PHYS3113: Stirling Engine

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[2] *Word count: 1669 words*
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This experiment showed the operation of a Stirling Engine and its reversible thermodynamic cycle. It qualitatively showed the relationship between rotational speed, temperature difference and applied load across both mechanical and electrical load applications failing to create reliable quantitative results. It qualitatively showed the thermodynamic cycle as a PV-diagram but failed to produce reliable quantitative data on power output. The experiment shows the operation of the Stirling engine as a refrigerator.

INTRODUCTION

The Stirling engine is an externally combusted heat engine which uses cyclic heating and cooling of a working fluid to produce work and drive the engine. The cycle can be broken down into 4 processes, I. an isothermal expansion with a heat addition. II. An isochoric depressurisation accompanied by a heat rejection. III. An isothermal compression with a heat rejection and IV. isochoric pressurisation accompanied with a heat addition. The difference between the heat addition is I and the heat rejection in III is the work that is created by the engine also presented as the area contained within the cycle.

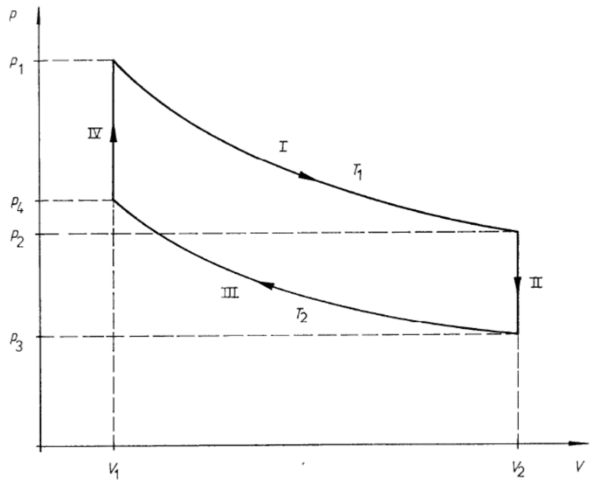

Figure 1: Thermodynamic cycle of the Ideal Stirling engine on a PV-diagram. [1]

(There's not a lot of background information for this experiment instead the experiment focuses on load cases and special operation configurations.)

PRE-WORK

Question 1: Derive expressions for the heat flow in (or out) and the work done by each step in the Stirling cycle.

For the Isothermal Expansion (1 → 2), work can be expressed as:

$$w_{12} = \int_1^2 P dv,$$

which from the ideal gas law can be expressed as;

$$w_{12} = kT_H \int_1^2 \frac{1}{v} dv\,,$$
$$= kT_H \ln\left(\frac{v_2}{v_1}\right).$$

As the process is isothermal, there is no change in internal energy thus heat addition can be expressed as;

$$0 = q_{12} - w_{12},$$

$$q_{12} = w_{12} = q_{in}.$$

This can be done similarly for the Isothermal Compression (3 → 4) thus;

$$w_{34} = kT_C \ln\left(\frac{v_4}{v_3}\right),$$

and;

$$q_{34} = w_{34} = q_{out}.$$

The isochoric processes produce no work thus assuming an ideal monatomic gas;

$$q_{23} = \Delta u_{23} = \frac{3}{2} k\Delta T_{23},$$

and;

$$q_{41} = \Delta u_{41} = \frac{3}{2} k\Delta T_{41},$$

The two isochoric heat additions cancel out as;

$$\Delta T_{23} = -\Delta T_{41}.$$

For a Stirling engine operating with an ideal monatomic gas, the temperature of the upper isotherm is $400K$ while the lower isotherm is $300K$. The maximal volume is $10\ m^3$ while the minimal volume is $2\ m^3$.

Question 2: Calculate the heat flow in each step.

$$q_{12} = w_{12} = kT_H \ln\left(\frac{v_2}{v_1}\right),$$
$$= (1.381 \times 10^{-23})(400) \ln\left(\frac{10}{2}\right),$$
$$= 8.891 \times 10^{-21}\ JN^{-1}.$$

$$q_{34} = w_{34} = kT_C \ln\left(\frac{v_3}{v_4}\right),$$
$$= (1.381 \times 10^{-23})(300) \ln\left(\frac{2}{10}\right),$$
$$= -6.668 \times 10^{-21}\ JN^{-1}.$$

$$q_{23} = -q_{41} = \frac{3}{2} k\Delta T_{23},$$
$$= \frac{3}{2}(1.381 \times 10^{-23})(300 - 400),$$
$$= -2.072 \times 10^{-21}\ JN^{-1}.$$

Question 3: Calculate the work done by the gas in the cycle.

