# 1 All Definitions for Numerical Versions

$$\mathcal{C}_2 = \frac{2\cos(k\Delta x) - 2}{\Delta x^2}$$

$$\mathcal{C}_4 = \frac{-2\cos(2k\Delta x) + 32\cos(k\Delta x) - 30}{12\Delta x^2}$$

$$\mathcal{G} = \left[H - \frac{H^3}{3}\mathcal{C}\right]$$

$$\mathcal{M}_3 = \frac{24}{26 - 2\cos(k\Delta x)}$$

$$\mathcal{M}_1 = \mathcal{M}_2 = 1$$

$$\mathcal{R}_1^+ = e^{ik\Delta x} \quad , \quad \mathcal{R}_1^- = 1$$

$$\mathcal{R}_2^- = 1 + \frac{i\sin(k\Delta x)}{2}$$

$$\mathcal{R}_2^+ = e^{ik\Delta x}\left(1 - \frac{i\sin(k\Delta x)}{2}\right)$$

$$\mathcal{R}_3^- = \frac{\mathcal{M}_3}{6}\left[5 + -e^{-ik\Delta x} + 2e^{ik\Delta x}\right]$$

$$\mathcal{R}_3^+ = \frac{\mathcal{M}_3 e^{ik\Delta x}}{6}\left[5 + 2e^{-ik\Delta x} - e^{ik\Delta x}\right]$$

$$\mathcal{R}_2^u = \frac{e^{ik\Delta x} + 1}{2}$$

$$\mathcal{R}_3^u = \frac{-3e^{2ik\Delta x} + 27e^{ik\Delta x} + 27 - 3e^{-ik\Delta x}}{48}$$

$$\mathcal{F}^{h,u} = H\mathcal{R}^u$$

$$\mathcal{F}^{h,h} = -\frac{\sqrt{gH}}{2}\left[\mathcal{R}^+ - \mathcal{R}^-\right]$$

$$\mathcal{F}^{u,u} = -\frac{\sqrt{gH}}{2}\mathcal{G}\left[\mathcal{R}^+ \ -\mathcal{R}^-\right]$$

$$\mathcal{F}^{u,h} = \frac{gH\mathcal{R}^- + gH\mathcal{R}^+}{2}$$

$\mathcal{D} = 1 - e^{-ik\Delta x}$

# 2 Taylor Expansions Of Analytic Values

We denote exact/analytic version with a subscript $a$

$$\mathcal{G}_a = H + \frac{H^3}{3}k^2$$

$$\mathcal{M}_a = 1 - \frac{1}{24}(k\Delta x)^2 + \frac{1}{192}(k\Delta x)^4 + O(x^6)$$

$$\mathcal{R}_a^+ = \mathcal{R}_a^- = \mathcal{R}_a = 1 + \frac{i}{2}k\Delta x - \frac{1}{8}k^2\Delta x^2 - \frac{i}{48}k^3\Delta x^3 + O(x^4)$$

For the fluxes I think its best to group the $\frac{\mathcal{D}}{\Delta x\mathcal{M}}\mathcal{F}$ because its collects all the terms using spatial approximations and has a nice form. In fact these terms approximate the derivative of the flux.

$$\frac{\mathcal{D}_a}{\Delta x\mathcal{M}_a}\mathcal{F}_a^{h,u} = ikH$$

$$\frac{\mathcal{D}_a}{\Delta x\mathcal{M}_a}\mathcal{F}_a^{h,h} = 0$$

$$\frac{\mathcal{D}_a}{\Delta x\mathcal{M}_a}\mathcal{F}_a^{u,h} = ikgH$$

$$\frac{\mathcal{D}_a}{\Delta x\mathcal{M}_a}\mathcal{F}_a^{u,u} = 0$$

So in particular for the mass equation

$$h_t + Hu_x = 0$$

then

$$i\omega h + \frac{\mathcal{D}_a}{\Delta x \mathcal{M}_a}\mathcal{F}_a^{h,u}u_j + \frac{\mathcal{D}_a}{\Delta x \mathcal{M}_a}\mathcal{F}_a^{h,h}h_j = 0$$

Indeed it is these the values of our approximations to these terms that confirm that our methods have the correct spatial accuracy.

# 3 Taylor Expansions Of First Order Values

$$\mathcal{G}_1 = H + \frac{H^3}{3}k^2 - \frac{H^3k^4(\Delta x)^2}{36} + O(x^4)$$

$$\mathcal{M}_1 = 1$$

$$\mathcal{R}_1^- = 1$$

$$\mathcal{R}_1^+ = 1 + ik\Delta x - \frac{1}{2}(k\Delta x)^2 - \frac{i}{3!}(k\Delta x)^3 + \frac{1}{4!}(k\Delta x)^4 + O((k\Delta x)^5)$$

$$\mathcal{R}_1^u = 1 + \frac{i}{2}k\Delta x - \frac{1}{4}(k\Delta x)^2 - \frac{i}{12}(k\Delta x)^3 + O(\Delta x^4)$$

