

1 Finite Element

Here are all the element wise matrices or vectors for the Finite Element Method described in my Thesis.

1.1 G

$$\int_{x_{j-1/2}}^{x_{j+1/2}} G \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} dx = \frac{\Delta x}{2} \int_{-1}^1 G \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} d\xi = \frac{\Delta x}{2} \begin{bmatrix} \frac{1}{3}G_{j-1/2}^+ \\ \frac{2}{3}G_{j-1/2}^+ + \frac{2}{3}G_{j+1/2}^- \\ \frac{1}{3}G_{j+1/2}^- \end{bmatrix}$$

1.2 uh

$$\int_{x_{j-1/2}}^{x_{j+1/2}} uh \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} dx = \frac{\Delta x}{2} \int_{-1}^1 uh \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} d\xi =$$

$$\frac{\Delta x}{2} \begin{bmatrix} \frac{7}{30}h_{j-1/2}^+ + \frac{1}{30}h_{j+1/2}^- & \frac{4}{30}h_{j-1/2}^+ & -\frac{1}{30}h_{j-1/2}^+ - \frac{1}{30}h_{j+1/2}^- \\ \frac{4}{30}h_{j-1/2}^+ & \frac{16}{30}h_{j-1/2}^+ + \frac{16}{30}h_{j+1/2}^- & \frac{4}{30}h_{j+1/2}^- \\ -\frac{1}{30}h_{j-1/2}^+ - \frac{1}{30}h_{j+1/2}^- & \frac{4}{30}h_{j+1/2}^- & \frac{1}{30}h_{j-1/2}^+ + \frac{7}{30}h_{j+1/2}^- \end{bmatrix} \begin{bmatrix} u_{j-1/2} \\ u_j \\ u_{j+1/2} \end{bmatrix}$$

1.3 $uh \frac{\partial b}{\partial x} \frac{\partial b}{\partial x}$

$$\int_{x_{j-1/2}}^{x_{j+1/2}} uh \frac{\partial b}{\partial x} \frac{\partial b}{\partial x} \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} dx = \frac{2}{\Delta x} \int_{-1}^1 uh \frac{\partial b}{\partial \xi} \frac{\partial b}{\partial \xi} \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} d\xi$$

$$= \frac{2}{\Delta x} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u_{j-1/2} \\ u_j \\ u_{j+1/2} \end{bmatrix}$$

