# PHYS2114 Laboratory 2: Magnetic Hysteresis

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[1] *Thursday B 2-5 Class*
[2] *Word count: 1937 words*
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This experiment explores the magnetisation of an iron core in a toroid, exploring the relationship between the applied magnetic field, and the magnetic field produced by the magnetised iron core which presents as a Hysteresis loop. The energy dissipation per cycle of the volume of the iron core was found to be $(E_{diss})_{old} = 9.1 \pm 1.4 \times 10^{-3}\,J$ using the 'old school' method and $(E_{diss})_{new} = 9.51 \pm 0.34 \times 10^{-3}\,J$ using the 'new school' method. The value for saturated remanent magnetisation was found to be $(B_{re})_{sat} = 7.89 \pm 0.21 \times 10^{-3}\,T$ and the minimum applied field intensity to produce maximum magnetisation was found to be $H_m = 974 \pm 33\,Am^{-1}$.

## INTRODUCTION

When a magnetic material is placed in an alternating auxiliary field $H$, the magnetic material will be periodically magnetised. Once the auxiliary field is removed, the magnetisation does not disappear but instead it retains its magnetisation. Thus, placing a magnetic material in an alternating auxiliary field will create a magnetic field in the material, which is phased shifted to the applied field, alternating between positive and negative. Once the auxiliary field has gone to zero, there is a remaining magnetic field from the magnetised material which is seen in FIG.1 as $B_{re}$. The auxiliary field in essence has to magnetise and then demagnetise the magnetic material every cycle. By plotting the produced magnetic field against the applied magnetic field, a hysteresis loop is produced seen in FIG. 1.

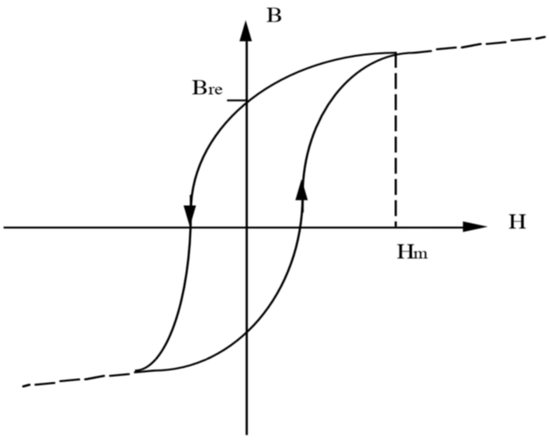


FIG. 1: An example of a hysteresis loop showing the remanent magnetisation and the saturation point.

The magnetisation of the material cannot increase indefinitely as the magnetisation is tied to the amount of material that is present within the auxiliary field. At a certain applied field strength, the magnetic material will become saturated, and the produced magnetic field will not appreciably increase for an increasing applied field. This is called the saturation point and can be seen on FIG. 1 as $H_m$.

Through this process of magnetisation and demagnetisation, eddy currents are produced within the material which dissipates energy as heat thus for every cycle of the hysteresis loop energy is dissipated. The area within the hysteresis loop is equal to the work done to the material per unit volume and thus is equal to the dissipated energy per cycle per unit volume of the material. It can be calculated by;

$$\oint H dB. \qquad [1]$$

## PRE-WORK

**Question 1.**

What hysteresis loop shape would you require for a magnetic material to be used in (a) a permanent magnet (b) a transformer core? Give reasons for your choice.

For a permanent magnet, the magnetisation remains stable as even after the applied auxiliary field $H$ is removed the material retains a high level of magnetic field $B$. Thus, the hysteresis loop would look like a square.

For a transformer core, the ideal hysteresis loop is narrow as possible to reduce the amount of energy dissipated by the core which is proportional to the area contained within the loop.

