**Response:**

I have compiled the examiners reports and responded to each issue raised. The following report has all the raised issues highlighted in green with the responses highlighted in blue. Each issue also has an accompanying red header and has been separated from the original text to make them easy to identify. For the final minor comments, ticks and crosses where used to denote whether the issue was resolved, with ticks demonstrating that the issue was resolved and crosses demonstrating that it was not. For each cross there is also a further justification.

**Examiner Reports**

**Examiner 1:**

I have now completed my review of this PhD research work and I recommend that

the thesis be awarded subject to minor changes being made to the satisfaction of the

delegated authority. I thoroughly enjoyed reading this thesis. It is well-written and the

chapters flow in a logical sequence.

The development of an experimentally calibrated computational model that can

predict complex flow behaviour, the presence of steep gradients, interactions of

dispersive waves with varying bathymetry, wave-breaking, and the wetting and

drying of a sloping beach is a very challenging research topic. The candidate and his

supervisory team should be congratulated on their excellent efforts in advancing the

current knowledge in this field. I believe the outcomes of this work could have a

significant impact on informing the development of future modelling capabilities in

higher spatial dimensions. Furthermore, the work from the thesis has been published

in high-impact international journals. The publications make noteworthy contributions

to the literature and clearly demonstrate the academic ability of the candidate.

The thesis is presented over seven chapters. In chapter 1, the background and

motivation for the research is given and the objectives of the thesis are stated

clearly. The motivation of using the Serre equations over the shallow water

equations to model water waves is also made clear. The candidate then summarises

the original contributions that the thesis research makes to the literature and

provides an overview of the four publications that arose from the work. Then, in

chapter 2, the literature related to the Serre equations is reviewed. At first the Serre

equations are introduced and the one-dimensional model is derived. Next, the Serre

equations are written in conservational form with a source term so that the finite

volume method can be applied. To conclude this chapter, a range of analytic

solutions are presented that are used to assess the accuracy and performance of the

proposed numerical methods. The forced Serre equations and the concept of forced

solutions are also provided.

In chapter 3, the full details of the second-order finite element method (FEVM) are

given. The full description of the linear analysis for FEVM is provided in chapter 4,

where the convergence and dispersion properties are investigated. Similar results

are deduced for the five other methods considered. I found this chapter very

informative and clearly presented.

In chapter 5, the analytic and forced solutions are used to validate the numerical

methods presented throughout the previous chapters to ensure the solutions behave

in a consistent and conservative manner. Convergence measures are introduced to

assess the numerical schemes. The defined measures are used at first to compare

the numerical methods for the solitary travelling wave solution, and then the

convergence and conservative properties are compared for the lake at rest solution.

The presentation of the results is well done. I felt the analysis is rigorous, correct and

presented in a consistent manner. The results were also convincing and were as

expected for these types of schemes. They also highlight the need for high-order

accuracy in the numerical approximations. The conclusion that second order

accuracy is sufficient is certainly something I’ve found in the past as well.

In chapter 6, the second-order FDVM and second-order FEVM schemes are

validated using experimental data. This data is used to assess the capability of each

method for simulating different physical scenarios. The results are impressive, with

both methods recovering the main distinguishing features of the experimental data.

These findings demonstrate the accuracy of the numerical methods in the presence

of steep gradients in the free surface. For the periodic waves case study, the

simulations show the importance of the dispersion terms in the Serre equations as

being crucial for capturing the experimental observation.

In chapter 7, the main results and outcomes of the research are summarised and the

potential future directions of the work are given.

**Issue:**

“One suggestion for improving chapter 7 would be to give further insight on the

challenges that might be faced in extending the mathematical and computational

foundations presented throughout the thesis to higher dimensions. An immediate

question that arises when implementing the schemes on unstructured grids is how

the flux limiter might be implemented and also how the interpolation schemes would

be extended to higher dimensions, and the impact this would have on the solver and

overall computation time.”

Response:

I have expanded the future work section, including some thoughts about the work required for higher dimensions. I focused on relating the current knowledge for the SWWE to the Serre equations and what aspects of the method require the most work, as this allows some insight into what is possible without falling outside the scope of the thesis.

**Issue:**

I now provide further feedback for the candidate to consider when preparing the final

version of the thesis for submission to the university.

“As a general comment, please check the style of using “I” and “we” throughout and

consider writing in the third person.”

Response:

I have rewritten everything in third person, excluding certain areas where my contribution was necessarily made explicit.

**Issue:**

**“Chapter 1**: is well written. Perhaps the literature related to the numerical solution of

these types of conservative equations could be broaden. For example, what

numerical methods/strategies have been employed previously for the shallow water

equations?”