Work done by the cycle can be expressed as;

$$w_{net} = w_{12} + w_{34},$$

which from the result in Question 1 can be expressed as;

$$\begin{aligned} w_{net} &= q_{12} + q_{34}, \\ &= (8.891 - 6.668) \times 10^{-21} JN^{-1}, \\ &= 2.223 \times 10^{-21} JN^{-1}. \end{aligned}$$

Question 4: Calculate the efficiency of the engine.

The efficiency of the engine can be expressed as;

$$\begin{aligned} \eta = \frac{w_{net}}{q_{in}} &= \frac{2.223 \times 10^{-21}}{8.891 \times 10^{-21}}, \\ &= 0.2500 = 25\%. \end{aligned}$$

This can be confirmed as the efficiency of the engine can be expressed as;

$$\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{400} = 0.25.$$

Question 5: Explain how the torque meter works.

When the torque meter is tightened to the spinning shaft, the friction applies a torque to the shaft. The deflection of the torque meter lifts a weight which is equal to the torque applied to the shaft. The deflection is measured on a calibrated scale which shows the magnitude of the load applied.

METHOD

Experiment 1: Mechanical Power

1. Setup the Stirling engine as seen in Figure 3 of the operating instructions. [1]
2. Ensure the torque meter is loosely fit to the shaft.
3. Ignite the flame under the hot side of the engine.
4. Spin the flywheel manually to kick-start the engine.
5. Allow the engine to reach its maximum speed.
6. Slowly tighten the torque meter and record the angular velocity, applied load and temperature difference.
7. Repeat step 6 incrementally until the engine is close to stalling.

Experiment 2: Electrical Power

1. Remove the torque meter and scale.
2. Place the generator into position and attach the belt to the large pulley.
3. Connect the variable resistor to the generator.
4. Connect the multimeters, one in series and one in parallel to record current and voltage respectively.
5. Apply a resistance of 10kΩ.
6. Ignite the flame under the hot side of the engine.
7. Spin the flywheel manually to kick-start the engine.
8. Slowly reduce the resistance on the variable resistor incrementally and record voltage, current, angular velocity, and temperature difference.
9. Repeat step 8 until the engine is close to stalling.

Experiment 3: PV-Diagram

1. Using the same setup as experiment 2, log in to the computer and open the graphing software.
2. Repeat steps 6 to 9 of experiment 2 except now record and save the pressure volume data from the software at each resistance increment.

Experiment 4: Refrigeration

1. Remove all electric connections from the engine, leaving the generator.
2. Remove the burner.
3. Flip the generator into motor mode and attach the motor to the 12V output such the engine runs in reverse.
4. The motor will start driving the engine.
5. Record the temperature difference of the engine at 30 second increments until the temperature difference stops changing.

