STAT 532 Assignment 2
Due: Friday, Sept. 11th by 4:00

Show all work **neatly and in order** for full credit and only include computer code and output that is necessary to completely answer a question. Other well organized code and output can be included in the appendix so I can check your work and provide comments if needed. In other words, I do not want to have to search through code or unnecessarily turn pages to find your "answer." Any plots should appear with the corresponding exercise solution.

NOTE: I expect that you will not refer to the solutions of previous students or search for solutions on-line until after assignments are completed and handed in.

1. (3 pts) Read Chapter 1 in Gelman and answer the following questions:
   - What is the mathematical definition of probabilities? (You can quote book(s) on this)
   - Briefly summarize the different ways we use probabilities to describe the world around us.
2. (4 pts) In the debate we talked briefly about a frequentist's criticism of the likelihood principle (Thanks Jordan). Please describe the argument and give an example to back up the argument. (Explain to someone who has never heard of it and reference where you found the argument).
3. (4 pts) In making inference starting from a likelihood function we often appeal to the idea of sampling distributions. Explain (to another student in the class) how the sampling distribution concept can be connected to maximum likelihood estimators. You can use pictures and words.
4. (4 pts) Explain in detail what a likelihood function is (to someone with little mathematical background) and how it differs from a probability function. Think carefully about use of notation in your discussion.
5. (2 pts) In the debate, Wilson made a point using the duckling example. Using Wilson's example, argue for the use of Bayesian methods. (max half page answer)
6. (3 pts) Plot Beta distributions to get a sense of how the shape of the beta distribution depends on the parameters. Provide a pretty plot comparing the density for 4 different parameter values with $\alpha = \beta$. You should also play around with unequal values, but you do not have to include results with the homework. (I do not want to see R-code)

7. (4 pts) Repeat the previous problems with Gamma distributions. Pay attention to the parameterization given in Gelman et al. and look up how Gamma distributions have been used in the construction of prior distributions for variance parameters. Why is the Gamma distribution a natural choice for a prior in this situation? (I do not want to see R-code)

8. A doctor is trying to decide whether a patient suffers from a particular type of bacterial infection. The diagnostic test is stated to have a correct indication of the infection 92% of the time.

   (a) (4 pts) One doctor in a practice believes about 5% of patients around the location suffer from this type of infection and and another doctor in the same practice thinks it is closer to 10%. They would both be considered experts. Incorporate the doctors' beliefs to obtain the probability of this bacterial infection given a person tests positive for it. Briefly discuss the results.

   (b) (4 pts) Now, come up with a continuous probability distribution to reflect each Dr.'s belief. Compare them with a plot and justify your choice. Again compare the results from the two priors.

9. (4 pts) Laplace's first real-life application of the Beta-Binomial model to calculate an inverse probability was to estimate the proportion of girl births in a population. A total of 241,945 girls and 251,527 boys were born in Paris from 1745 to 1770. Laplace let θ be the probability that any birth is female, and based on these data calculated the probability that θ is less than 0.5. What conclusion would you make about whether the sex ratio is 0.5 based on the resulting piece of information? Show work of calculations and clearly justify your conclusion.

10. (8 pts) Use R to plot the likelihood function associated with the following scenarios. You can use the given distribution for the likelihood function, assuming you do not know the parameters of the distribution. In other words, suppose you have the data and the form of the likelihood function, but do not know the parameters that were used to generate the data. Calculate usual summary measures, add vertical lines to your plots showing the MLE, the true value, and ends of a confidence interval.

    (a) One realization of 20 observations from a $Poisson(\lambda = 5)$ distribution

    (b) One realization of 100 observations from a $Poisson(\lambda = 5)$ distribution

    (c) One realization of 15 observations from a $N(\mu = 10, \sigma^2 = 5)$ distribution

    (d) One realization of 5 observations from a $N(\mu = 10, \sigma^2 = 5)$ distribution

    (e) One realization of 1 observation from a $Binomial(m = 100, p = 0.2)$ distribution

    (f) One realization of 30 observations from a $Binomial(m = 100, p = 0.2)$ distribution

(g) Briefly comment on anything that surprised you or observations that reinforced something you already knew.

11. (4 pts) Exercise 1.1 (page 27 - Use R to "sketch" the density in part (a))

12. (2 pts) Exercise 1.6

13. (4 pts) Exercise 1.8

14. (6 pts) Problem 2.5 (b), (c), and (d) on page 58.

15. Investigating Jeffreys ideas about noninformative priors:

    (a) (3 pts) Suppose $\theta \in (0, \infty)$ and we would like to reflect our ignorance about θ by using a prior distribution uniform over θ's support such that $p(\theta) = 1I_{(0,\infty)}(\theta)$. We often prefer to transform such variables so that they are defined on the whole real line using a natural log transformation. Show via transformation what prior is induced on $\log(\theta)$ by placing the uniform prior on θ. Does it appear to be obviously "non-informative?"'

    (b) (2 pts) Problem 2.12

16. (4 pts) For the Beta-Binomial situation and the following cases look at the sensitivity of the posterior distribution to the following prior distributions: Beta(1,1), Beta(0.5,0.5), Beta(2,2), and the improper Beta(0,0). Show your work and briefly describe your conclusions.

    (a) $m = 6, y = 2$

    (b) $m = 30, y = 10$

    (c) $m = 90, y = 30$

17. (3 pts) Explain the meaning of the word "subjective" (contrast with "objective"): Write a detailed list of steps involved in Statistical Inference (from research question formulation through dissemination of statements made from statistical inference) and classify each one as either more subjective or more objective (not allowed to say the degree of subjectivity and objectivity are equal). Very briefly justify each choice (you do not even have to use complete sentences).

18. (0 pts) Be reading Chapters 7, 8, and 9 in Willful Ignorance.