1 Cell Averaged G

We have:

$$G = uh - \frac{\partial}{\partial x}\left(\frac{h^3}{3}\frac{\partial u}{\partial x}\right)$$

To take the cell average integrate over cell and divide by its measure (length). We will denote cell averages with an over bar.

$$\frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} G dx = \frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} \left[uh - \frac{\partial}{\partial x}\left(\frac{h^3}{3}\frac{\partial u}{\partial x}\right)\right] dx$$

$$\overline{G} = \frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} uh dx - \frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} \frac{\partial}{\partial x}\left(\frac{h^3}{3}\frac{\partial u}{\partial x}\right) dx$$

Leibenitz rule gives (bounds of integral dont depend on x):

$$\overline{G} = \frac{1}{\Delta x}\int_{x_i}^{x_{i+1}} uh dx - \frac{1}{\Delta x}\frac{\partial}{\partial x}\int_{x_i}^{x_{i+1}} \left(\frac{h^3}{3}\frac{\partial u}{\partial x}\right) dx$$

I dont think anything nice is going to fall out from here.