**Question 2.**

Using Eqn.11 from Student Notes [1] show if $V_{in} = V\cos(\omega t)$ for the circuit in Fig.5 [1] then $V_C = V_0 \sin(\omega t)$ and $V_0 = \frac{V}{RC\omega}$.

$$V_c = \frac{1}{RC}\int V_{in} dt. \qquad [2]$$

Using $V_{in} = V\cos(\omega t)$;

$$V_c = \frac{1}{RC}\int V\cos(\omega t)\, dt, \qquad [3]$$

$$= \frac{V}{RC}\frac{\sin(\omega t)}{\omega} + C. \qquad [4]$$

As there is no voltage across the capacitor at $t = 0$, $C = 0$;

$$\therefore V_c = \left(\frac{V}{RC\omega}\right)\sin(\omega t), \qquad [5]$$

which can be written as;

$$V_C = V_0 \sin(\omega t), \qquad [6]$$

where;

$$V_0 = \frac{V}{RC\omega}. \qquad [7]$$

**Question 3.**

Show that the component values shown in Fig. 3 [1] are suitable for the integration of a 50 Hz signal.

The condition for integration is that;

$$\frac{1}{\omega C} \ll R. \qquad [8]$$

If $C = 8 \times 10^{-6}\, F$, $R = 22000\, \Omega$ and $\omega = 50$ then;

$$\frac{1}{(8 \times 10^{-6})(50)} \ll 22000, \qquad [9]$$

$$2500 \ll 22000. \qquad [10]$$

The condition is satisfied and can be integrated.

**Question 4.**

Derive Eqn. 14 [1], specifically deriving expressions for the scaling factors $k_1$ and $k_2$. Ensure your scaling factors give a final value for $E$ with the correct units: energy dissipated per cycle, and per unit volume of the specimen. You will need to show a dimensional analysis or use units throughout your calculation.

$$E = k_1 k_2 S. \qquad [11]$$

where $S = V_x V_y$ having units of [Volt$^2$] and $E$ has units of [Joule $\times$ Metre$^{-3}$].

Start from;

$$W = \oint HdB, \quad [12]$$

$$= HB, \quad [13]$$

$$= \left(\frac{n_p}{2\pi r R_p} V_y\right)\left(\frac{R_s C}{n_s A} V_x\right) \quad [14]$$

$$= \frac{n_p}{n_s}\frac{R_s}{R_p}\frac{C}{2\pi r A}\, V_y\, V_x. \quad [15]$$

Therefore;

$$k_1 = \frac{n_p}{n_s}\frac{R_s}{R_p}, \quad [16]$$

And;

$$k_2 = \frac{C}{2\pi r A}. \quad [17]$$

$k_1$ is non-dimensional and $k_2$ has units of Farads $\times$ metre$^{-3}$ thus;

$$[k_1 k_2 S] = [\text{Farads}^1 \times \text{Metre}^{-3} \times \text{Volt}^2]. \quad [18]$$

Farads $F$ has units of Coulomb$^1$ $\times$ Volt$^{-1}$ thus;

$$[k_1 k_2 S] = [\text{Coulomb}^1 \times \text{Volt}^1 \times \text{Metre}^{-3}]. \quad [19]$$

Coulombs have units of Amp$^1$ $\times$ Seconds$^1$ thus;

$$[k_1 k_2 S] = [\text{Amp}^1 \times \text{Seconds}^1 \times \text{Volt}^1 \times \text{Metre}^{-3}]. \quad [20]$$

Energy has units of Joule $=$ Amp$^1$ $\times$ Seconds$^1$ $\times$ Volt$^1$ thus;

$$[k_1 k_2 S] = [\text{Joule} \times \text{Metre}^{-3}] = [E]. \quad [21]$$

**Question 5.**

Using the results from the previous section, how would you calculate the total energy dissipated by the iron sample over one hysteresis cycle, $E_{\text{Toroid}}$?

$$\frac{E_{tot}}{\nu} = \oint HdB = \frac{n_p}{n_s}\frac{R_s}{R_p}\frac{C}{2\pi r A}\oint V_x dV_y = k_1 k_2 \oint V_x dV_y, \quad [22]$$

$$\therefore E_{tot} = \nu \times k_1 k_2 \oint V_x dV_y. \quad [23]$$

## METHOD

### 1. Energy dissipation

1. Setup the circuit as seen in FIG. 3 in the student notes [1].
2. Connect the oscilloscope to the computer and open the recording software.
3. Turn on the 12V 50 Hz power supply.
4. Increase the variable resistor till the current through the circuit is 1 Amp.
5. Press ‘auto-scale’ on the oscilloscope and observe the hysteresis Loop.
6. Using the computer software, display the hysteresis loop.
7. Screenshot the hysteresis loop ensuring the grid lines are visible.
8. Print out the screen shot and cut out the hysteresis loop and unit square outside the data points.
9. Weigh the loop and the square and record.
10. Cut inside the lines, weigh the loop and square again. Use this weight difference as the uncertainty.
11. Save the hysteresis loop data for numerical analysis.

