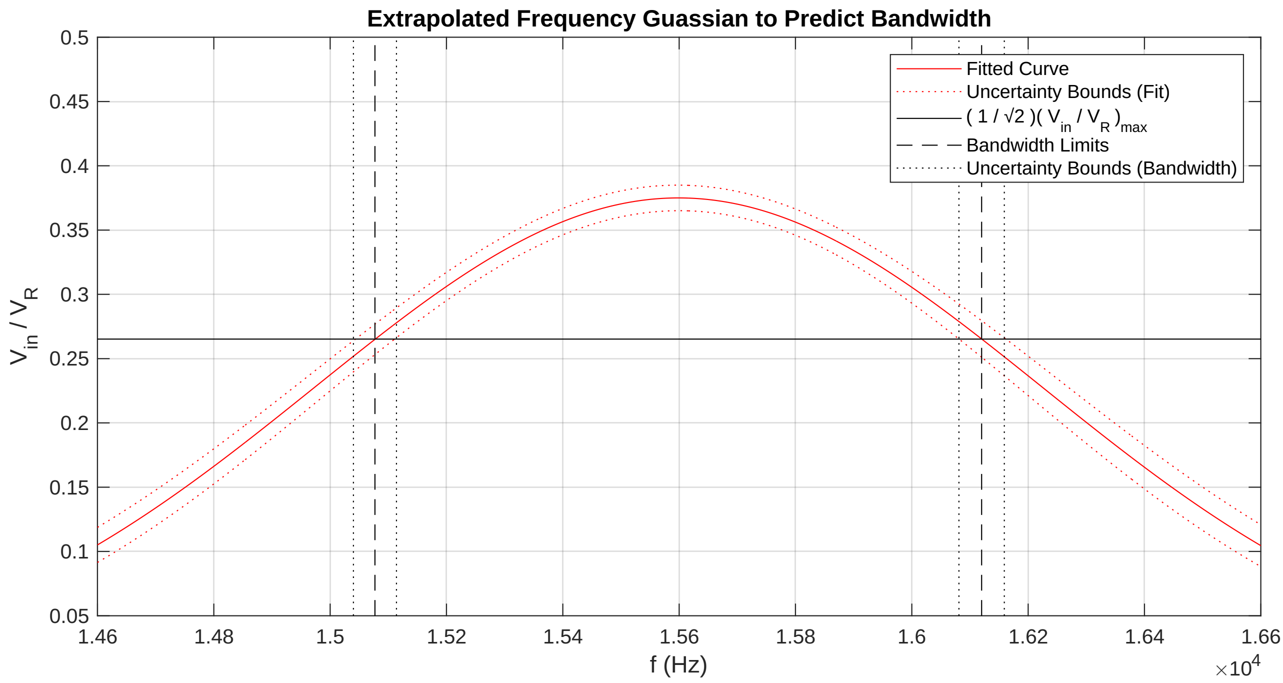


FIG. 9: A graph of the estimated relationship between voltage ratio and frequency, estimating the value of frequency for $\frac{1}{\sqrt{2}}\frac{V_{in}}{V_R}$ corresponding to bandwidth limits.

To find **bandwidth $\Delta\nu$,** of the circuit, the Gaussian used to estimate the relationship of Voltage ratio to frequency must be extrapolated to find when the Voltage ratio drops to $\frac{1}{\sqrt{2}}$ of its peak value seen in FIG. 9. The two frequencies found at this value are the bandwidth limits and the frequency difference between these limits is defined as the bandwidth.

$$\vec{\nu} = [15077 \pm 39, 16120\ \pm 39]\ Hz, \tag{52}$$

thus;

$$\Delta\nu = 1043 \pm 78\ Hz. \tag{53}$$

This can be compared to the theoretical value for bandwidth $\Delta\nu$;

$$\Delta\nu_{theo} = \frac{R_{circuit}}{2\pi L} = \frac{15.29 \pm 0.39}{2\pi(2.18 \pm 0.005) \times 10^{-3}} = 1114 \pm 34\ Hz, \tag{54}$$

which is within the uncertainty of the calculated experimental value of the bandwidth of the circuit.

The **Q-factor Q**, of the circuit can be found by;

$$Q = \frac{\nu_0}{\Delta\nu} = \frac{(15599 \pm 17)}{(1043 \pm 78)} = 15.0 \pm 1.2. \tag{55}$$

This can be compared to the theoretical value for the Q-factor Q;

$$Q_{theo} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{15.29 \pm 0.39}\sqrt{\frac{(2.18 \pm 0.005) \times 10^{-3}}{(48.4 \pm 0.05) \times 10^{-9}}} = 13.88 \pm 0.39, \tag{56}$$

which is close but lower than the calculated experimental value for the Q-factor of the circuit.