Response:

I added further literature surrounding numerical solvers for conservative equations, focusing mainly on the SWWE, as it is also a water wave equation which has had a significant amount of previous work.

**Issue:**

**“Chapter 3**: should include further discussion on use of flux limiting methods for

these types of conservative models. At the bottom of pp. 25, explain meaning of

“robust”. The treatment of boundary cells and general boundary conditions should be

expanded. On pp 28, remind the reader of what h, w, b, and G are. The introduction

of SSP is brief and could be expanded (as well as alerting the reader that further

details will be provided later in this chapter). Sometimes notation is not always clear,

for example on pp 29 what is $q\_{-1/2}^{+}$, etc? On pp 30, explain why this

particular reconstruction method was used. There are many other limiters published

in the literature and I recommend giving further background. On pp 32, I think it

would helpful to provide some details of the spaces introduced and then refer to

Appendix B for further details. In equation (3.7) typo “,”. On pp 37, elaborate on the

conditioning of the matrix and discuss whether pivoting strategies are required (this

becomes more apparent on pp 45). Was any structure of the matrix considered? On

pp 40-41, I thought the section on source terms could be improved by adding more

detail. It wasn’t made clear to the reader why this strategy was necessary. On pp 42,

more discussion of the temporal scheme should be provided. Why was this particular

scheme chosen and what is a typical value of time step that can achieved during the

simulations for the solver? On pp 44, I think this type of transformation (3.25) may

have been employed for solving Richard’s equation in groundwater modelling

scenarios, it would be useful to broaden the literature on this to provide the reader

with further insight.”

Response:

I included some further background material on the use of flux limiting methods for conservative equations (in the description of the flux approximation). I also included a definition of robust. I did not expand the SSP introduction, as it is made clear that further details of the steps are provided after the overview. I cleared up the notation, and made sure to perform some extra explanation when new quantities are introduced. I provided some reasons for why the particular reconstruction was used. I expanded the introduction to the spaces. I fixed equation (3.7). I mention that the particular matrix solve is discussed later, and drew attention to this while also expanding on the matrix solution technique. Yes, the band-diagonal structure was made use of and this is mentioned in the text. I connected the well-balancing modifications to the results to better demonstrate the necessity of the employed method. I provided some further details on the reasons for the particular time stepping method (second-order, preserves stability of first order method). There was no adaptive time stepping performed in the thesis and so I did not comment on it. I provided some additional references to how others have handled the dry bed in the SWWE.

**Issue:**

**“Chapter 4**: Please check the use of “elementwise” throughout. It should be one

word. On pp 63, there is some confusion with $\lambda$ used in different contexts

on that page.”

Response:

Made elementwise appear as one word everywhere. Changed $\lambda^\pm$ to $\tau^\pm$ to prevent confusion between eigenvalues of the evolution matrix and wavelengths of waves.

**Issue:**

**“Chapter 5**: The comments around the third-order SSP RK method seem to suggest

something else may have caused the issues experienced. I felt further investigation

on this might be warranted. The family of exponential integrators might be worth

consideration in the future. This chapter may also have provided a good opportunity

to make computation time comparisons between the different methods and highlight

any trends in the time step history (adaptation) of the methods considered.”

Response:

I did test this in a variety of ways, such as those mentioned in the thesis and also some others. My strongest evidence for this was that when I chose some numbers that are guaranteed to sum to 1 in floating point such as (1/2,1/2) and (3/8,5/8) this error in conservation did not appear (although of course accuracy was lost, as these coefficients have nothing to do with the accuracy of the RK step). However, since this only explains why the conserved quantities (h,G) have an increase in conservation error over time from round-off precision, the order of accuracy was not required.

While a comparison of the run-times would be very interesting. I am not completely satisfied with the efficiency of the current code, particularly for the FEVM. As such an appropriate comparison of the methods can not currently be made, hence the lack of such comparisons in the thesis. Since the main objective of this thesis is the development of the foundation work for FEVM because it can be extended to 2D on unstructured meshes and is more stable in the presence of steep gradients, optimising this code and comparing run-times is not a pressing issue and so was not performed.

As for the time step history, delta t is kept constant for these results. I will made this more clear in the paper

**Issue:**

**“Chapter 6**: Possibly consider another read of this chapter as some smoothing of the

English presentation is still required. For example, on pp 97, the sentence starting

“While all the conserved quantities...” could be reworded. Same for the sentence

“Where a higher resolution...” seems incomplete. On page 116, the sentence

“Where the technique...” could also be reworded.”

Action :

I have edited the specific examples as well as the whole chapter.

