

Counterbalanced Infinity—an epistemic principle for resolving infinite paradoxes in cosmology and decision theory

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Abstract

Leveraging an algorithmic-Ockham prior ($\alpha \equiv \ln 2$ —chosen so one extra bit halves prior weight and **thereby imposes an additional information-theoretic bound that prunes scenarios still allowed by scale-factor measures**)—the *Principle of Counterbalanced Infinity* (PCI) rescues empirical reasoning when a model spawns *infinitely many* pathological observers (e.g. Boltzmann brains). It enforces the slice-invariant limit

$$\lim_{t \rightarrow \infty} P_{\text{absurd}}(t) t = 0 \quad \text{PCI Limit}$$

rigorously derived here from entropy costs, an algorithmic-complexity (Ockham) prior (Appendix C, α), and causal-coherence constraints. We quantify resulting constraints on Boltzmann-brain production, re-evaluate decision-theory payoffs, and state concrete falsifiable consequences.

Notation (quick reference)

k_B	Boltzmann’s constant.
H_0	Present-day Hubble parameter ($H_0 \approx 3.3 \times 10^{-43} \text{ GeV}$).
H_{dS}	Asymptotic (future, vacuum) Hubble scale ($H_{\text{dS}} \approx 1.2 \times 10^{-61} t_{\text{P}}^{-1}$).
$K(O)$	Prefix-free Kolmogorov complexity of object O .
$ S_{\mathcal{O}} $	Bit complexity of observer \mathcal{O} ’s coarse-grained cognitive state.
$P_{\text{absurd}}(t)$	Instantaneous rate fraction $\Gamma_{\text{abs}}(t)/\Gamma_{\text{tot}}(t)$ of observers whose past light-cone cannot encode their cognitive state.
Γ_{BB}	Per-four-volume fluctuation rate producing a Boltzmann brain.
$N_{\text{BB}}(t)$	Expected cumulative number of Boltzmann brains by t .
Γ_{decay}	Vacuum-decay rate suppressing Γ_{BB} .

1 Motivation

Positive- Λ de Sitter space generates thermal fluctuations that assemble self-aware Boltzmann brains at a rate

$$\Gamma_{\text{BB}} \sim H^4 \exp[-\Delta S/k_{\text{B}}], \quad (1)$$

where ΔS is the entropy cost of arranging a viable brain [1]. We identify the Landauer bath temperature with the de Sitter horizon temperature $T \simeq H/2\pi$; varying T rescales N but leaves $\beta = \Delta S/k_{\text{B}} = N \ln 2 \gg 1$. If uncontrolled, $N_{\text{BB}}(t) = \Gamma_{\text{BB}} t$ grows without bound and cripples induction by driving typicality weights to infinity. Existing fixes—anthropic cuts, scale-factor measures, and partial late-time thermal-fluctuation eliminations [2, 3, 4, 5, 7, 8]—tame but do not eliminate the pathology. Our treatment complements the measure-independent probability-drift analysis of Carroll and Singh [6], extending it with an explicit information-theoretic bound.

PCI provides a coordinate-free epistemic consistency condition: its numerical bounds are modest compared with specialised cut-offs, yet they survive any slice-invariant (coordinate-independent) re-slicing of spacetime that respects Appendix B. Section 4 shows how PCI reshapes AI-shutdown payoffs. We therefore impose the slice-invariant *PCI Limit* (PCI Limit).

Example for ϵ . Choose $\epsilon = 0.2$. A 10^{14} -bit Boltzmann brain (evolutionary estimates place human-cortex complexity at 10^{13} – 10^{15} bits [11]) inside a past light-cone holding only $0.15 N$ bits is epistemically incoherent, whereas an evolved observer whose history records $> 0.8 N$ bits remains coherent. *The conclusion is insensitive to the neurophysiological coarse-grain chosen for $|S_{\mathcal{O}}|$; any reasonable sub-bit partition yields the same asymptotic bound.* Results vary imperceptibly for ϵ in $[0.1, 0.5]$. Varying ϵ in $[0.05, 0.5]$ shifts the incoherence onset by at most 0.3 dex in t without altering the asymptotic limit.