where

$$\begin{aligned}
A_{0,0} = & \frac{1333}{1344} h_{j-1/2}^+ b_{j-1/2} b_{j-1/2} + \frac{443}{6720} h_{j+1/2}^- b_{j-1/2} b_{j-1/2} - \frac{3141}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/2} \\
& - \frac{171}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/2} + \frac{1161}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/2} + \frac{27}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/2} \\
& - \frac{145}{1344} h_{j-1/2}^+ b_{j+1/2} b_{j-1/2} - \frac{11}{6720} h_{j+1/2}^- b_{j+1/2} b_{j-1/2} - \frac{3141}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j-1/6} \\
& - \frac{171}{2240} h_{j+1/2}^- b_{j-1/2} b_{j-1/6} + \frac{4617}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/6} + \frac{243}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/6} \\
& - \frac{1863}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/6} - \frac{81}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/6} + \frac{387}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j-1/6} \\
& + \frac{9}{2240} h_{j+1/2}^- b_{j+1/2} b_{j-1/6} + \frac{1161}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j+1/6} + \frac{27}{2240} h_{j+1/2}^- b_{j-1/2} b_{j+1/6} \\
& - \frac{1863}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/6} - \frac{81}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/6} + \frac{891}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/6} \\
& + \frac{81}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/6} - \frac{27}{320} h_{j-1/2}^+ b_{j+1/2} b_{j+1/6} - \frac{27}{2240} h_{j+1/2}^- b_{j+1/2} b_{j+1/6} \\
& - \frac{145}{1344} h_{j-1/2}^+ b_{j-1/2} b_{j+1/2} - \frac{11}{6720} h_{j+1/2}^- b_{j-1/2} b_{j+1/2} + \frac{387}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/2} \\
& + \frac{9}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/2} - \frac{27}{320} h_{j-1/2}^+ b_{j+1/6} b_{j+1/2} - \frac{27}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/2} \\
& + \frac{131}{6720} h_{j-1/2}^+ b_{j+1/2} b_{j+1/2} + \frac{13}{1344} h_{j+1/2}^- b_{j+1/2} b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,1} = & \frac{103}{336}h_{j-1/2}^+b_{j-1/2}b_{j-1/2} + \frac{3}{70}h_{j+1/2}^-b_{j-1/2}b_{j-1/2} - \frac{183}{560}h_{j-1/2}^+b_{j-1/6}b_{j-1/2} \\
& - \frac{3}{140}h_{j+1/2}^-b_{j-1/6}b_{j-1/2} + \frac{3}{112}h_{j-1/2}^+b_{j+1/6}b_{j-1/2} - \frac{3}{140}h_{j+1/2}^-b_{j+1/6}b_{j-1/2} \\
& - \frac{11}{1680}h_{j-1/2}^+b_{j+1/2}b_{j-1/2} - 0h_{j+1/2}^-b_{j+1/2}b_{j-1/2} - \frac{183}{560}h_{j-1/2}^+b_{j-1/2}b_{j-1/6} \\
& - \frac{3}{140}h_{j+1/2}^-b_{j-1/2}b_{j-1/6} + \frac{243}{560}h_{j-1/2}^+b_{j-1/6}b_{j-1/6} + 0h_{j+1/2}^-b_{j-1/6}b_{j-1/6} \\
& - \frac{81}{560}h_{j-1/2}^+b_{j+1/6}b_{j-1/6} - 0h_{j+1/2}^-b_{j+1/6}b_{j-1/6} + \frac{3}{80}h_{j-1/2}^+b_{j+1/2}b_{j-1/6} \\
& + \frac{3}{140}h_{j+1/2}^-b_{j+1/2}b_{j-1/6} + \frac{3}{112}h_{j-1/2}^+b_{j-1/2}b_{j+1/6} - \frac{3}{140}h_{j+1/2}^-b_{j-1/2}b_{j+1/6} \\
& - \frac{81}{560}h_{j-1/2}^+b_{j-1/6}b_{j+1/6} - 0h_{j+1/2}^-b_{j-1/6}b_{j+1/6} + \frac{81}{560}h_{j-1/2}^+b_{j+1/6}b_{j+1/6} \\
& + 0h_{j+1/2}^-b_{j+1/6}b_{j+1/6} - \frac{3}{112}h_{j-1/2}^+b_{j+1/2}b_{j+1/6} + \frac{3}{140}h_{j+1/2}^-b_{j+1/2}b_{j+1/6} \\
& - \frac{11}{1680}h_{j-1/2}^+b_{j-1/2}b_{j+1/2} - 0h_{j+1/2}^-b_{j-1/2}b_{j+1/2} + \frac{3}{80}h_{j-1/2}^+b_{j-1/6}b_{j+1/2} \\
& + \frac{3}{140}h_{j+1/2}^-b_{j-1/6}b_{j+1/2} - \frac{3}{112}h_{j-1/2}^+b_{j+1/6}b_{j+1/2} + \frac{3}{140}h_{j+1/2}^-b_{j+1/6}b_{j+1/2} \\
& - \frac{1}{240}h_{j-1/2}^+b_{j+1/2}b_{j+1/2} - \frac{3}{70}h_{j+1/2}^-b_{j+1/2}b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,2} = & \frac{443}{6720} h_{j-1/2}^+ b_{j-1/2} b_{j-1/2} - \frac{13}{1344} h_{j+1/2}^- b_{j-1/2} b_{j-1/2} + \frac{171}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/2} \\
& + \frac{27}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/2} - \frac{27}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/2} - \frac{9}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/2} \\
& + \frac{11}{6720} h_{j-1/2}^+ b_{j+1/2} b_{j-1/2} + \frac{11}{6720} h_{j+1/2}^- b_{j+1/2} b_{j-1/2} + \frac{171}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j-1/6} \\
& + \frac{27}{2240} h_{j+1/2}^- b_{j-1/2} b_{j-1/6} - \frac{243}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/6} - \frac{81}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/6} \\
& + \frac{81}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/6} + \frac{81}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/6} - \frac{9}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j-1/6} \\
& - \frac{27}{2240} h_{j+1/2}^- b_{j+1/2} b_{j-1/6} - \frac{27}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j+1/6} - \frac{9}{2240} h_{j+1/2}^- b_{j-1/2} b_{j+1/6} \\
& + \frac{81}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/6} + \frac{81}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/6} - \frac{81}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/6} \\
& - \frac{243}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/6} + \frac{27}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j+1/6} + \frac{171}{2240} h_{j+1/2}^- b_{j+1/2} b_{j+1/6} \\
& + \frac{11}{6720} h_{j-1/2}^+ b_{j-1/2} b_{j+1/2} + \frac{11}{6720} h_{j+1/2}^- b_{j-1/2} b_{j+1/2} - \frac{9}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/2} \\