RESULTS & ANALYSIS

Experiment 1: Mechanical Power

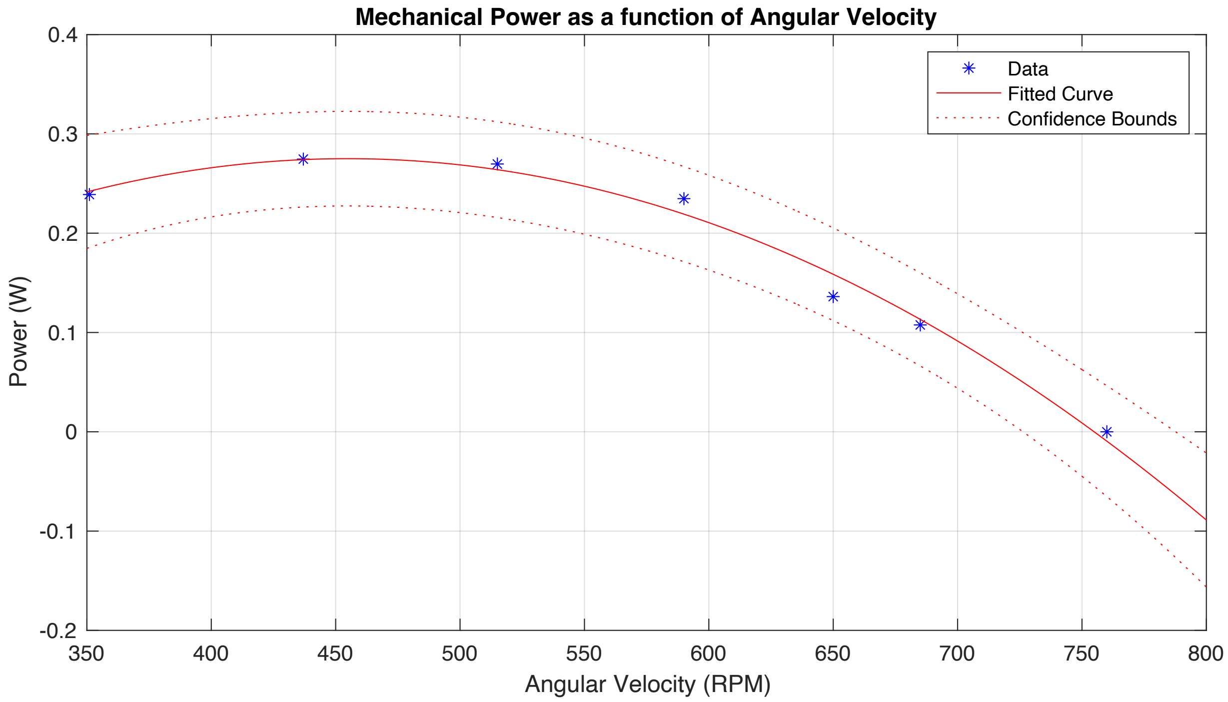

Figure 2: Graphed power curve of the torque meter setup of the Stirling engine.

The optimal angular frequency for power output is estimated by maximising a quadratic fit of the results and error taken from the upper and lower bounds of the 95% confidence interval. The estimated value is;