**Issue:**

**“Chapter 7**: In its current form his chapter is too brief. In the first paragraph the

candidate states the work resulted in “new behaviours and the resolution of

differences” (in what exactly?). These are important statements (and contributions)

that should be further elaborated. I felt that the major contributions summarized on

pp 130 could be linked to the research questions and original research objectives

stated in chapter 1. Each bullet point could be expanded to refer to a specific

outcome achieved in an earlier chapter. More details on future work could also be

provided.”

Action:

I have expanded certain points that were too vague such as the one pointed out. Additionally, I have matched the contributions to the goals of the thesis and the chapters/references in which that work appears and I have expanded the future work section.

In closing, I would like to take this opportunity to wish the candidate well for their

future academic endeavours.

**Examiner 2:**

The thesis with title “Simulation of rapidly varying and dry bed flow using the Serre

equations solved by finite element volume methods”, written by Jordan Peter Antony

Pitt, concerns the mathematical and numerical modelling of surface water waves and

specifically the theoretical and numerical investigation of a specific mathematical

model of water wave theory known to as the Serre equations. The theory of water

waves is a major topic in the mathematical and engineering sciences due to its

important applications such as the impact of tsunamis in coastal areas. The physical

equations describing the propagation of water waves are the Euler equations for

incomressible fluid flow with free surface elevation. Although the Euler equations

consist of the exact equations of mass and momentum equations it is very hard to be

studied by theoretical and numerical means. Moreover, due to their complexity it is

almost impossible to have any use in the studies of the water waves runup on

beaches. For this reason scientists developed various approximations. These

approximations consist of nonlinear and dispersive partial differential equations.

Among the various mathematical models that approximate the Euler equations the

most accurate model so far seems to be the so called the Serre equations which

approximate the momentum equations of the Euler equation while the mass

conservation remains exact. The Serre equations currently attracts the attention of

most of the research groups in the area of water waves, including the strongests

groups in France and the United States. This is because of their complicated

formulation that makes their theoretical and numerical study challenging along with

the fact that the properties of the solutions of Serre equations remained unexplored

for decades. The scientific area of water waves remains one of the hot topics in

mathematical sciences. There are more than 20 international journal publishing

research articles in the area of waves. There are more than five international

conferences and several workshops dedicated in water waves taking place every

year while hundreds of publications appear annually. For all these reasons I found

the topic of the thesis not only extremely interesting but also timely accurate and

very successful.

The thesis is very well written that I hardly found typos or corrections to make. It is also very well-organised and reading the thesis of Mr Pitt was a real pleasure. The

problem and the objectives of the thesis are clearly stated and the new results in the

thesis along with the publications in scientific journals that occurred during this study

are clearly indicated.

The first chapter of the thesis consists of a bried and very simple to understand

introduction in the theory of water waves with emphasis on the asymptotic wave

regimes. The problem and the objectives of the thesis are stated clearlry making

obvious the importance of the study. The publications of the author are also listed,

including the abstracts and the contribution of the candidate on them. There are two

publications in prestigious international scientific journals and two publications in the

proceedings of two international conferences while I am sure more can be written in

the near future. This is an impressive achivement by a PhD student in the specific

field.

The thesis continues with the second chapter and an analytical description of the

Serre equations. Various formulations of the Serre equations are presented along

with some of its properties such as the conservation of energy, its dispersion

characteristics, the existence of analytical solutions and asymptotic solutions. The

focus of this chapter is on a specific formulation of the Serre equation that appears

ideal for applying his numerical methods. The candidate shows excellent knowledge

of the background of the equations and the relevant literature. The chapter closes

with a brief description of the so called “dam-break problem” or else the Riemann

problem for the Serre equations and presents some related new results that haven’t

been discovered by previous asymptotic techniques.

Chapter three is dedicated on a new numerical method for the approximation of the

solutions of the Serre equations. In my opinion this chapter is perhaps the most

important chapter of his thesis because the numerical method is new and highly

sophisticated. The method is called by the author “Finite element volume method”

and combines for the semi-discretisation the flexibility and locality of the finite volume

method and the high-accuracy of the finite element methods. Specifically, the finite

volume method is used to discretise the hyperbolic part of the equation while the

elliptic part is solved using the finite element method for the discretisation of the

elliptic operator. This combination appeared to be ideal. As far as it concerns the

time-integration a second-order Strong Stability Preserving Runge-Kutta method is

being used. The temporal discretisation by such method is the standard way if

someone wants to study the runup problem. The thesis also employs the state of the

art technique to achieve well-balancing of the numerical fluxes and solve also the

wet-dry bed problem, which is the hydrostatic reconstruction of the bottom adapted

to the Serre equations. The numerical method is described with great detail but of

course due to the high-complexity of the method requires an experienced

reader. The numerical methods of choice are appropriate for the purposes of this

study and scientifically sound.