### 2. Remanent magnetisation

1. Ensure steps 1 – 4 of the previous method are complete.
2. Save the hysteresis data for 1 Amp using the computer software.
3. Using the variable resistor, decrease the current through the circuit by 0.1 Amps and save the produced hysteresis loop data using the computer software.
4. Repeat step 3 until the current through the circuit reaches 0 Amps.

## RESULTS & ANALYSIS

The given values for the experiment are;

$$n_p = 710 \text{ turns}, \qquad n_s = 800 \text{ turns}, \qquad r = 50\,mm, \qquad C = 10.201\,\mu F,$$

$$A_T = 60\,mm^2, \qquad V_T = 18800\,mm^3, \qquad R_p = 1.9742\,\Omega, \qquad R_s = 21.846\,k\Omega.$$

(These values are taken from Frank Yang's Experimental Setup)

### 1. Energy dissipation

#### 1.1. The "old school" Method

**Table 1: Mass of printed hysteresis loop and a unit area.**

| | $m_{max}\,(g)$ | $m_{min}\,(g)$ | $\frac{\Delta m}{2}\,(g)$ | $m_{avg}\,(g)$ |
|---|---|---|---|---|
| **Unit Area** | 0.0508 | 0.0448 | 0.0030 | 0.0478 |
| **Hysteresis Loop** | 0.2397 | 0.2038 | 0.018 | 0.2218 |

The number of unit areas within the hysteresis area is;

$$n = \frac{0.222 \pm 0.018}{0.0478 \pm 0.0030} = 4.64 \pm 0.71 \tag{24}$$

The unit area has units of $20\,V^2$ thus area $S$, of the hysteresis loop is;

$$S_{\text{old}} = 20(4.64 \pm 0.71) = 93 \pm 14\,V^2 \tag{25}$$

The energy dissipation per cycle per unit volume of the core of the toroid is;

$$e_{\text{old}} = \oint HdB = \left(\frac{n_p}{2\pi r R_p}\right)\left(\frac{R_s C}{n_s A}\right) S \tag{26}$$

$$= \left(\frac{(710)}{2\pi(0.05)(1.9742)}\right)\left(\frac{(21.846)(10.201 \times 10^{-6})}{(800)(60 \times 10^{-6})}\right)(93 \pm 14) \tag{27}$$

$$= 486 \pm 73.18\,Jm^{-3}. \tag{28}$$

Therefore, the total energy dissipated by the volume of the core of the toroid per cycle is;

$$E_{\text{old}} = e_{\text{old}} V_{\text{core}}, \tag{29}$$

$$= (486 \pm 73)(18800 \times 10^{-9}) = 9.1 \pm 1.4 \times 10^{-3}\,J. \tag{30}$$