& - \frac{27}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/2} + \frac{27}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/2} + \frac{171}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/2} \\
& - \frac{13}{1344} h_{j-1/2}^+ b_{j+1/2} b_{j+1/2} - \frac{443}{6720} h_{j+1/2}^- b_{j+1/2} b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,0} = & \frac{103}{336}h_{j-1/2}^+b_{j-1/2}b_{j-1/2} + \frac{3}{70}h_{j+1/2}^-b_{j-1/2}b_{j-1/2} - \frac{183}{560}h_{j-1/2}^+b_{j-1/6}b_{j-1/2} \\
& - \frac{3}{140}h_{j+1/2}^-b_{j-1/6}b_{j-1/2} + \frac{3}{112}h_{j-1/2}^+b_{j+1/6}b_{j-1/2} - \frac{3}{140}h_{j+1/2}^-b_{j+1/6}b_{j-1/2} \\
& - \frac{11}{1680}h_{j-1/2}^+b_{j+1/2}b_{j-1/2} - 0h_{j+1/2}^-b_{j+1/2}b_{j-1/2} - \frac{183}{560}h_{j-1/2}^+b_{j-1/2}b_{j-1/6} \\
& - \frac{3}{140}h_{j+1/2}^-b_{j-1/2}b_{j-1/6} + \frac{243}{560}h_{j-1/2}^+b_{j-1/6}b_{j-1/6} + 0h_{j+1/2}^-b_{j-1/6}b_{j-1/6} \\
& - \frac{81}{560}h_{j-1/2}^+b_{j+1/6}b_{j-1/6} - 0h_{j+1/2}^-b_{j+1/6}b_{j-1/6} + \frac{3}{80}h_{j-1/2}^+b_{j+1/2}b_{j-1/6} \\
& + \frac{3}{140}h_{j+1/2}^-b_{j+1/2}b_{j-1/6} + \frac{3}{112}h_{j-1/2}^+b_{j-1/2}b_{j+1/6} - \frac{3}{140}h_{j+1/2}^-b_{j-1/2}b_{j+1/6} \\
& - \frac{81}{560}h_{j-1/2}^+b_{j-1/6}b_{j+1/6} - 0h_{j+1/2}^-b_{j-1/6}b_{j+1/6} + \frac{81}{560}h_{j-1/2}^+b_{j+1/6}b_{j+1/6} \\
& + 0h_{j+1/2}^-b_{j+1/6}b_{j+1/6} - \frac{3}{112}h_{j-1/2}^+b_{j+1/2}b_{j+1/6} + \frac{3}{140}h_{j+1/2}^-b_{j+1/2}b_{j+1/6} \\
& - \frac{11}{1680}h_{j-1/2}^+b_{j-1/2}b_{j+1/2} - 0h_{j+1/2}^-b_{j-1/2}b_{j+1/2} + \frac{3}{80}h_{j-1/2}^+b_{j-1/6}b_{j+1/2} \\
& + \frac{3}{140}h_{j+1/2}^-b_{j-1/6}b_{j+1/2} - \frac{3}{112}h_{j-1/2}^+b_{j+1/6}b_{j+1/2} + \frac{3}{140}h_{j+1/2}^-b_{j+1/6}b_{j+1/2} \\
& - \frac{1}{240}h_{j-1/2}^+b_{j+1/2}b_{j+1/2} - \frac{3}{70}h_{j+1/2}^-b_{j+1/2}b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,1} = & \frac{101}{420}h_{j-1/2}^+b_{j-1/2}b_{j-1/2} + \frac{29}{420}h_{j+1/2}^-b_{j-1/2}b_{j-1/2} - \frac{6}{35}h_{j-1/2}^+b_{j-1/6}b_{j-1/2} \\
& - \frac{3}{35}h_{j+1/2}^-b_{j-1/6}b_{j-1/2} - \frac{3}{28}h_{j-1/2}^+b_{j+1/6}b_{j-1/2} - \frac{3}{140}h_{j+1/2}^-b_{j+1/6}b_{j-1/2} \\
& + \frac{4}{105}h_{j-1/2}^+b_{j+1/2}b_{j-1/2} + \frac{4}{105}h_{j+1/2}^-b_{j+1/2}b_{j-1/2} - \frac{6}{35}h_{j-1/2}^+b_{j-1/2}b_{j-1/6} \\
& - \frac{3}{35}h_{j+1/2}^-b_{j-1/2}b_{j-1/6} + \frac{27}{28}h_{j-1/2}^+b_{j-1/6}b_{j-1/6} + \frac{27}{28}h_{j+1/2}^-b_{j-1/6}b_{j-1/6} \\
& - \frac{27}{35}h_{j-1/2}^+b_{j+1/6}b_{j-1/6} - \frac{27}{35}h_{j+1/2}^-b_{j+1/6}b_{j-1/6} - \frac{3}{140}h_{j-1/2}^+b_{j+1/2}b_{j-1/6} \\
& - \frac{3}{28}h_{j+1/2}^-b_{j+1/2}b_{j-1/6} - \frac{3}{28}h_{j-1/2}^+b_{j-1/2}b_{j+1/6} - \frac{3}{140}h_{j+1/2}^-b_{j-1/2}b_{j+1/6} \\
& - \frac{27}{35}h_{j-1/2}^+b_{j-1/6}b_{j+1/6} - \frac{27}{35}h_{j+1/2}^-b_{j-1/6}b_{j+1/6} + \frac{27}{28}h_{j-1/2}^+b_{j+1/6}b_{j+1/6} \\
& + \frac{27}{28}h_{j+1/2}^-b_{j+1/6}b_{j+1/6} - \frac{3}{35}h_{j-1/2}^+b_{j+1/2}b_{j+1/6} - \frac{6}{35}h_{j+1/2}^-b_{j+1/2}b_{j+1/6} \\
& + \frac{4}{105}h_{j-1/2}^+b_{j-1/2}b_{j+1/2} + \frac{4}{105}h_{j+1/2}^-b_{j-1/2}b_{j+1/2} - \frac{3}{140}h_{j-1/2}^+b_{j-1/6}b_{j+1/2} \\
& - \frac{3}{28}h_{j+1/2}^-b_{j-1/6}b_{j+1/2} - \frac{3}{35}h_{j-1/2}^+b_{j+1/6}b_{j+1/2} - \frac{6}{35}h_{j+1/2}^-b_{j+1/6}b_{j+1/2} \\
& + \frac{29}{420}h_{j-1/2}^+b_{j+1/2}b_{j+1/2} + \frac{101}{420}h_{j+1/2}^-b_{j+1/2}b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,2} = & \frac{3}{70}h_{j-1/2}^+b_{j-1/2}b_{j-1/2} - \frac{1}{240}h_{j+1/2}^-b_{j-1/2}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j-1/6}b_{j-1/2} \\
& - \frac{3}{112}h_{j+1/2}^-b_{j-1/6}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j+1/6}b_{j-1/2} + \frac{3}{80}h_{j+1/2}^-b_{j+1/6}b_{j-1/2} \\
& + 0h_{j-1/2}^+b_{j+1/2}b_{j-1/2} - \frac{11}{1680}h_{j+1/2}^-b_{j+1/2}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j-1/2}b_{j-1/6} \\
& - \frac{3}{112}h_{j+1/2}^-b_{j-1/2}b_{j-1/6} - 0h_{j-1/2}^+b_{j-1/6}b_{j-1/6} + \frac{81}{560}h_{j+1/2}^-b_{j-1/6}b_{j-1/6} \\
& + 0h_{j-1/2}^+b_{j+1/6}b_{j-1/6} - \frac{81}{560}h_{j+1/2}^-b_{j+1/6}b_{j-1/6} - \frac{3}{140}h_{j-1/2}^+b_{j+1/2}b_{j-1/6} \\
& + \frac{3}{112}h_{j+1/2}^-b_{j+1/2}b_{j-1/6} + \frac{3}{140}h_{j-1/2}^+b_{j-1/2}b_{j+1/6} + \frac{3}{80}h_{j+1/2}^-b_{j-1/2}b_{j+1/6} \\
& + 0h_{j-1/2}^+b_{j-1/6}b_{j+1/6} - \frac{81}{560}h_{j+1/2}^-b_{j-1/6}b_{j+1/6} - 0h_{j-1/2}^+b_{j+1/6}b_{j+1/6} \\
& + \frac{243}{560}h_{j+1/2}^-b_{j+1/6}b_{j+1/6} - \frac{3}{140}h_{j-1/2}^+b_{j+1/2}b_{j+1/6} - \frac{183}{560}h_{j+1/2}^-b_{j+1/2}b_{j+1/6} \\
& - 0h_{j-1/2}^+b_{j-1/2}b_{j+1/2} - \frac{11}{1680}h_{j+1/2}^-b_{j-1/2}b_{j+1/2} - \frac{3}{140}h_{j-1/2}^+b_{j-1/6}b_{j+1/2} \\
& + \frac{3}{112}h_{j+1/2}^-b_{j-1/6}b_{j+1/2} - \frac{3}{140}h_{j-1/2}^+b_{j+1/6}b_{j+1/2} - \frac{183}{560}h_{j+1/2}^-b_{j+1/6}b_{j+1/2} \\
& + \frac{3}{70}h_{j-1/2}^+b_{j+1/2}b_{j+1/2} + \frac{103}{336}h_{j+1/2}^-b_{j+1/2}b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,0} = & \frac{443}{6720} h_{j-1/2}^+ b_{j-1/2} b_{j-1/2} - \frac{13}{1344} h_{j+1/2}^- b_{j-1/2} b_{j-1/2} + \frac{171}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/2} \\