**Issue:**

“In Section 3.3 the candidate presents an estimate

related to the CFL condition, which apparently seems valid. Here I would say that

there might be a good idea to include some references (if there are any) related to

the estimation of the CFL condition. On the other hand it might not be necessary as

for nonlinear dispersive waves it is known that the “real” CFL depends nonlinearly on

the amplitude (speed) of the waves that it is usually impossible to get an analytical

expression for it.”

Response:

Given that the CFL only requires the maximum and minimum wavespeed, this isn't necessary (as alluded to in the comment).

The chapter closes with the description of the dry bed handling.

Chapter four presents the analysis of the numerical method for the linearised equations and some convergence results for the numerical method. This analysis

provides with information about the stability and dispersion properties of the

numerical method and for dispersive equations is not an easy task. Moreover, the

analysis shows that the current numerical method has favourable convergence

properties and is more efficient compared to standard finite difference methods. For

new numerical methods it is always necessary to develop supportive theory since it

is important to guarantee that the method will provide correct and accurate solutions.

The specific chapter is a plus for the thesis and of course for the theoretical

background of the new numerical method.

Chapter five contains and experimental validation of the numerical method described

and analysed in the previous chapters. In addition, a comparison between other

numerical methods such as mixed finite difference-finite volume of various orders of

accuracy is presented. These methods also are extended from their original

formulation using hydrostatic reconstruction of the bottom to handle dry beds. The

chapter starts with the numerical convergence rates and computational errors for the

case of an analytical traveling wave solution. This experiment doesn’t require the

activation of the dry-bed techniques or the wet-dry algorithms but focuses on the

propagation of water waves.

**Issue:**

“Other error indicators such as phase and shape errors

could have been explored to support the numerical method even better. On the other

hand the numerical errors are very small, which proves that the method is highly

accurate.”

Response:

These are good points, although for the purposes of this thesis the current errors are sufficient in terms of confirming the order of accuracy and the conservation.

The convergence rates appeared also to be optimal.

**Issue:**

“There is one point that requires some extra attention. Because the mass conservation equation is

hyperbolic, it is expected a drop on the global convergence rate of the numerical

method on the boundaries of the domain. I think this has been demonstrated in the

Section 5.3 where the convergence rates and errors are presented in the case of

“lake at rest”.”

Response:

From the paper “The Convergence Rate for Difference Approximations to Mixed Initial Boundary Value Problems ” (Gustaffason,1975) a drop in the convergence rate at the boundaries will not affect the global convergence rate for this finite volume based method. This has been added in the boundary conditions section.

The poor convergence results for lake at rest are due to lack of well-balancing and not the loss of convergence rate at the boundary. While the poor conservation results are due to the initial discretisation and not the loss of convergence rate at the wet/dry fronts.

In this chapter one can find even more experiments. Some of the

experiments are original and they all demonstrate the efficiency of the new numerical

method.

After studying the convergence properties of the new numerical method, the

candidate presents a validation of the computational model against laboratory data in

Chapter six. A detailed comparison of numerical solutions with laboratory data for

four different experimental settings is presented. All these experiments are

chalenging, demonstrate the ability of the computational model to handle wet-dry

interfaces and highly nonlinear and dispersive effects. All the numerical results

presented in this chapter are of very good quality and support the new numerical

method.

Chapter seven consists of the conclusions and plans for future research. The thesis

also includes four appendices. The first two appendices consist of background

material related to the mathematical model and the numerical method. The third

appendix contains very interesting linear analysis for some numerical methods

closely related to the new one presented in the thesis that have been used for

comparisons. The last appendix contains the abstracts and additional information of

the published papers of the author.

The thesis is very well-written and in standard English. It is written in a balanced way

so it contains enough technical details so as the reader can have a complete

understanding of the methods and be able to reproduce the results. The

mathematical and computational tools employed by the author are of high quality

and significant difficulty. The candidate appears to have a complete understanding of the field, with extensive knowledge in the mathematical theory (differential equations,

computational mathematics and asymptotic analysis) and the physics (classical and

fluid mechanics) relevant to the subject of the thesis. The candidate has an in depth

knowledge of the field and has published two papers in highly ranked, international,

scientific journals and one in conferences proceedings, I concluded that he is

capable to carry out independent research and continue his academic career.

The main novelties of the thesis are the following: The rigorous derivation of a new

finite element/finite volume method for the numerical solution of the Serre equations

capable to handle wed/dry interface. The regorous study of the linear stability and

convergence of the new numerical method. The regorous study of the linear stability

and convergence of other numerical methods such as the finite difference and the

hybrid finite difference/finite volume methods for the numerical solution of the Serre

equations. The discovery of new dynamical behaviour for the flow of the Riemann

problem (dam-break problem). All achievements are of current scientific interest and

consist important advances for the mathematical studies of the theory and numerical

analysis of mathematical models of water wave theory.