### 1.2 The “new school” Method

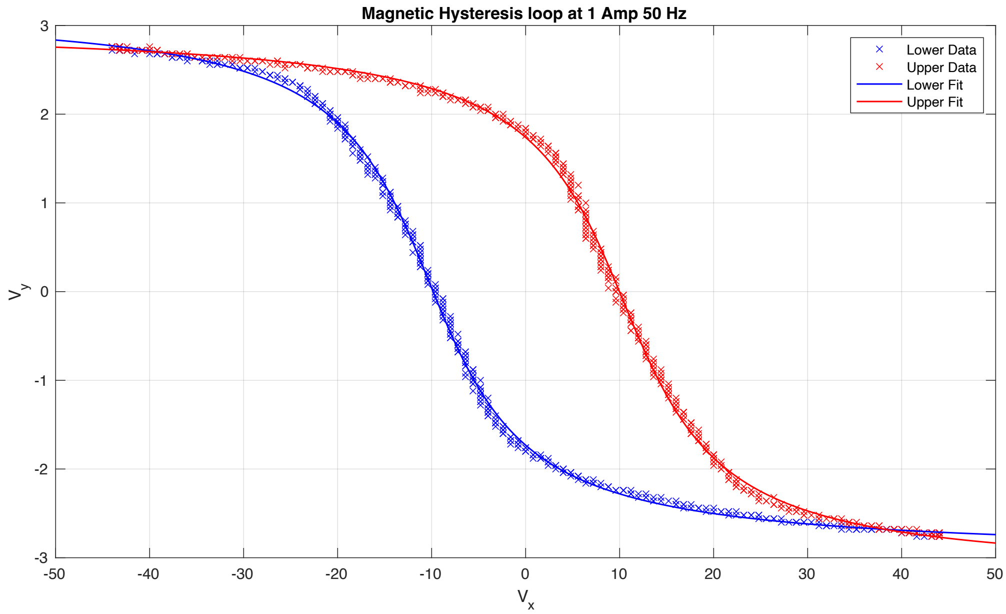


FIG. 2: Plotted and fitted data of the upper and lower bounds of the hysteresis loop.

The upper and lower bounds were fitted with MATLAB’s fit function to an inverse tan function taking the form of;

$$f(x) = a \cdot \text{atan}\big(b \cdot (x - c)\big) + d. \quad [31]$$

The estimated function values and uncertainties are;

**Table 2: Function values of fitted upper and lower data.**

| | ***a*** | ***b*** | ***c*** | ***d*** |
|---|---|---|---|---|
| $\boldsymbol{F}_{\text{low}}$ | $-1.972 \pm 0.011$ | $0.1318 \pm 0.0019$ | $-10.27 \pm 0.07$ | $0.1109 \pm 0.0093$ |
| $\boldsymbol{F}_{\text{high}}$ | $-1.982 \pm 0.013$ | $0.1291 \pm 0.0022$ | $10.38 \pm 0.08$ | $-0.105 \pm 0.011$ |

Using MATLAB’s fzero function, the integration bounds were estimated from the functions’ points of intersection. The integration bounds are;

$$[-39.1 \pm 2.4, 38.3 \pm 1.4]. \quad [32]$$

The following integral was numerically estimated using MATLAB’s quad function which uses recursive adaptive Simpson quadrature to find the area $S$, of the hysteresis loop;

$$S_{\text{new}} = \int_{-39.1\pm2.4}^{38.3\pm1.4} \big(F_{\text{high}}(x) - F_{\text{low}}(x)\big)\, dx = 95.2 \pm 3.3\, V^2. \quad [33]$$

The energy dissipation per cycle per unit volume of the core of the toroid is;

$$e_{\text{new}} = \oint HdB = \left(\frac{n_p}{2\pi r R_p}\right)\left(\frac{R_s C}{n_s A}\right) S \quad [34]$$

$$= \left(\frac{(710)}{2\pi(0.05)(1.9742)}\right)\left(\frac{(21.846)(10.201 \times 10^{-6})}{(800)(60 \times 10^{-6})}\right)(95.2 \pm 3.3) \quad [35]$$

$$= 506 \pm 18\, Jm^{-3}. \quad [36]$$

Therefore, the total energy dissipated by the volume of the core of the toroid per cycle is;

$$E_{\text{new}} = e_{\text{new}} V_{\text{core}}, \quad [37]$$

$$= (506 \pm 18\,)(18800 \times 10^{-9}) = 9.51 \pm 0.34\ \times 10^{-3}\, J. \quad [38]$$