& + \frac{27}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/2} - \frac{27}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/2} - \frac{9}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/2} \\
& + \frac{11}{6720} h_{j-1/2}^+ b_{j+1/2} b_{j-1/2} + \frac{11}{6720} h_{j+1/2}^- b_{j+1/2} b_{j-1/2} + \frac{171}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j-1/6} \\
& + \frac{27}{2240} h_{j+1/2}^- b_{j-1/2} b_{j-1/6} - \frac{243}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/6} - \frac{81}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/6} \\
& + \frac{81}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/6} + \frac{81}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/6} - \frac{9}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j-1/6} \\
& - \frac{27}{2240} h_{j+1/2}^- b_{j+1/2} b_{j-1/6} - \frac{27}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j+1/6} - \frac{9}{2240} h_{j+1/2}^- b_{j-1/2} b_{j+1/6} \\
& + \frac{81}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/6} + \frac{81}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/6} - \frac{81}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/6} \\
& - \frac{243}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/6} + \frac{27}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j+1/6} + \frac{171}{2240} h_{j+1/2}^- b_{j+1/2} b_{j+1/6} \\
& + \frac{11}{6720} h_{j-1/2}^+ b_{j-1/2} b_{j+1/2} + \frac{11}{6720} h_{j+1/2}^- b_{j-1/2} b_{j+1/2} - \frac{9}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/2} \\
& - \frac{27}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/2} + \frac{27}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/2} + \frac{171}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/2} \\
& - \frac{13}{1344} h_{j-1/2}^+ b_{j+1/2} b_{j+1/2} - \frac{443}{6720} h_{j+1/2}^- b_{j+1/2} b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,1} = & \frac{3}{70}h_{j-1/2}^+b_{j-1/2}b_{j-1/2} - \frac{1}{240}h_{j+1/2}^-b_{j-1/2}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j-1/6}b_{j-1/2} \\
& - \frac{3}{112}h_{j+1/2}^-b_{j-1/6}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j+1/6}b_{j-1/2} + \frac{3}{80}h_{j+1/2}^-b_{j+1/6}b_{j-1/2} \\
& + 0h_{j-1/2}^+b_{j+1/2}b_{j-1/2} - \frac{11}{1680}h_{j+1/2}^-b_{j+1/2}b_{j-1/2} + \frac{3}{140}h_{j-1/2}^+b_{j-1/2}b_{j-1/6} \\
& - \frac{3}{112}h_{j+1/2}^-b_{j-1/2}b_{j-1/6} - 0h_{j-1/2}^+b_{j-1/6}b_{j-1/6} + \frac{81}{560}h_{j+1/2}^-b_{j-1/6}b_{j-1/6} \\
& + 0h_{j-1/2}^+b_{j+1/6}b_{j-1/6} - \frac{81}{560}h_{j+1/2}^-b_{j+1/6}b_{j-1/6} - \frac{3}{140}h_{j-1/2}^+b_{j+1/2}b_{j-1/6} \\
& + \frac{3}{112}h_{j+1/2}^-b_{j+1/2}b_{j-1/6} + \frac{3}{140}h_{j-1/2}^+b_{j-1/2}b_{j+1/6} + \frac{3}{80}h_{j+1/2}^-b_{j-1/2}b_{j+1/6} \\
& + 0h_{j-1/2}^+b_{j-1/6}b_{j+1/6} - \frac{81}{560}h_{j+1/2}^-b_{j-1/6}b_{j+1/6} - 0h_{j-1/2}^+b_{j+1/6}b_{j+1/6} \\
& + \frac{243}{560}h_{j+1/2}^-b_{j+1/6}b_{j+1/6} - \frac{3}{140}h_{j-1/2}^+b_{j+1/2}b_{j+1/6} - \frac{183}{560}h_{j+1/2}^-b_{j+1/2}b_{j+1/6} \\
& - 0h_{j-1/2}^+b_{j-1/2}b_{j+1/2} - \frac{11}{1680}h_{j+1/2}^-b_{j-1/2}b_{j+1/2} - \frac{3}{140}h_{j-1/2}^+b_{j-1/6}b_{j+1/2} \\
& + \frac{3}{112}h_{j+1/2}^-b_{j-1/6}b_{j+1/2} - \frac{3}{140}h_{j-1/2}^+b_{j+1/6}b_{j+1/2} - \frac{183}{560}h_{j+1/2}^-b_{j+1/6}b_{j+1/2} \\
& + \frac{3}{70}h_{j-1/2}^+b_{j+1/2}b_{j+1/2} + \frac{103}{336}h_{j+1/2}^-b_{j+1/2}b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,2} = & \frac{13}{1344} h_{j-1/2}^+ b_{j-1/2} b_{j-1/2} + \frac{131}{6720} h_{j+1/2}^- b_{j-1/2} b_{j-1/2} - \frac{27}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/2} \\
& - \frac{27}{320} h_{j+1/2}^- b_{j-1/6} b_{j-1/2} + \frac{9}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/2} + \frac{387}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/2} \\
& - \frac{11}{6720} h_{j-1/2}^+ b_{j+1/2} b_{j-1/2} - \frac{145}{1344} h_{j+1/2}^- b_{j+1/2} b_{j-1/2} - \frac{27}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j-1/6} \\
& - \frac{27}{320} h_{j+1/2}^- b_{j-1/2} b_{j-1/6} + \frac{81}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j-1/6} + \frac{891}{2240} h_{j+1/2}^- b_{j-1/6} b_{j-1/6} \\
& - \frac{81}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j-1/6} - \frac{1863}{2240} h_{j+1/2}^- b_{j+1/6} b_{j-1/6} + \frac{27}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j-1/6} \\
& + \frac{1161}{2240} h_{j+1/2}^- b_{j+1/2} b_{j-1/6} + \frac{9}{2240} h_{j-1/2}^+ b_{j-1/2} b_{j+1/6} + \frac{387}{2240} h_{j+1/2}^- b_{j-1/2} b_{j+1/6} \\
& - \frac{81}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/6} - \frac{1863}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/6} + \frac{243}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/6} \\
& + \frac{4617}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/6} - \frac{171}{2240} h_{j-1/2}^+ b_{j+1/2} b_{j+1/6} - \frac{3141}{2240} h_{j+1/2}^- b_{j+1/2} b_{j+1/6} \\
& - \frac{11}{6720} h_{j-1/2}^+ b_{j-1/2} b_{j+1/2} - \frac{145}{1344} h_{j+1/2}^- b_{j-1/2} b_{j+1/2} + \frac{27}{2240} h_{j-1/2}^+ b_{j-1/6} b_{j+1/2} \\
& + \frac{1161}{2240} h_{j+1/2}^- b_{j-1/6} b_{j+1/2} - \frac{171}{2240} h_{j-1/2}^+ b_{j+1/6} b_{j+1/2} - \frac{3141}{2240} h_{j+1/2}^- b_{j+1/6} b_{j+1/2} \\
& + \frac{443}{6720} h_{j-1/2}^+ b_{j+1/2} b_{j+1/2} + \frac{1333}{1344} h_{j+1/2}^- b_{j+1/2} b_{j+1/2},
\end{aligned}$$