Concluding, I believe that the thesis with title “Simulation of Rapidly Varying and Dry

Bed Flow using the Serre equations solved by a Finite Element Volume Method”

written by Jordan Peter Anthony Pitt contains highly advanced applied mathematical

tools, techniques and results, and is of excellent quality. It requires extensive

knowledge of various fields in mathematics and in physics. It is also characterised by

its interdisciplinary nature. There are are at least four important new results that lead

to various good quality publications, published in top- ranked international scientific

journals. The overal quality of the thesis is very satisfactory.

**Examiner 3:**

The main results of this thesis are the development and validation of a numerical technique

for the solution of the Serre equations which describe the long-wave limit of finite-amplitude

surface waves. The particular method uses finite-volumes for the unsteady components and

finite-elements for the solution of a boundary-value problem. This method is compared with

a number of other methods, but primarily with a similar method that uses finite-differences

for the boundary-value problem.

These two methods are extended beyond the constant coefficients Serre equations to variable bottom topography and dry beds where the fluid depth approaches zero. The main conclusion of the research is that the proposed method is the

best-performing method out a suite of methods for the Serre equations, in particular when

the solution has extreme gradients.

**Issue:**

“However, the final test case appears to be flawed, as the

solutions are compared against experimental observations in which the Serre equations are

not valid. In these scenarios I would anticipate that given sufficient resolution the solutions of

the Serre equations should remain bounded. This may not be the case, as there are examples

such as the fifth-order Korteweg–de Vries equation which develop singularities in finite time.

This point is discussed in more detail below.”

Response:

This is true, what these results highlight however while not being appropriate for the chosen

equations is the greater robustness of the FEVM over the FDVM. I agree that sufficient resolution should see this instability disappear. However, robustness should be maintained even for low resolutions. Therefore, my conclusions as stated in the thesis are correct. I have made this clearer in the thesis.

The thesis present a comprehensive analysis of various methods for the Serre equations,

and Mr Pitt has shown a very good understanding of the numerical methods.

**Issue:**

“His numerical analysis of the methods is sound, though in some places in Chapter 4 some further explanations could have been included.”

Response:

I have expanded some points that were vague, as suggested.

The choice of test cases for the two main numerical methods

is very appropriate, and clearly shows the strengths and weaknesses of all the methods that

are considered.

Although I would question which is the most useful method for one-dimensional calcu-

lations, I agree that that FEVM would be the preferred method for typical simulations in

two-dimensions. Hopefully further work can be done to extend this to run on unstructured

meshes in two-dimensions and to develop a parallel version of the code. I would foresee that

one of the major difficulties would be coupling this to a shallow-water code that applies in

the open ocean, but this should be surmountable.

**Issue:**

“The main aspect of the thesis which lets it down is the presentation. The problems related to this are: organization, reference to the work done by Mr Pitt, punctation and flow of text,and use of casual expressions. The organization of the content makes the thesis quite confusing to follow. I understand

the justification for this in that Mr Pitt wants to make it clear the work that he primarily undertook for this thesis. However, that then means that important results appear out

of place. For example, the observations of the dam-break problem in §2 could have been

expanded and put as a larger section in §5 as part of the validation and comparison of the

various methods. Similarly, since the thesis focusses on the comparison of FEVM and FDVM 2 ,

and it appears that Mr Pitt performed the analysis of FDVM 2 , it would have made more sense

to include this work in Chapter 4 or as a separate chapter.”

Response:

Thank you for your notes on the general punctuation and flow of text, I have read through and fixed this issue throughout the thesis. While I think the organisation of the thesis is a legitimate criticism, the current format was chosen for a number of reasons.

Firstly, while the results of the dam-break problem are very interesting, the analysis performed in that paper is very different to the other analyses in this thesis, and as a more exploratory investigation it lacks some of the rigour of the tests performed in this paper. Given that the paper provides this background and the analysis results we did not think duplication was required here, and intended further reading of the paper for those so inclined.

Secondly, in chapter 4 we present the derivation of the operators for the FEVM in detail. The derivation of the FDVM is remarkably similar, and only differs in the calculation of the $\mathcal{G}$ operator. Since this operator is provided in the appendix using the methods described in Chapter 4, we did not think a duplication of the derivation for the FDVM was necessary. This is to balance the requirement that the work be reproducible without the thesis being overly long.

These were definitely something I struggled with, but ultimately these were the decisions made and I think that the current organisation is good and justified.