## 2. Remanent magnetisation

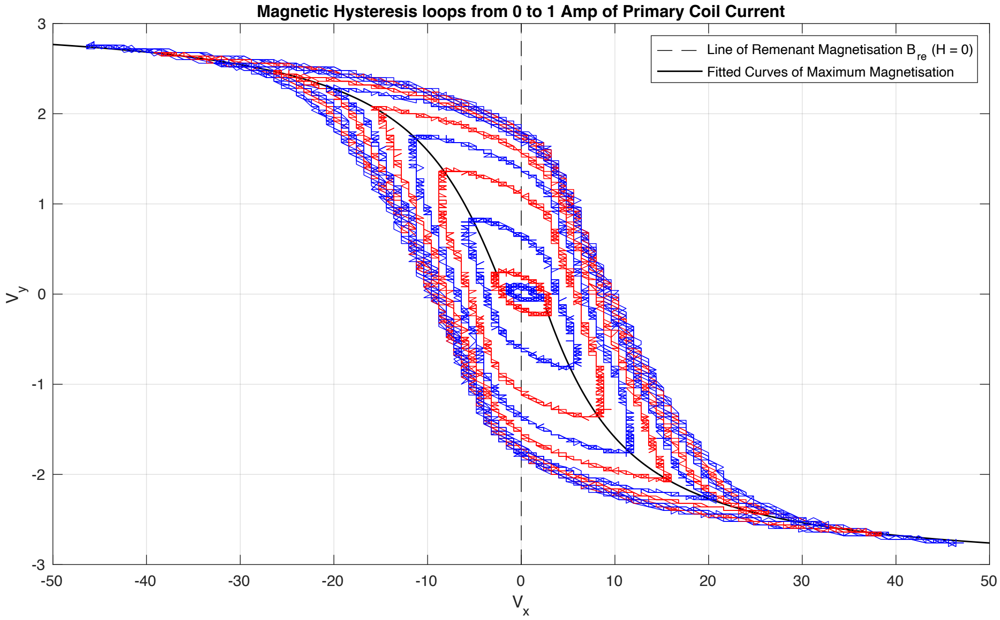


FIG. 3: Plot of Magnetic Hysteresis loops ranging from 0 to 1 Amp of primary coil current with the line of remanent magnetisation and saturation trend (virgin curve) shown. (Alternating colours indicate adjacent current intervals)

The remanent magnetisation $B_{re}$, in the iron core of the toroid can be determined by observing the magnetic field when the applied magnetic field from the primary coil is 0, when $H = 0$ and thus $V_x = 0$. (This is indicated in FIG. 3 as the line of Remanent Magnetisation). To determine the maximum remanent magnetisation, remanent magnetisation is plotted against current ($I$ is proportional to $H$) and found where $(B_{re})_{max}$ is found through the 'Knee' Method.

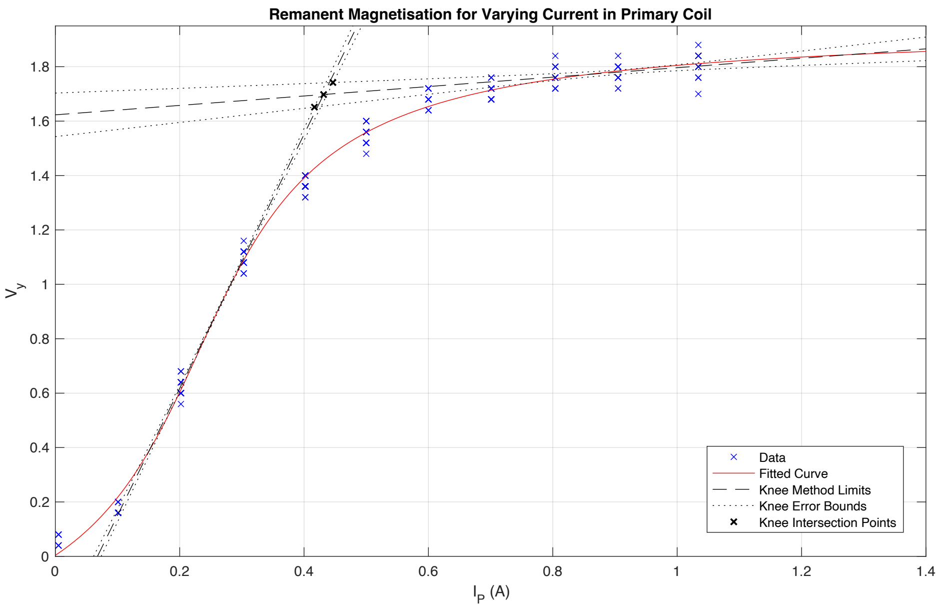


FIG. 4: Plot of how remanent magnetisation voltage changes with current, using the knee method to find the value of saturated remnant magnetisation.