1.4 $h^3 \frac{\partial u}{\partial x}$

$$\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{h^3}{3} \frac{\partial u}{\partial x} \frac{\partial}{\partial x} \left(\begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} \right) dx = \frac{2}{3\Delta x} \int_{-1}^1 h^3 \frac{\partial u}{\partial \xi} \begin{bmatrix} \frac{\partial \phi_{j-1/2}}{\partial \xi} \\ \frac{\partial \phi_j}{\partial \xi} \\ \frac{\partial \phi_{j+1/2}}{\partial \xi} \end{bmatrix} d\xi$$

$$= \frac{2}{3\Delta x} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u_{j-1/2} \\ u_j \\ u_{j+1/2} \end{bmatrix}$$

where

$$A_{0,0} = \frac{79}{120} h_{j-1/2}^+ h_{j-1/2}^+ h_{j-1/2}^+ + \frac{13}{120} h_{j+1/2}^- h_{j-1/2}^+ h_{j-1/2}^+ + \frac{13}{120} h_{j-1/2}^+ h_{j+1/2}^- h_{j-1/2}^+ \\ + \frac{1}{24} h_{j+1/2}^- h_{j+1/2}^- h_{j-1/2}^+ + \frac{13}{120} h_{j-1/2}^+ h_{j-1/2}^+ h_{j+1/2}^- + \frac{1}{24} h_{j+1/2}^- h_{j-1/2}^+ h_{j+1/2}^- \\ + \frac{1}{24} h_{j-1/2}^+ h_{j+1/2}^- h_{j+1/2}^- + \frac{7}{120} h_{j+1/2}^- h_{j+1/2}^- h_{j+1/2}^-,$$