**Issue:**

“Throughout the thesis there are many references to ‘I’ or ‘my’. For example §1.2 com-

mences with the sentence ‘My research . . . ’, which could be better expressed as ‘The research

undertaken for this thesis . . . ’. On p. 7 there is the sentence ‘This paper was produced

by me with the support of my coauthors based on my own work’. This could be better

expressed as ‘This paper was primarily prepared by me in collaboration with my coathors,

based on research that I primarily undertook’. The are many other occurrences referrring to

‘my research’. This detracts significantly from the contribution made by Mr Pitt’s supervi-

sors, in amongst other things, suggesting the original research topic, checking on calculations,

providing contacts and suggesting various avenues of research.”

Response:

This is a great point, that I did not consider with my use of first person. I have corrected this throughout the thesis.

**Issue:**

“The punctuation and flow of the writing is confusing and in places creates possible am-

biguities. For example on p. 1 at the beginning of the second paragraph ‘The physics of

water can be described using Newton’s second law. From which . . . ’. As a new sentence has

commenced it is not clear what ‘which’ refers to. In this case the full-stop after ‘law’ should

be replaced by a comma, which removes the ambiguity. On p. 8 in the first paragraph the

sentence ‘Followed . . . ’ should a continuation of the previous sentence or written as ‘This

is followed . . . ’. On p. 9, line 4, the semi-colon should be removed. On p. 14 the sentence

‘Additionally, the Green–Naghdi equations [21] which are equivalent to the Serre equations

for one-dimensional flows were . . . ’ should have commas after the reference and ‘were’. On

p. 15 in the first line a comma should be inserted rather than commencing a new sentence

or ‘Where’ should be removed. This connection between sentences is a common problem

throughout the thesis. However, a thorough proofreading of the thesis of the final version of

the thesis should occur to correct all the punctation.”

Response:

I have read through and addressed the sentence flow issue, and removed possible ambiguities.

**Issue:**

“Two very common expressions that occur in this thesis are ‘we get’ and ‘So that’. The

first could be replaced by ‘we obtain’, ‘gives’ or ‘gives rise to’. The second could be replaced

by ‘Consequently’, ‘Therefore’ or ‘Hence’.”

Response:

I have replaced these expressions as suggested, and as a result the thesis is a much better read, thank you.

**Issue:**

“In Chapter 3 the variable w = h + b is introduced. This is a derived variable and does not

occur in the Serre equations (2.6), hence I do not follow why it is carried through for all the

calculations . For example, in point (iv) of p. 29 S j only needs to be a function of h, b and

u. The only point that w is explicitly written is in (3.19)–(3.20), and could be defined just

for this calculation.”

Response:

I had a similar thought when I was writing it. However, I did want to be explicit about what variables were required, even derived ones, to calculate a given approximation. Hence the usage in the thesis.

**Issue:**

“The justification for FEVM compared to FDVM 2 largely relies on §6.3. As mentioned

earlier, I believe there are significant problems with this. Firstly, I think the figures for

the FDVM 2 , Figures 6.19–6.20 are incorrect. It looks as the experimental results have been

plotted in blue rather than the numerical results. The main reason I think this is that the

description in the text does not concur with the plots, there are dynamics due to reflected

waves which should not be present in the numerical results and in all other simulations the

dynamics of FEVM and FDVM 2 are very similar. Secondly, it is known that the Serre equa-

tions do not model breaking, therefore it is pointless to compare the solutions after breaking.

I would except that close to breaking dispersion must act to smooth any discontinuities, and

create solitary waves, dispersive wavetrains or undular bores. I would therefore suggest that

the breakdown of FEVM or FDVM 2 is due to the solutions being under-resolved. FEVM may not become singular, but it would be diverging from the solution of the Serre equations

in this limit. Hence to say from this test that FEVM is superior to FDVM 2 is unrealistic.”

Response:

Firstly, I can assure you that the figures and there labels are not incorrect. I have made certain that the explanation for the reflected waves in the actual experiment is clear in the text. These reflections are not present in the numerical solution, as the domain is so large that no boundary effects impact the results.

The Serre equations do not model wave breaking, as pointed out and so breaking waves in experimental results and numerical solutions of the Serre equations may be quite different, as observed for this experiment. Despite these shortcomings the utility of this experiment is that it demonstrates the robustness of the method up to and after wave breaking. The FEVM is capable of handling the sudden water jump with the usedresolution while the FDVM is not, making the FEVM more desirable for physical modelling where robustness of a method is vital as the behaviour can be quite complicated to predict prior to running the model. I have made this point clear in the text.