$$\omega_{mech} = 454 \pm 32\, rpm.$$

Experiment 2: Electrical Power

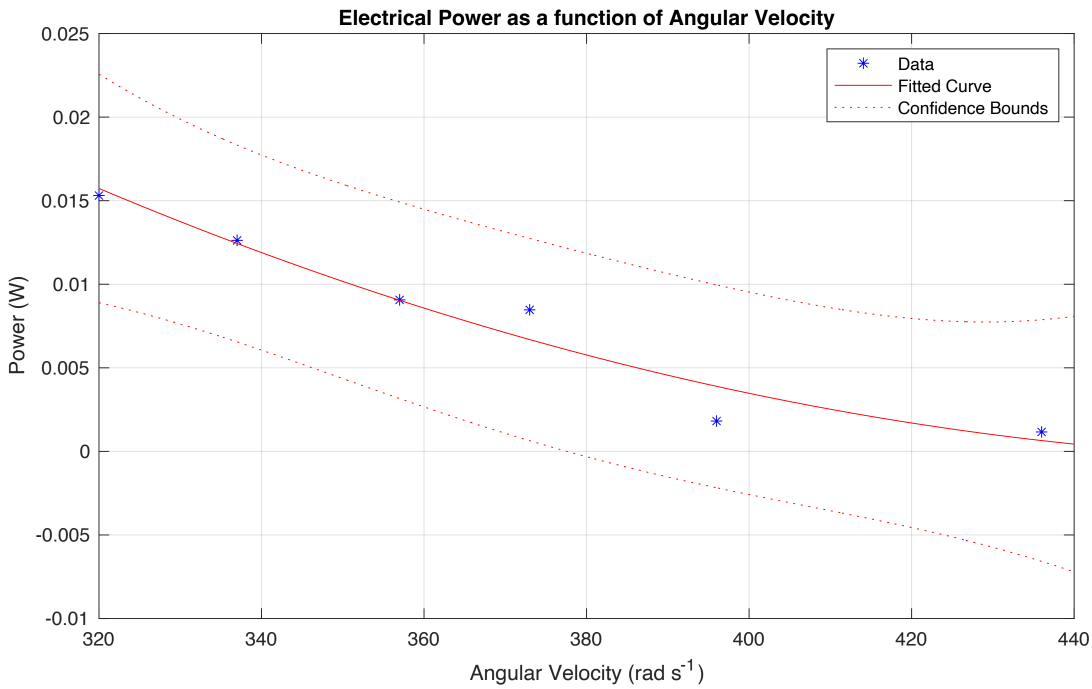

Figure 3: Graphed power curve of the electrical resistance setup of the Stirling engine.

The trials that were conducted produced no discernible maximum to find an optimal angular frequency. This experiment failed.