$$A_{0,1} = \frac{23}{30} h_{j-1/2}^+ h_{j-1/2}^+ h_{j-1/2}^+ - \frac{1}{10} h_{j+1/2}^- h_{j-1/2}^+ h_{j-1/2}^+ - \frac{1}{10} h_{j-1/2}^+ h_{j+1/2}^- h_{j-1/2}^+ \\ - \frac{1}{30} h_{j+1/2}^- h_{j+1/2}^- h_{j-1/2}^+ - \frac{1}{10} h_{j-1/2}^+ h_{j-1/2}^+ h_{j+1/2}^- - \frac{1}{30} h_{j+1/2}^- h_{j-1/2}^+ h_{j+1/2}^- \\ - \frac{1}{30} h_{j-1/2}^+ h_{j+1/2}^- h_{j+1/2}^- - \frac{1}{6} h_{j+1/2}^- h_{j+1/2}^- h_{j+1/2}^-,$$

$$A_{0,2} = \frac{13}{120} h_{j-1/2}^+ h_{j-1/2}^+ h_{j-1/2}^+ - \frac{1}{120} h_{j+1/2}^- h_{j-1/2}^+ h_{j-1/2}^+ - \frac{1}{120} h_{j-1/2}^+ h_{j+1/2}^- h_{j-1/2}^+ \\ - \frac{1}{120} h_{j+1/2}^- h_{j+1/2}^- h_{j-1/2}^+ - \frac{1}{120} h_{j-1/2}^+ h_{j-1/2}^+ h_{j+1/2}^- - \frac{1}{120} h_{j+1/2}^- h_{j-1/2}^+ h_{j+1/2}^- \\ - \frac{1}{120} h_{j-1/2}^+ h_{j+1/2}^- h_{j+1/2}^- + \frac{13}{120} h_{j+1/2}^- h_{j+1/2}^- h_{j+1/2}^-,$$

$$\begin{aligned}
A_{1,0} = & \frac{23}{30}h_{j-1/2}^+h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{10}h_{j+1/2}^-h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{10}h_{j-1/2}^+h_{j+1/2}^-h_{j-1/2}^+ \\
& - \frac{1}{30}h_{j+1/2}^-h_{j+1/2}^-h_{j-1/2}^+ - \frac{1}{10}h_{j-1/2}^+h_{j-1/2}^+h_{j+1/2}^- - \frac{1}{30}h_{j+1/2}^-h_{j-1/2}^+h_{j+1/2}^- \\
& - \frac{1}{30}h_{j-1/2}^+h_{j+1/2}^-h_{j+1/2}^- - \frac{1}{6}h_{j+1/2}^-h_{j+1/2}^-h_{j+1/2}^-,
\end{aligned}$$

$$\begin{aligned}
A_{1,1} = & \frac{14}{15}h_{j-1/2}^+h_{j-1/2}^+h_{j-1/2}^+ + \frac{2}{15}h_{j+1/2}^-h_{j-1/2}^+h_{j-1/2}^+ + \frac{2}{15}h_{j-1/2}^+h_{j+1/2}^-h_{j-1/2}^+ \\
& + \frac{2}{15}h_{j+1/2}^-h_{j+1/2}^-h_{j-1/2}^+ + \frac{2}{15}h_{j-1/2}^+h_{j-1/2}^+h_{j+1/2}^- + \frac{2}{15}h_{j+1/2}^-h_{j-1/2}^+h_{j+1/2}^- \\
& + \frac{2}{15}h_{j-1/2}^+h_{j+1/2}^-h_{j+1/2}^- + \frac{14}{15}h_{j+1/2}^-h_{j+1/2}^-h_{j+1/2}^-,
\end{aligned}$$

$$\begin{aligned}
A_{1,2} = & \frac{1}{6}h_{j-1/2}^+h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{30}h_{j+1/2}^-h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{30}h_{j-1/2}^+h_{j+1/2}^-h_{j-1/2}^+ \\
& - \frac{1}{10}h_{j+1/2}^-h_{j+1/2}^-h_{j-1/2}^+ - \frac{1}{30}h_{j-1/2}^+h_{j-1/2}^+h_{j+1/2}^- - \frac{1}{10}h_{j+1/2}^-h_{j-1/2}^+h_{j+1/2}^- \\
& - \frac{1}{10}h_{j-1/2}^+h_{j+1/2}^-h_{j+1/2}^- - \frac{23}{30}h_{j+1/2}^-h_{j+1/2}^-h_{j+1/2}^-,
\end{aligned}$$

$$\begin{aligned}
A_{2,0} = & \frac{13}{120}h_{j-1/2}^+h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{120}h_{j+1/2}^-h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{120}h_{j-1/2}^+h_{j+1/2}^-h_{j-1/2}^+ \\
& - \frac{1}{120}h_{j+1/2}^-h_{j+1/2}^-h_{j-1/2}^+ - \frac{1}{120}h_{j-1/2}^+h_{j-1/2}^+h_{j+1/2}^- - \frac{1}{120}h_{j+1/2}^-h_{j-1/2}^+h_{j+1/2}^- \\
& - \frac{1}{120}h_{j-1/2}^+h_{j+1/2}^-h_{j+1/2}^- + \frac{13}{120}h_{j+1/2}^-h_{j+1/2}^-h_{j+1/2}^-,
\end{aligned}$$