The data was fitted to an inverse tan function (annex 1) and the Knee limits were fitted to a linear function (annex 2). The intersection points of the Knee limits determine the point of saturation of the core;

$$(V_y)_{\text{sat}} = 1.700 \pm 0.044\,V. \quad [39]$$

The saturation value for remanent Magnetisation is therefore;

$$(B_{re})_{sat} = \left(\frac{R_s C}{n_s A}\right)(V_y)_{\mathrm{sat}} \tag{40}$$

$$= \left(\frac{(21.846)(10.201 \times 10^{-6})}{(800)(60 \times 10^{-6})}\right)(1.700 \pm 0.044\ ) \tag{41}$$

$$= 7.89 \pm 0.21 \times 10^{-3}\ T. \tag{42}$$

The current that $(V_y)_{\mathrm{sat}}$ occurs at was determined from the Knee intersection points;

$$I_{sat} = 0.431 \pm 0.015. \tag{43}$$

From the value of $I_{sat}$, the minimum applied magnetic field strength can be determined by;

$$H_m = \frac{n_p}{2\pi r} I_{sat} = \left(\frac{(710)}{2\pi(0.05)}\right)(0.431 \pm 0.015) = 974 \pm 33\ Am^{-1}. \tag{44}$$

## DISCUSSION

The uncertainty in the report was made to a relative error of 2 sig.fig. and the value the uncertainty is associated with was taken to the absolute error of its uncertainty. The uncertainty was propagated through calculations by taking the absolute value of the maximum deviation from the calculated value, a symmetrical form of the upper-lower bound method of uncertainty propagation. Most data were approximately fitted to a function through the MatLab 'fit' function. The uncertainty of the function values was taken as a 95% confidence interval (approximately a $2\sigma$ error), which is what is propagated through the calculations.

The uncertainty for the 'old school' method was produced by taking half the difference in value from the inner cut and outer cuts of the loop and unit square. This method is surprisingly accurate as the difference in value found for energy dissipation only differed by 4.3% with both in the correct order of magnitude of the theoretical value. The uncertainty for the 'old school' method was however 400% greater than that of the 'new school' method. The value found for remanent magnetisation found is the correct order of magnitude. However, the value found for field intensity is about 3 times lower than expected.

The main source of error in the calculations is that due to some human error, the actual values for the resistances, capacitance and number of turns in the primary and secondary loop was lost and another individuals' values were used. The values were similar but not the same and such the calculations on the data are inaccurate to an unknown degree.

## CONCLUSION

This experiment was successful in exploring the magnetisation of an iron core in a toroid, finding the relationship between the applied magnetic field, and the magnetic field produced by the magnetised iron core which presented as a Hysteresis loop. The energy dissipation per cycle of the volume of the iron core was found to be $(E_{diss})_{old} = 9.1 \pm 1.4 \times 10^{-3}\ J$ using the 'old school' method and $(E_{diss})_{new} = 9.51 \pm 0.34\ \times 10^{-3}\ J$ using the 'new school' method. The value for saturated remanent magnetisation was found to be $(B_{re})_{sat} = 7.89 \pm 0.21 \times 10^{-3}\ T$ and the minimum applied field intensity to produce maximum magnetisation was found to be $H_m = 974 \pm 33\ Am^{-1}$.

## REFERENCES

[1] - PHYS2114 Experimental Physics Student Notes - Hysteresis

## APPENDIX

**1.** MATLAB's Numerical estimation of $B_{re}$ vs $I$ curve using an inverse tan function;

$$f(x) = a \cdot \text{atan}\big(b \cdot (x - c)\big) + d. \quad [1]$$

**Table 1: Function values of fitted $B_{re}$ curve.**

| | $\boldsymbol{a}$ | $\boldsymbol{b}$ | $\boldsymbol{c}$ | $\boldsymbol{d}$ |
|---|---|---|---|---|
| $\boldsymbol{F}_{\text{low}}$ | $-1.972 \pm 0.011$ | $0.1318 \pm 0.0019$ | $-10.27 \pm 0.07$ | $0.1109 \pm 0.0093$ |

**2.** MATLAB's Numerical estimation of Knee limits curves using a linear function;

$$f(x) = m \cdot x + b \quad [2]$$

| | $\boldsymbol{a}$ | $\boldsymbol{b}$ |
|---|---|---|
| $\boldsymbol{F}_{\text{low}}$ | $4.67 \pm 0.15$ | $-0.3166 \pm 0.0038$ |
| $\boldsymbol{F_{high}}$ | $0.173 \pm 0.087$ | $1.623 \pm 0.080$ |