**Issue:**

“I think that the boundary conditions in §6.2 are incorrect, in particular the equation on

p. 106. Since this is a periodic wave this should be

u(x, t) = (ω /k) ((h(x, t) − h\_0 )/ h(x,t))

For the low frequency case the wave

√ speed should be 1.84 m/s and for the high frequency this

should be 1.64, m/s, rather than gh 0 = 1.98 m/s. As the error for the high frequency case

seems to be larger than for the low frequency case, this could explain some of the difference

between the numerical and experimental results.”

Response:

I reran these experiments with the suggested changes and it makes no difference to the experimental results. I have amended the boundary conditions but no other changes were necessary.

**Issue:**

“Chapter 4 and §2.2.2 would be much simplified by showing that the linearised Serre

equations are just the Boussinesq equations:

[]

where X = x − U t.”

Response:

While this is an interesting point and it would aid some readers, I have decided to maintain a consistent set of equations and therefore not make this suggested change.

**Issue:**

“Finally, there are two comments for consideration, that do not necessarily need to be

addressed in the final version of the thesis.

Regarding Figure 2.2 and the related text. Does the behaviour in the middle of the

domain oscillate between the node structure and the growth structure as the two wavetrains

pass through each other? Since it is concluded that the dynamics in the vicinity of this region

are linear, it possible that this structure is just superposition or annihilation of wavetrains.”

Response:

There is no oscillation between structures, just a decay from the growth to the node to the flat structure as time passes. The conclusion is not that the dynamics in the vicinity of this region are linear, but that they are non-linear over short time spans as linear theory predicts separate wave trains. This is a fascinating result and we really are interested to see any progress that can be made to understanding this behaviour.

**Issue:**

On p. 79 the error in FDVM 3 was attributed to the fractional coefficients not being able

to be represented exactly in finite-precision. Were other third-order Runge–Kutta schemes

tested for which the coefficients can be represented exactly?

Response:

I did not try other third order methods, but I determined that this was the correct reason as when I altered the coefficients for the third-order RK scheme the conservation was exact. Although, owing to changing just the 1/3 and 2/3, the order of accuracy was lost. To me this demonstrated relatively clearly that it is these coefficients that were troublesome for conservation.

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**Issues:**

Minor points

An incomplete list of minor points that should be addressed in the final version of the thesis

are:

• Units for parameters are typically in roman font, e.g., 1.84 m/s. **X**

• p. 4, line -7: ‘whose results , and the analysis and results for this method’. ✓

• p. 8: ‘It’s’ should be ‘Its’. ✓

• p. 10, lines 11–13: This sentence is ambiguous and should be clarified. This could

interpreted to mean that in a numerical method the Serre equations are solved without

bottom friction and then the results are post-processed to include bottom friction. ✓

• p. 10, line -8: ‘average horizontal velocity’. ✓

• p. 10, line -4: ‘a fluid particle’. ✓

• p. 10, line -1: ‘vertical momentum equation of the Euler equations’. ✓

• p. 11, line -4: ‘dispersion of waves on the free surface’. ✓

• p. 12, lines (-2)–(-1): ‘By rewriting the Serre equations and introducing’. ✓

• p. 13, lines (-2)–(-1): ‘and a dispersion relation’. ✓

• p. 17, line -12: ‘by summing integrating’. ✓

• p. 17, line -4: ‘of for the Serre equations’. ✓

• p. 19 line 9–10: ‘initial condition value problems’.✓

• p. 25, line -1: This statement that the finite-volume method only requires values within

the cell is not technically correct, as it is dependent through G on values outside the

cell. This is clear from the top of p. 37 where it is stated that the matrix that must be

inverted is penta-diagonal. ✓

• p. 26, line -9: What equations are to be solved when it is stated ‘solving the equations’ ? ✓

• p. 32, line -9: ‘Integrating this equation by parts’. ✓

• p. 44, lines (-13)–(-12): ‘small chosen positive parameter. The analytical error intro-

duced by this transformation is smallest decreases with h base is smallest.’ ✓

• p. 44, line -2: ‘also serves’. ✓

• p. 45, line -10: Font for ‘LU’ should be consistent. ✓

• p. 45, line -5: ‘We possess obtain’. ✓

• p. 46, line 2: New sentence. ✓

• p. 48, line 7: ‘We analysed analyse’. ✓

• p. 48: Third paragraph should be the concluding sentence of the second paragraph. ✓

• p. 48, line -5: ‘and which is independent’. ✓

• p. 48, line -3: ‘propagating Fourier modes through the numerical scheme representing

the numerical solution in terms of Fourier modes’. ✓

• p. 52: For the second and third equations, general typesetting convention is to start

each line with an operator, so second line in both should start with an ‘=’.

