Australian National University

Anonymous examination report for candidate

Candidate and candidature details

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Examination report

The thesis with title “Simulation of rapidly varying and dry bed flow using the Serre equations solved by finite element volume methods”, written by Jordan Peter Antony Pitt, concerns the mathematical and numerical modelling of surface water waves and specifically the theoretical and numerical investigation of a specific mathematical model of water wave theory known to as the Serre equations. The theory of water waves is a major topic in the mathematical and engineering sciences due to its important applications such as the impact of tsunamis in coastal areas. The physical equations describing the propagation of water waves are the Euler equations for incomressible fluid flow with free surface elevation. Although the Euler equations consist of the exact equations of mass and momentum equations it is very hard to be studied by theoretical and numerical means. Moreover, due to their complexity it is almost impossible to have any use in the studies of the water waves runup on beaches. For this reason scientists developed various approximations. These approximations consist of nonlinear and dispersive partial differential equations. Among the various mathematical models that approximate the Euler equations the most accurate model so far seems to be the so called the Serre equations which approximate the momentum equations of the Euler equation while the mass conservation remains exact. The Serre equations currently attracts the attention of most of the research groups in the area of water waves, including the strongests groups in France and the United States. This is because of their complicated formulation that makes their theoretical and numerical study challenging along with the fact that the properties of the solutions of Serre equations remained unexplored for decades. The scientific area of water waves remains one of the hot topics in mathematical sciences. There are more than 20 international journal publishing research articles in the area of waves. There are more than five international conferences and several workshops dedicated in water waves taking place every year while hundreds of publications appear annually. For all these reasons I found the topic of the thesis not only extremely interesting but also timely accurate and very successful.

The thesis is very well written that I hardly found typos or corrections to make. It is also very well-organised and reading the thesis of Mr Pitt was a real pleasure. The problem and the objectives of the thesis are clearly stated and the new results in the thesis along with the publications in scientific journals that occurred during this study are clearly indicated.

The first chapter of the thesis consists of a bried and very simple to understand introduction in the theory of water waves with emphasis on the asymptotic wave regimes. The problem and the objectives of the thesis are stated clearlry making obvious the importance of the study. The publications of the author are also listed, including the abstracts and the contribution of the candidate on them. There are two publications in prestigious international scientific journals and two publications in the proceedings of two international conferences while I am sure more can be written in the near future. This is an impressive achivement by a PhD student in the specific field.

The thesis continues with the second chapter and an analytical description of the Serre equations. Various formulations of the Serre equations are presented along with some of its properties such as the conservation of energy, its dispersion characteristics, the existence of analytical solutions and asymptotic solutions. The focus of this chapter is on a specific formulation of the Serre equation that appears ideal for applying his numerical methods. The candidate shows excellent knowledge of the background of the equations and the relevant literature. The chapter closes with a brief description of the so called “dam-break problem” or else the Riemann problem for the Serre equations and presents some related new results that haven’t been discovered by previous asymptotic techniques.

Chapter three is dedicated on a new numerical method for the approximation of the solutions of the Serre equations. In my opinion this chapter is perhaps the most important chapter of his thesis because the numerical method is new and highly sophisticated. The method is called by the author “Finite element volume method” and combines for the semi-discretisation the flexibility and locality of the finite volume method and the high-accuracy of the finite element methods. Specifically, the finite volume method is used to discretise the hyperbolic part of the equation while the elliptic part is solved using the finite element method for the discretisation of the elliptic operator. This combination appeared to be ideal. As far as it concerns the time-integration a second-order Strong Stability Preserving Runge-Kutta method is being used. The temporal discretisation by such method is the standard way if someone wants to study the runup problem. The thesis also employs the state of the art technique to achieve well-balancing of the numerical fluxes and solve also the wet-dry bed problem, which is the hydrostatic reconstruction of the bottom adapted to the Serre equations. The numerical method is described with great detail but of course due to the high-complexity of the method requires an experienced reader. The numerical methods of choice are appropriate for the purposes of this study and scientifically sound. In Section 3.3 the candidate presents an estimate related to the CFL condition, which apparently seems valid. Here I would say that there might be a good idea to include some references (if there are any) related to the estimation of the CFL condition. On the other hand it might not be necessary as for nonlinear dispersive waves it is known that the “real” CFL depends nonlinearly on the amplitude (speed) of the waves that it is usually impossible to get an analytical expression for it. The chapter closes with the description of the dry bed handling.