$$\mathcal{D}\mathcal{F}_1^{h,u} = Hik\Delta x - \frac{Hi}{6}(k\Delta x)^3 + O(\Delta x^4)$$

$$\mathcal{D}\mathcal{F}_1^{h,h} = -\frac{\sqrt{gH}}{2}k^2\Delta x + O(\Delta x^3)$$

$$\mathcal{D}\mathcal{F}_1^{u,h} = igHk - \frac{igHk^3(\Delta x)^2}{6} + O(\Delta x^3)$$

$$\mathcal{D}\mathcal{F}_1^{u,u} = \frac{i\sqrt{gH}H}{6}(H^2+3)k^2(\Delta x) - \frac{\sqrt{gH}H}{72}(2H^2+3)k^4(\Delta x)^3 + O(\Delta x^5)$$

# 4 Taylor Expansions Of Second Order Values

$$\mathcal{G}_2 = H + \frac{H^3}{3}k^2 - \frac{H^3k^4(\Delta x)^2}{36} + O(x^4)$$

$$\mathcal{M}_2 = 1$$

$$\mathcal{R}_2^- = 1 + \frac{i}{2}(k\Delta x) - \frac{i}{12}(k\Delta x)^3 + O(x^4)$$

$$\mathcal{R}_2^+ = 1 + \frac{i}{2}k\Delta x + \frac{i}{6}k^3\Delta x^3 + O\left(\Delta x^4\right)$$

$$\mathcal{R}_2^u = 1 + \frac{i}{2}k\Delta x - \frac{1}{4}(k\Delta x)^2 - \frac{i}{12}(k\Delta x)^3 + O(\Delta x^4)$$

$$\mathcal{DF}_2^{h,u} = Hik - \frac{iH}{6}k^3(\Delta x)^2 + \frac{iH}{120}k^5(\Delta x)^4 + O(\Delta x^5)$$

$$\mathcal{DF}_2^{h,h} = \frac{\sqrt{gH}}{8}k^4(\Delta x)^3 - \frac{\sqrt{gH}}{48}k^6(\Delta x)^5 + O(\Delta x^7)$$

$$\mathcal{DF}_2^{u,u} = \frac{\sqrt{gH}}{24}H(H^2+3)k^4\Delta x^3 - \frac{\sqrt{gH}}{96}H(H^2+2)k^6\Delta x^5 + O(\Delta x^6)$$

$$\mathcal{DF}_2^{u,h} = igHk + \frac{igH}{12}k^3\Delta x^2 - \frac{13igH}{240}k^5\Delta x^4 + O(\Delta x^6)$$

# 5 Taylor Expansions Of Third Order Values

$$\mathcal{G}_3 = H + \frac{H^3k^2}{3} - \frac{H^3k^6(\Delta x)^4}{270} + O(\Delta x^6)$$

$$\mathcal{M}_3 = 1 + \frac{1}{24}(k\Delta x)^2 - \frac{1}{288}(k\Delta x)^4 + O(x^6)$$

$$R_3^- = 1 + \frac{i}{2}k\Delta x - \frac{1}{8}(k\Delta x)^2 - \frac{5}{48}(k\Delta x)^3 + O(\Delta x^4)$$

$$R_3^+ = 1 + \frac{i}{2}k\Delta x - \frac{1}{8}(k\Delta x)^2 + \frac{1}{16}(k\Delta x)^3 + O(\Delta x^4)$$

$$R_3^u = 1 + \frac{i}{2}k\Delta x - \frac{1}{8}(k\Delta x)^2 - \frac{i}{48}(k\Delta x)^3 + O(\Delta x^4)$$

$$\frac{\mathcal{D}}{\Delta x \mathcal{M}_3}\mathcal{F}_3^{h,u} = ikH - \frac{9iH}{320}k^5\Delta x^4 - \frac{iH}{448}k^7\Delta x^6 + O(\Delta x^9)$$

$$\frac{\mathcal{D}}{\Delta x \mathcal{M}_3}\mathcal{F}_3^{h,h} = \frac{\sqrt{gH}}{12}k^4\Delta x^3 - \frac{\sqrt{gH}}{72}k^6\Delta x^5 + \frac{\sqrt{gH}}{960}k^8\Delta x^7 + O(\Delta x^9)$$

$$\frac{\mathcal{D}}{\Delta x \mathcal{M}_3}\mathcal{F}_3^{u,u} = \frac{H\sqrt{gH}}{36}(H^2+3)k^4\Delta x^3 - \frac{iH\sqrt{gH}}{144}(H^2+3)k^5\Delta x^4 + O(\Delta x^5)$$

$$\frac{\mathcal{D}}{\Delta x \mathcal{M}_3}\mathcal{F}_3^{u,h} = igkH - \frac{igH}{30}k^5\Delta x^4 + O(\Delta x^6)$$

$$\frac{\mathcal{D}}{\Delta x \mathcal{M}_3}\mathcal{F}_3^{u,h} = igkH - \frac{igH}{30}k^5\Delta x^4 + O(\Delta x^6)$$