• p. 52, line -1: ‘the this’. ✓

• p. 56, line 2: ‘When the flow is flowing to’. ✓

• p. 58: Last paragraph reads like dot points. Needs connections between sentences to

improve the flow. ✓

• p. 59, line 9: ‘g’ should be in math font. ✓

• p. 59, line -8: ‘This is different differs’. ✓

• p. 59, line -3: ‘enough sufficient’. ✓

• p. 63, line 4: ‘determines the error in the speed’. **X**

• p. 72: In §5.1.2 the initial quantity that should be used is the numerical representation

of the initial condition, rather than the actual initial condition. This is acknowledged

at the end of the section, and borne out in §5.3. **X**

• p. 73, line 17: ‘will be useful’. ✓

• p. 74, line -5: ‘over as a function of’. ✓

• p. 77, line 1: ‘h’s evolution equation the evolution equation for h’. ✓

• p. 77, lines 2–3: ‘; with the error in u being dominated by the error in G . The error in

the calculation of u is then dominated by the error in G.’ ✓

• p. 77, line -11: ‘the uh’. ✓

• p. 80, para. 4: In the third sentence I do not follow why the second-order increase in

the error follows as ∆x → 0. ✓

• p. 83, line -3: ‘numerical solutions reproduction’. ✓

• p. 86, lines 5–6: ‘an a 1 high Gaussian bump a Gaussian bump of amplitude a 1 ’. ✓

• p. 87, line 17: ‘in Figure 5.12 for’. ✓

• p. 95, line -2: ‘in on the water surface’. ✓

• p. 97, line 15: ‘observe generate oscillations’. ✓

• p. 103, lines 8–9: ‘except mass; which is conserved exactly . The exception is the mass,

which is conserved exactly’. ✓

• p. 103, line 9: ‘machine epsilon precision’. ✓

• p. 116, line -1: ‘and these’. ✓

• p. 123, line 8: ‘methods robustness of the numerical methods’. ✓

• p. 124, lines 11–12: ‘method simulation during the run-up process due to the methods

handling’. ✓

• p. 129, line -12: Font for D and W should be calligraphic. ✓

• p. 130, line -2: Including wave breaking is not a natural extension of this model, as

the Serre equations do not apply. The formulation of the model would have to change

completely. ✓

• p. 143, equation (C.5): Subsequent to this equation F η,G is not mentioned. Should this

term be zero following from (C.2a)? **X**

Response:

I made most of these suggestions, as is indicated by the checkmarks. I have also not done some, and I will the reasoning for not making the suggested changes here.

“• Units for parameters are typically in roman font, e.g., 1.84 m/s.”

I did not make this change because units were already consistently in one font. I have also not been suggested this in my publications which use non roman fonts for the units. Since consistency is paramount, I decided to not make this change and thus avoid any inconsistencies.

“• p. 63, line 4: ‘determines the error in the speed’.”

I did not make this change because this statement is false, although I have attempted to make the context clearer to stop confusion into the future.

“• p. 72: In §5.1.2 the initial quantity that should be used is the numerical representation

of the initial condition, rather than the actual initial condition. This is acknowledged

at the end of the section, and borne out in §5.3.”

Both are acceptable forms of demonstrating conservation, the added benefit of the analytic solution is that it also supports the convergence results as well. For this reason testing the conservation with the analytic solution is also a more difficult test of the method. Therefore, I have chosen to demonstrate it against the analytic solution, and deferring to the numerical initial conditions where necessary.

“• p. 143, equation (C.5): Subsequent to this equation F η,G is not mentioned. Should this

term be zero following from (C.2a)? .”

This term is mentioned. I have however attempted to clear up the following sentences.

Action :

--Addressed all these points--

✓ -indicates action taken as directed

**X –** indicates direct action was not taken

**Additional Changes:**

Abstract : modified to improve sentence flow, fixed spelling mistakes: ‘parallelisable’ and made the goal of the thesis more clear.

Chapter 1: Improve ‘flow’, restrict to third person, more formal. Expanded on current methods used to solve the SWWE and some background as to why (parallel supercomputers)

Chapter 2: edited sentence structure, made it third person and more formal.

Chapter 3: edited sentence structure, made it third person and more formal. Addressed Examiner 1s points

Chapter 4: edited sentence structure, made it third person and more formal. Added additional explanation of the reasoning as suggested by Examiner 3.

Chapter 5: edited sentence structure, made it third person and more formal. Addressed the concerns of Examiner 1 and 3.

Chapter 6: edited sentence structure, made it third person and more formal. Addressed the concerns of Examiner 1 and 3.

Chapter 7: edited sentence structure, made it third person and more formal. Addressed the concerns of Examiner 1.

Abstracts: edited sentence structure, made it third person and more formal.

Equation 3.7: brought comma outside the brackets (p.33)

Equation for the reconstructing cubic for the bed subscripts corrected (p.38)