$$\begin{aligned}
A_{2,1} = & \frac{1}{6}h_{j-1/2}^+h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{30}h_{j+1/2}^-h_{j-1/2}^+h_{j-1/2}^+ - \frac{1}{30}h_{j-1/2}^+h_{j+1/2}^-h_{j-1/2}^+ \\
& - \frac{1}{10}h_{j+1/2}^-h_{j+1/2}^-h_{j-1/2}^+ - \frac{1}{30}h_{j-1/2}^+h_{j-1/2}^+h_{j+1/2}^- - \frac{1}{10}h_{j+1/2}^-h_{j-1/2}^+h_{j+1/2}^- \\
& - \frac{1}{10}h_{j-1/2}^+h_{j+1/2}^-h_{j+1/2}^- - \frac{23}{30}h_{j+1/2}^-h_{j+1/2}^-h_{j+1/2}^-,
\end{aligned}$$

$$\begin{aligned}
A_{2,2} = & \frac{7}{120} h_{j-1/2}^+ h_{j-1/2}^+ h_{j-1/2}^+ + \frac{1}{24} h_{j+1/2}^- h_{j-1/2}^+ h_{j-1/2}^+ + \frac{1}{24} h_{j-1/2}^+ h_{j+1/2}^- h_{j-1/2}^+ \\
& + \frac{13}{120} h_{j+1/2}^- h_{j+1/2}^- h_{j-1/2}^+ + \frac{1}{24} h_{j-1/2}^+ h_{j-1/2}^+ h_{j+1/2}^- + \frac{13}{120} h_{j+1/2}^- h_{j-1/2}^+ h_{j+1/2}^- \\
& + \frac{13}{120} h_{j-1/2}^+ h_{j+1/2}^- h_{j+1/2}^- + \frac{79}{120} h_{j+1/2}^- h_{j+1/2}^- h_{j+1/2}^-,
\end{aligned}$$

1.5 $h^2 u \frac{\partial b}{\partial x}$

$$\begin{aligned}
-\int_{x_{j-1/2}}^{x_{j+1/2}} \frac{h^2}{2} u \frac{\partial b}{\partial x} \frac{\partial}{\partial x} \left(\begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} \right) dx &= -\frac{1}{\Delta x} \int_{-1}^1 h^2 u \frac{\partial b}{\partial \xi} \begin{bmatrix} \frac{\partial \phi_{j-1/2}}{\partial \xi} \\ \frac{\partial \phi_j}{\partial \xi} \\ \frac{\partial \phi_{j+1/2}}{\partial \xi} \end{bmatrix} d\xi \\
&= -\frac{1}{\Delta x} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u_{j-1/2} \\ u_j \\ u_{j+1/2} \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
A_{0,0} = & \frac{31}{42} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{19}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{19}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{23}{1680} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{537}{560} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{33}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{33}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} - \frac{3}{280} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{39}{140} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} + \frac{3}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{97}{1680} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{1}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{1}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& - \frac{1}{120} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,1} = & \frac{71}{210} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{15} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{15} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{1}{84} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{3}{20} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{9}{70} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} - \frac{3}{14} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{6}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{6}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{27}{140} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{11}{420} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{2}{105} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{2}{105} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{11}{210} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,2} = & \frac{19}{280} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} - \frac{23}{1680} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} - \frac{23}{1680} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{43}{1680} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} + \frac{33}{560} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} + \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& + \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{3}{28} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{3}{280} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{3}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{3}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{21}{80} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{1}{560} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{1}{120} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{1}{120} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{19}{105} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,0} = & \frac{193}{210}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} - \frac{8}{105}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} - \frac{8}{105}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& - \frac{1}{84}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} + \frac{171}{140}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} + \frac{9}{140}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& + \frac{9}{140}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} + 0h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} - \frac{27}{70}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& - 0h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} - 0h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} - \frac{9}{140}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& + \frac{1}{12}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} + \frac{1}{84}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} + \frac{1}{84}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& + \frac{8}{105}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,1} = & \frac{41}{105}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} - \frac{3}{35}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} - \frac{3}{35}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& - \frac{4}{105}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} + \frac{12}{35}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} + \frac{3}{35}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& + \frac{3}{35}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} + \frac{3}{35}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} + \frac{3}{35}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& + \frac{3}{35}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} + \frac{3}{35}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} + \frac{12}{35}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& - \frac{4}{105}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} - \frac{3}{35}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} - \frac{3}{35}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& - \frac{41}{105}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,2} = & \frac{8}{105} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{84} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{84} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{1}{12} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{9}{140} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - 0 h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - 0 h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} - \frac{27}{70} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} - 0 h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{9}{140} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{9}{140} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} + \frac{171}{140} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{1}{84} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} - \frac{8}{105} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} - \frac{8}{105} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& - \frac{193}{210} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,0} = & \frac{19}{105} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{120} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{120} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{1}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{21}{80} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{3}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{3}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{3}{280} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{3}{28} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} + \frac{33}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{43}{1680} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} - \frac{23}{1680} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} - \frac{23}{1680} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& - \frac{19}{280} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,1} = & \frac{11}{210} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{2}{105} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{2}{105} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{11}{420} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{27}{140} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{6}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{6}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} - \frac{3}{14} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{9}{70} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{3}{20} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{1}{84} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{1}{15} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{1}{15} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{71}{210} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,2} = & \frac{1}{120} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{97}{1680} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} + \frac{3}{560} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{39}{140} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} - \frac{3}{280} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{33}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{33}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{537}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{23}{1680} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{19}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{19}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{31}{42} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