The assumption that the circuit is lightly damped can now be verified by confirming that;

$$\frac{1}{LC} \gg \frac{R^2}{4L^2} \quad \Rightarrow \quad \omega_0^2 \gg \frac{(\Delta\omega)^2}{4}, \tag{57}$$

$$(15599)^2 \gg \frac{(1043)^2}{4} \quad \Rightarrow \quad 2.43 \times 10^8 \gg 2.72 \times 10^5, \tag{58}$$

which verifies that the circuit is lightly damped.

DISCUSSION

The uncertainty in the report was made to a relative error of 2 sig.fig. and the value the uncertainty is associated with was taken to the absolute error of its uncertainty. The uncertainty was propagated through calculations by taking the absolute value of the maximum deviation from the calculated value, a symmetrical form of the upper-lower bound method of uncertainty propagation. All data was approximately fitted to a function through the MatLab 'fit' function. The uncertainty of the function values was taken as a 95% confidence interval (approximately a 2σ error), which is what is propagated through the calculations.

The values for resistance, inductance and capacitance were given/measured directly and are taken to a relative error of their significant figures. This means the calculated theoretical values have uncertainties. In the calculation of the damping coefficient k, a data point was taken as an outlier (seen in FIG. 6) as its approximately outside the 3σ range of the data without it included. The error approximation in the estimate of the period T was taken to the relative error of its significant figures ($\pm 0.5\ \mu s$) as that is the limit of the measuring apparatus. In reality the error should be half of the width of the peak that was being measured but the data collected did not include enough information to be able to do so. The error in the calculated value for k was taken from the error in the approximation of the gradient propagated through with the discussed error in the period. The error in the value for resonant frequency and bandwidth were taken through the error in approximation of their values from the fitted Gaussian and propagated through to the calculation of the Q-Factor.

The experimental values found were generally accurate to their theoretical counterpart but where the values deviated is where the resistance of the circuit was a major component of the calculation. This is mainly seen in the theoretical value of the damping coefficient k. After this, the value for resistance of the circuit, was estimated from the experimental calculation of resistance from the found value of k. This made the values for bandwidth and Q-Factor which both have resistance as a major component of their calculations significantly more accurate to their theoretical counterpart. The circuit resistance should have been measured directly to avoid this issue. The results recorded for the resonant section of the experiment should also have been done over a larger frequency range so that the data could be more accurately fitted to a gaussian. This would have largely increased the accuracy of experimental value for bandwidth.

CONCLUSION

This experiment successfully explored the transient and steady-state response of a Resistor, Inductor and Capacitor (RLC) circuit, verifying the response driving values and confirming theoretical relationships. The values of bandwidth $\Delta\nu = 1043 \pm 78\ Hz$ and period $T = 64 \pm 0.5\ \mu s$ were both within the uncertainty from the theoretical value with an accuracy of 6.4% and 0.84% respectively. The values for resonant frequency $\nu_0 = 15599 \pm 17\ Hz$ and Q-Factor $Q = 15.0 \pm 1.2$ are close but higher than their calculated theoretical value with an accuracy of 0.67% and 7.5% respectivly. The most inaccurate value is that of $k = 3507 \pm 82$ which has an inaccuracy of 25%. This is due to the inaccuracy of the value for circuit resistance which is in turn why the accuracy for bandwidth and Q-Factor are so low.

REFERENCES

PHYS2114 Experimental Physics Student Notes: RLC Circuits

RLC Circuits OPERATING INSTRUCTIONS

R-L-C Circuits and Resonant Circuits by the Ohio State University - https://www.asc.ohio-state.edu/gan.1/teaching/summer05/Lec4.pdf

APPENDIX

1.

Table 1: Circuit Properties

Inductance L	$2.18\ mH$
Resistance R	$5\ \Omega$
Capacitance C	$48.4\ nF$
Resistance of Inductor V_L	$6.5\ \Omega$

2.

Table 2: Results for RLC Transient Response, Peak voltage vs Time.

Peak Voltage (V)	Time (μs)	Peak Number n
1.80	16	1
1.48	80	2
1.20	144	3
0.96	208	4
0.76	272	5
0.60	336	6
0.48	400	7
0.40	464	8
0.32	528	9
0.24	592	10
0.20	656	11
0.16	720	12
0.12	784	13
0.08	848	14
0.08	912	15

3.

Table 3: Results for RLC Resonant Response, Frequency vs Voltage Ratio.

Frequency (kHz)	V_{in} (mV)	V_R (mV)	$\frac{V_{in}}{V_R}$
15.0000	120	512	0.2344
15.2500	124	408	0.3039
15.5000	124	336	0.3690
15.5625	124	336	0.3690
15.6000	122	336	0.3631
15.6100	122	328	0.3720
15.6125	122	320	0.3813
15.6200	122	336	0.3631
15.6500	122	336	0.3631

15.6875	122	336	0.3631
15.7500	122	336	0.3631
15.8000	122	352	0.3466
16.0000	120	408	0.2941