

Contents

1	Introduction	1
1.1	Objectives of the Thesis	1
1.2	Original Contribution of the Thesis	1
1.3	Organisation of the Thesis	1
2	The Serre Equations	3
2.1	The Serre Equations	3
2.1.1	Alternative form of the Serre Equations	4
2.2	Properties of the Serre Equations	5
2.2.1	Conservation Properties	5
2.2.2	Dispersion Relation	5
2.2.3	Analytic Solutions	6
2.2.4	Asymptotic Results	6
3	Finite Element Volume Method	7
3.1	Notation for Numerical Grids	7
3.2	Structure Overview	8
(I)	Reconstruct the Quantities Inside the Cells	10
(I).1	Reconstruction for h , w and G	10
(I).2	Reconstruction for b	11
(II)	Calculate the Velocity Over the Domain	12
(II).1	Basis Function Approximations	13
(II).2	Calculation of Element-wise Matrices	14
(II).3	Assembly of the Global Matrix	17
(III)	Calculate All the Fluxes Across the Cell Interfaces	17
(III).1	Calculation of Derivatives	18
(III).2	Modification to h to Enforce Well Balancing	20
(IV)	Calculate All the Source Terms for the Cells	20

(V) Update All the Cell Averages Using a Forward Euler Approximation	21
(VI) Update All the Cell Averages Using a Second-Order SSP Runge-Kutta Method	22
3.7 CFL condition	22
3.8 Boundary Conditions	23
3.9 Dry Beds	24
4 Linear Analysis of the Numerical Methods	27
4.1 Linearised Serre equations with horizontal bed	28
4.2 Evolution Matrix	29
4.2.1 Overview of the Evolution Step	30
(I) Reconstruct the Quantities Inside the Cells	31
(II) Calculate the Velocity Over the Domain	32
(III) Calculate All the Fluxes Across the Cell Interfaces	34
(IV) Calculate All the Source Terms for the Cells	37
(V) Update All the Cell Averages Using a Forward Euler Approximation	37
(VI) Update All the Cell Averages Using a Second-Order SSP Runge-Kutta Method	38
4.3 Convergence Analysis	39
4.3.1 Stability	39
4.3.2 Consistency	41
4.4 Dispersion Analysis	46
5 Validation	55
5.1 Analytic Validation	55
5.1.1 Soliton	55
5.1.2 Lake at Rest	55
5.2 Forced Solution Validation	55
5.2.1 Travelling Gaussian	55
5.3 Experimental Validation	55
5.3.1 Beji	55
5.3.2 Synolakis	55
5.3.3 Roeber	55
Bibliography	59