Chapter four presents the analysis of the numerical method for the linearised equations and some convergence results for the numerical method. This analysis provides with information about the stability and dispersion properties of the numerical method and for dispersive equations is not an easy task. Moreover, the analysis shows that the current numerical method has favourable convergence properties and is more efficient compared to standard finite difference methods. For new numerical methods it is always necessary to develop supportive theory since it is important to guarantee that the method will provide correct and accurate solutions. The specific chapter is a plus for the thesis and of course for the theoretical background of the new numerical method.

Chapter five contains and experimental validation of the numerical method described and analysed in the previous chapters. In addition, a comparison between other numerical methods such as mixed finite difference-finite volume of various orders of accuracy is presented. These methods also are extended from their original formulation using hydrostatic reconstruction of the bottom to handle dry beds. The chapter starts with the numerical convergence rates and computational errors for the case of an analytical traveling wave solution. This experiment doesn't require the activation of the dry-bed techniques or the wet-dry algorithms but focuses on the propagation of water waves. Other error indicators such as phase and shape errors could have been explored to support the numerical method even better. On the other hand the numerical errors are very small, which proves that the method is highly accurate. The convergence rates appeared also to be optimal. There is one point that requires some extra attention. Because the mass conservation equation is hyperbolic, it is expected a drop on the global convergence rate of the numerical method on the boundaries of the domain. I think this has been demonstrated in the Section 5.3 where the convergence rates and errors are presented in the case of "lake at rest". In this chapter one can find even more experiments. Some of the experiments are original and they all demonstrate the efficiency of the new numerical method.

After studying the convergence properties of the new numerical method, the candidate presents a validation of the computational model against laboratory data in Chapter six. A detailed comparison of numerical solutions with laboratory data for four different experimental settings is presented. All these experiments are chalenging, demonstrate the ability of the computational model to handle wet-dry interfaces and highly nonlinear and dispersive effects. All the numerical results presented in this chapter are of very good quality and support the new numerical method.

Chapter seven consists of the conclusions and plans for future research. The thesis also includes four appendices. The first two appendices consist of background material related to the mathematical model and the numerical method. The third appendix contains very interesting linear analysis for some numerical methods closely related to the new one presented in the thesis that have been used for comparisons. The last appendix contains the abstracts and additional information of the published papers of the author.

The thesis is very well-written and in standard English. It is written in a balanced way so it contains enough technical details so as the reader can have a complete understanding of the methods and be able to reproduce the results. The mathematical and computational tools employed by the author are of high quality and significant difficulty. The candidate appears to have a complete understanding of the field, with extensive knowledge in the mathematical theory (differential equations, computational mathematics and asymptotic analysis) and the physics (classical and fluid mechanics) relevant to the subject of the thesis. The candidate has an in depth knowledge of the field and has published two papers in highly ranked, international, scientific journals and one in conferences proceedings, I concluded that he is capable to carry out independent research and continue his academic career.

The main novelties of the thesis are the following: The rigorous derivation of a new finite element/finite volume method for the numerical solution of the Serre equations capable to handle wed/dry interface. The regorous study of the linear stability and convergence of the new numerical method. The regorous study of the linear stability and convergence of other numerical methods such as the finite difference and the hybrid finite difference/finite volume methods for the numerical solution of the Serre equations. The discovery of new dynamical behaviour for the flow of the Riemann problem (dam-break problem). All achievements are of current scientific interest and consist important advances for the mathematical studies of the theory and numerical analysis of mathematical models of water wave theory.

Concluding, I believe that the thesis with title "Simulation of Rapidly Varying and Dry Bed Flow using the Serre equations solved by a Finite Element Volume Method" written by Jordan Peter Anthony Pitt contains highly advanced applied mathematical tools, techniques and results, and is of excellent quality. It requires extensive knowledge of various fields in mathematics and in physics. It is also characterised by its interdisciplinary nature. There are are at least four important new results that lead to various good quality publications, published in top- ranked international scientific journals. The overal quality of the thesis is very satisfactory.