Experiment 3: PV-Diagram

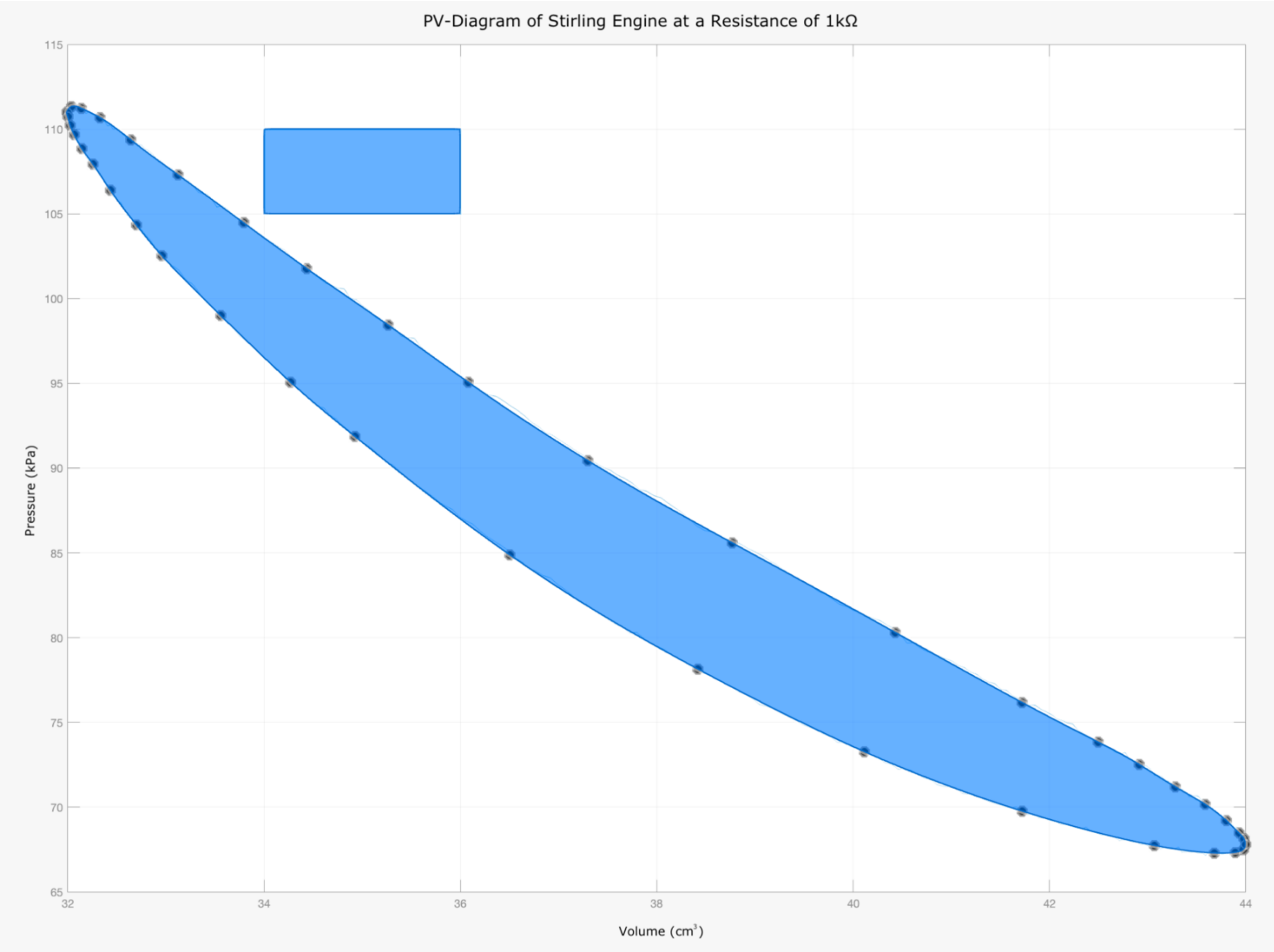

Figure 4: Image of the PV-diagram at 1kΩ resistance traced in Fusion360 to obtain relative area.

The area of the cycle was traced in Fusion360 using splines which produces an estimate of the area. The tracing was repeated multiple times and the results averaged. The uncertainty in the area was taken as the maximum difference to the average. The same process was used to estimate the area of a unit square on the grid. The estimated areas are;

$$\text{Unit Square} = 2.204 \pm 0.006\, u^2, \qquad \text{Cycle Area} = 17.99 \pm 0.74\, u^2.$$

From the units of the axis, kPa for pressure and cm^3 for volume, the work per unit area can be calculated;

$$\begin{aligned}\text{Unit Area} &= \frac{5000\text{ Pa} \times 2\text{ x }10^{-6}\text{ m}^3}{2.204\, u^2}, \\ &= (4.537 \pm 0.025) \times 10^{-3}\, J\, u^{-2}.\end{aligned}$$

Thus, an estimation of work per cycle can be calculated;

$$\begin{aligned}w_{cyc} &= \left((4.537 \pm 0.025) \times 10^{-3}\right)(17.99 \pm 0.74) \\ &= 0.0816 \pm 0.0038\, J\, cyc^{-1}.\end{aligned}$$

The angular velocity of the engine was 482 $rpm = 8.033\ Hz$ thus the power output of the engine can be calculated;

$$\begin{aligned}P_{mech} &= (0.0816 \pm 0.0038)(8.033), \\ &= 0.655 \pm 0.031\, W.\end{aligned}$$

This result is extremely confusing as the estimated electrical power output is;

$$\begin{aligned}P_{elec} = VI &= (3.6)(3.2 \times 10^{-3}), \\ &= 0.01152\text{ W}.\end{aligned}$$

