Title:

Robust Computational Models for Water Waves

Summary of Presentation:

The presentation consists of three main parts: the motivation for building robust computational models for water waves, the history of this project at the ANU and the major contributions of my thesis to this project. I will summarise all three parts here, separately.

Motivation:

A significant portion of the worlds population and therefore critical infrastructure is located on or around the coast, putting it at risk of ocean wave hazards such as tsunamis or storm surges. In fact tsunamis and storm surges rank among some of the deadliest and costliest natural disasters in human history. Therefore, detailed knowledge about the risks posed to coastal areas by these hazards is incredibly valuable. There are also a variety of physical phenomena that aren’t the waves themselves but instead a direct consequence of the waves that are of interest to a variety of fields. For instance; the transport of nutrients in shallow coastal waters; the erosion of beaches and the breakup of sea-ice.

History:

The ANU has a long standing relationship with Geoscience Australia (GA) to develop the robust computational model ANUGA. ANUGA was originally focused on modelling the inundation caused by storm surges but development has been focused on tsunamis as a result of the 2004 Indian Ocean tsunami which is the deadliest natural disaster of the 21st century thus far. This has not restricted its scope however, and ANUGA has been employed to model a wide array of natural hazards from riverine flooding of catchments in Mozambique to storm surges in Western Australia.

ANUGA is based on a mathematical model; a set of partial differential equations, known as the Shallow Water Wave Equations (SWWE). The derivation of the SWWE relies on a set of assumptions about the physics of water that explains most of the behaviour of water waves but neglects other effects; such as dispersion. Dispersion is the process by which waves of different frequencies travel at different speeds, which is true for real water but not for the SWWE. Recent papers have concluded that dispersion plays a significant role in the evolution of a tsunami [1,2] and so the inclusion of this effect in our computational model is important. Therefore, to improve our computational model a new mathematical model is required.

The Serre equations were chosen as the new mathematical model as they extend the SWWE to include dispersion, are considered one of the best models for water waves and have numerical methods that can fit within the ANUGA framework. As part of his PhD thesis Chris Zoppou developed a numerical method for the 1D Serre equations that combined a Finite Volume Method (FVM) and a Finite Different Method (FDM). The use of a FVM allows this numerical method to fit well within the current ANUGA program, however the FDM does not. This previous work also did not include a validation of the numerical methods for the Serre equations for steep gradient and dry bed problems.

Contribution:

My thesis extends Chris Zoppou’s work by replacing the FDM with a Finite Element Method (FEM) which fits well within the current ANUGA program. The thesis also validated this new numerical method for the steep gradient and the dry bed problem; thus solving all the major remaining hurdles for the 1D Serre equations. The result of this thesis is that we now possess a robust computational model for the 1D Serre equations that can be extended to the 2D Serre equations within the ANUGA framework.

In the presentation I describe the methods two central parts; the FVM and the FEM and how they interact, at an abstract level. The FVM is described using the intuition of the physics of conservation and then related to the general partial differential equations in conservation law form with a source term. While the FEM is described using the standard example, that you will find in most of the literature surrounding these methods.

I then describe the validation techniques of the steep gradient and the dry bed problem.

The steep gradient problem can be reduced to just the dam-break problem; which looks at the evolution of an initially still body of water on a flat bed with a discontinuous jump in the water depth. Currently, there are no known analytic solutions to this problem for the Serre equations. There are some comparisons with experiments and some numerical solutions present in the literature; however previously there was no consensus on which numerical solutions were correct, with a variety of observed behaviours. We observed a new behaviour too and then set about to demonstrate its legitimacy through the consistent convergent behaviour of numerical solutions for a variety of numerical methods. This produced the most extensive study of this problem by numerical solution thus far and provided very strong evidence of the validity of the developed numerical method for the steep gradient problem. Additionally, we were able to provide the community with information for when they should expect to observe the different behaviours given the resolution of the numerical method. This work was published in Wave Motion in 2018 [5].

The dry bed problem investigates the wetting and drying of beds in the numerical method. The inclusion of dry beds was another major contribution of the thesis, but the particulars of this were omitted from the presentation as they are quite technical. The validation of the numerical method for the dry bed problem relied on the use of forced solutions. These are solutions that are ‘forced’ by modifying the Serre equations with a source term that cancels the other terms precisely. By choosing appropriate functions we can assess the validity of all terms in the numerical method for solving the the dry bed problem for the Serre equations. This was required because there are currently no analytic solutions for the dry bed problem in the literature. We also compared our numerical solutions to experimental results to demonstrate the agreement of the computational model and the underlying physics.

The thesis provided another major contribution to the project and that is an analysis of the developed methods for the linearised Serre equations to determine their consistency, stability and dispersion properties. Some of this work was published in Applied Mathemtical Modelling in 2017 [4] . These results were important but were too technical for a more general audience and so were omitted from the presentation.

References:

1. J.T. Kirby, F. Shi, B. Tehranirad, J.C. Harris, S.T. Grilli, (2013), Dispersive tsunami waves in the ocean: Model equations and sensitivity to dispersion and Coriolis effects, Ocean Modelling, 62.

2. Kulikov, E. (2006), Dispersion of the Sumatra Tsunami waves in the Indian Ocean detected by satellite altimetry, Russian Journal of Earth Sciences, 8.

3. C. Zoppou. Numerical Solution of the One-dimensional and Cylindrical Serre

Equations for Rapidly Varying Free Surface Flows. PhD thesis, Australian

National University, Mathematical Sciences Institute, College of Physical

and Mathematical Sciences, Australian National University, Canberra, ACT

2600, Australia, 2014.

4. C. Zoppou, J. Pitt, and S. Roberts. Numerical solution of the fully non-

linear weakly dispersive Serre equations for steep gradient flows. Applied

Mathematical Modelling, 48:70–95, 2017.

5. J.P.A. Pitt, C. Zoppou, and S.G. Roberts. Behaviour of the serre equations

in the presence of steep gradients revisited. Wave Motion, 76(1):61–77, 2018.