Title: Numerically Solving The 1D Serre Equations in the Presence of Discontiniuities

Abstract:

The Serre equations are a shallow water approximation to the incompressible Euler equations that retain the terms of the Shallow Water Wave Equations while introducing dispersive terms that make the Serre equations more relevant when wave amplitude is significant compared to water depth. Most of the literature numerically solves these equations for smooth initial conditions, however, in real world applications such as the Dam-Break problem it is important to handle discontinous initial conditions.

Thus the numerical scheme proposed in [Hank] has been extended to second- and third-order to investigate the capabilities of this scheme in the presence of discontinuities, which we expect to be good since it utilises a Finite Volume Method.

These methods were validated and their order of convergene was confirmed for smooth initial conditions using the analytic soliton solution. The methods also compared well with experimental results from [] containing a discontinuity.

To further investigate the behaviour of discontinuities a smooth approximation of the Dam-Break problem was used to observe how the solutions to the smooth dam break problem behaved as the initial conditions became more discontinous. The results of these methods were compared to the results of two second-order finite difference methods. One which is simply the second-order centered finite difference approximation to the Serre equations and the other from [Grimshaw]. These schemes showed the same behaviour in the presence of steep gradients as those derived from the numerical scheme proposed by [Hank] and included all the observed behaviour for discontinuous and smooth initial conditions thus observed far in the literature.

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