Experiment 4: Refrigeration

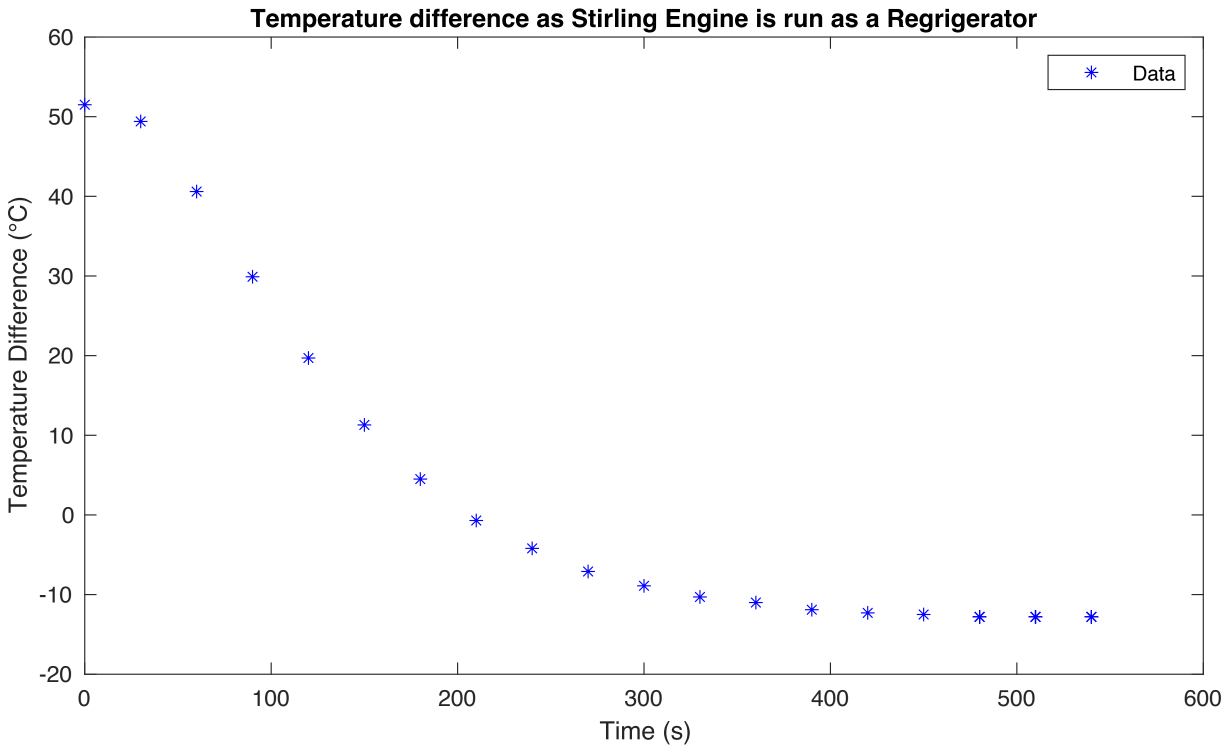

Figure 5: Graphed curve of reducing temperature difference of the engine due to its refrigeration configuration.

By running the Stirling engine in by a motor, the isothermal expansion is no longer heated by the burner and thus sucks heat from the 'hot' arm. The expanded gas then moves towards the tight piston transferring heat away from the 'hot' side. The engine in reverse serves to transfer heat from one reservoir to the other which means after running for a period of time, the cooling ability will reduce. As the temperature difference reduces, the heat addition and rejection approach similarity and thus the refrigerator will reach an equilibrium where it cannot cool the system anymore. For the experiment conducted, the temperature difference asymptotes to $-12.8°C$.

DISCUSSION

For the mechanical power experiment, the data obtained gave a curve which showed the maximum power output across a range of angular velocities. The data was fitted in MATLAB to a quadratic function which was maximised to find the optimal angular velocity. The lower and higher 95% confidence bounds were then also maximised to find the error range. A few trials were completed but never could the same data set be recreated. The values would swing wildly as temperature difference would never stay constant. Perturbation in the air surrounding the engine and the burner would cause drastic swings in power output where the first 30 mins of the experiment was spent attempting to start the engine with no luck. While these results produced the desired curve and obvious maximum, the specific value found varied significantly across trials and thus only qualitative conclusions can be read from this experiment.

For the electrical power experiment, the variation across temperature difference was more severe hindering the possibility to get a range of valid data with controlled variables. During this experiment, the engine stalled many times and only reached approximately 60% of the highest angular velocity of the previous experiment. The air conditioning turned on in between the first two experiments which is believed to be the reason. No conclusions can be read from this experiment as the desired curve was not produced and thus no quantitative data can be drawn.

The PV-diagram produced, successfully showed the engine cycle and the method of finding area seemingly was quite accurate and reliable. The image was traced multiple times in Fusion360 with a maximum 4% difference in area found across trials. The results of the power output vary significantly compared to that of electrical power which is very confusing as the individual methods of obtaining power output seem reliable. My math might be wrong.

The refrigeration experiment was very successful in showing the operation of the Stirling engine as a refrigerator. The quantitative data retrieved, the minimum temperature, was the only consistent result of the entire experiment and this is due to the elimination of the randomness of the heating ability of the burner.

CONCLUSION

This experiment was moderately successful in showing the operation of a Stirling Engine and its reversible thermodynamic cycle. It qualitatively showed the relationship between rotational speed, temperature difference and applied load across both mechanical and electrical load applications failing to create reliable quantitative results. It qualitatively showed the thermodynamic cycle as a PV-diagram but failed to produce reliable quantitative data on power output. The experiment successfully showed the operation of the Stirling engine as a refrigerator.

REFERENCES

[1] - Stirling_Engine_Student Notes_V8-1.pdf