1.6 $h^2 \frac{\partial u}{\partial x} \frac{\partial b}{\partial x}$

$$\begin{aligned}
- \int_{x_{j-1/2}}^{x_{j+1/2}} \frac{h^2}{2} \frac{\partial u}{\partial x} \frac{\partial b}{\partial x} \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} dx &= - \frac{1}{\Delta x} \int_{-1}^1 h^2 \frac{\partial u}{\partial \xi} \frac{\partial b}{\partial \xi} \begin{bmatrix} \phi_{j-1/2} \\ \phi_j \\ \phi_{j+1/2} \end{bmatrix} d\xi \\
&= - \frac{1}{\Delta x} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \begin{bmatrix} u_{j-1/2} \\ u_j \\ u_{j+1/2} \end{bmatrix}
\end{aligned}$$

where

$$\begin{aligned}
A_{0,0} = & \frac{31}{42}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} + \frac{19}{280}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} + \frac{19}{280}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& + \frac{23}{1680}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} - \frac{537}{560}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} - \frac{33}{560}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& - \frac{33}{560}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} - \frac{3}{280}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} + \frac{39}{140}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& - \frac{3}{280}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} - \frac{3}{280}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} + \frac{3}{560}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& - \frac{97}{1680}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} + \frac{1}{560}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} + \frac{1}{560}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& - \frac{1}{120}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,1} = & \frac{193}{210}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} - \frac{8}{105}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} - \frac{8}{105}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& - \frac{1}{84}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} + \frac{171}{140}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} + \frac{9}{140}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& + \frac{9}{140}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} + 0h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} - \frac{27}{70}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& - 0h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} - 0h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} - \frac{9}{140}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& + \frac{1}{12}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} + \frac{1}{84}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} + \frac{1}{84}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& + \frac{8}{105}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{0,2} = & \frac{19}{105} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{120} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{120} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{1}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{21}{80} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{3}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{3}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{3}{280} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{3}{28} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} + \frac{33}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{43}{1680} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} - \frac{23}{1680} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} - \frac{23}{1680} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& - \frac{19}{280} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,0} = & \frac{71}{210} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{15} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{15} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{1}{84} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{3}{20} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{9}{70} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} - \frac{3}{14} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{6}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{6}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{27}{140} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{11}{420} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{2}{105} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{2}{105} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{11}{210} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,1} = & \frac{41}{105} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} - \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} - \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{4}{105} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} + \frac{12}{35} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{3}{35} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{3}{35} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} + \frac{12}{35} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& - \frac{4}{105} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} - \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} - \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& - \frac{41}{105} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{1,2} = & \frac{11}{210} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{2}{105} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{2}{105} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& + \frac{11}{420} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} - \frac{27}{140} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{6}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{6}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} - \frac{3}{14} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} + \frac{9}{70} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& + \frac{3}{35} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} + \frac{3}{35} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{3}{20} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{1}{84} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{1}{15} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{1}{15} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{71}{210} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,0} = & \frac{19}{280}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} - \frac{23}{1680}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} - \frac{23}{1680}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& - \frac{43}{1680}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} + \frac{33}{560}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} + \frac{3}{280}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& + \frac{3}{280}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} + \frac{3}{28}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} + \frac{3}{280}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& - \frac{3}{560}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} - \frac{3}{560}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} - \frac{21}{80}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& - \frac{1}{560}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} + \frac{1}{120}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} + \frac{1}{120}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& + \frac{19}{105}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,1} = & \frac{8}{105}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/2} + \frac{1}{84}h_{j+1/2}^-h_{j-1/2}^+b_{j-1/2} + \frac{1}{84}h_{j-1/2}^+h_{j+1/2}^-b_{j-1/2} \\
& + \frac{1}{12}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/2} - \frac{9}{140}h_{j-1/2}^+h_{j-1/2}^+b_{j-1/6} - 0h_{j+1/2}^-h_{j-1/2}^+b_{j-1/6} \\
& - 0h_{j-1/2}^+h_{j+1/2}^-b_{j-1/6} - \frac{27}{70}h_{j+1/2}^-h_{j+1/2}^-b_{j-1/6} - 0h_{j-1/2}^+h_{j-1/2}^+b_{j+1/6} \\
& + \frac{9}{140}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/6} + \frac{9}{140}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/6} + \frac{171}{140}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/6} \\
& - \frac{1}{84}h_{j-1/2}^+h_{j-1/2}^+b_{j+1/2} - \frac{8}{105}h_{j+1/2}^-h_{j-1/2}^+b_{j+1/2} - \frac{8}{105}h_{j-1/2}^+h_{j+1/2}^-b_{j+1/2} \\
& - \frac{193}{210}h_{j+1/2}^-h_{j+1/2}^-b_{j+1/2},
\end{aligned}$$

$$\begin{aligned}
A_{2,2} = & \frac{1}{120} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/2} + \frac{1}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/2} + \frac{1}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/2} \\
& - \frac{97}{1680} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/2} + \frac{3}{560} h_{j-1/2}^+ h_{j-1/2}^+ b_{j-1/6} - \frac{3}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j-1/6} \\
& - \frac{3}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j-1/6} + \frac{39}{140} h_{j+1/2}^- h_{j+1/2}^- b_{j-1/6} - \frac{3}{280} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/6} \\
& - \frac{33}{560} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/6} - \frac{33}{560} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/6} - \frac{537}{560} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/6} \\
& + \frac{23}{1680} h_{j-1/2}^+ h_{j-1/2}^+ b_{j+1/2} + \frac{19}{280} h_{j+1/2}^- h_{j-1/2}^+ b_{j+1/2} + \frac{19}{280} h_{j-1/2}^+ h_{j+1/2}^- b_{j+1/2} \\
& + \frac{31}{42} h_{j+1/2}^- h_{j+1/2}^- b_{j+1/2},
\end{aligned}$$