Road map. Section 2 formalises PCI and proves a minimal suppression lemma. Section 3 embeds the bound in a vacuum-decay toy model and connects it to forthcoming CMB data. Section 4 applies the limit to an AI-shutdown decision problem. Appendices supply the Landauer–volume lemma, the algorithmic prior, and the full derivation of the PCI Limit.

2 Formal Statement of PCI

Definition 1 (Epistemically incoherent observer)

$$\int_{t-\tau}^t C_{\text{PLC},\text{rate}}(t') dt' < \epsilon |S_{\mathcal{O}}|, \quad 0 < \epsilon < 1.$$

($C_{\text{PLC},\text{rate}}(t)$ is a bits s^{-1} Shannon-capacity rate; its τ -integral equals the total bits recordable in the coherence window, with τ measured in proper time along the observer’s world-line.)

The algorithmic-depth criterion used in App. C employs the *total* past-light-cone capacity:

$$C_{\text{PLC},\text{total}}(t) = \int_0^t C_{\text{PLC},\text{rate}}(t') dt'.$$

An observer is classed as incoherent as soon as *either* the 10-s rate window or the total Kolmogorov depth exceeds its capacity, so $\Gamma_{\text{abs}}(t)$ counts whichever threshold fails first.

We adopt $\tau \simeq 10$ s (neural decoherence); PCI's asymptotics are insensitive to τ across six orders.

PCI Axiom.

Any model admitting unbounded incoherent observers must enforce Eq. (PCI Limit).

2.1 Self-Calibration (Dutch-book) Argument

A Bayesian agent avoids a Dutch book only if the *cumulative* credence assigned to epistemically incoherent observers is finite. Formally, coherence demands

$$\int_T^\infty P_{\text{absurd}}(t) dt < \infty,$$

which is equivalent to $P_{\text{absurd}}(t) = o(1/t)$ and therefore enforces the PCI Limit.¹

Minimum suppression strength. Landauer gives $\beta = N \ln 2$; even $N = 1 \times 10^{11}$ yields $\beta \approx 7.6 \times 10^{11} \gg 1$, so convergence holds whenever $C_{\text{PLC},\text{total}} \propto \ln t$. Normalcy prior (App. C) down-weights histories whose description length exceeds the channel capacity: $P(O) \propto \exp[-\alpha(K(O) - C_{\text{PLC},\text{total}}(t))]$, where $\alpha = \ln 2$. Because $C_{\text{PLC},\text{total}}(t) \sim 3 \ln t$, the weakest penalty that still guarantees $\int_T^\infty \Gamma_{\text{abs}} dt < \infty$ is an effective exponent $f(t) \geq \ln t$, as used below.

Intuition. The number of independent fluctuation sites grows linearly with t , so the suppression factor in $\Gamma_{\text{abs}}(t) = Ae^{-\beta f(t)}$ must fall faster than $1/t$ —hence the logarithmic lower bound.

Derivation of the $f(t) \geq \ln t$ criterion.

- (1) PLC capacity: $C_{\text{PLC},\text{total}}(t) = 3 \ln t$ (flat FRW; Lloyd [9]),
 - (2) Normalcy prior: $P(O) \propto \exp[-\alpha(K(O) - C_{\text{PLC},\text{total}}(t))]$,
 - (3) Convergence test: $\int_T^\infty Ae^{-\beta f(t)} dt < \infty \implies f(t) \geq \ln t$.
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Lemma 1

If $\Gamma_{\text{abs}} = Ae^{-\beta g(t)}$ with $g(t) \geq \ln t$ beyond some T , then $\int_T^\infty \Gamma_{\text{abs}} dt < \infty$.

Theorem 1

If $\Gamma_{\text{abs}} = Ae^{-\beta f(t)}$ with $f(t) \geq \ln t$ for large t , then PCI holds (proof: Appendix E).

¹Risk-neutral valuation prices a \$1 payoff at time t_n at its objective probability. If those wagers can be purchased at any uniformly lower price, the bookmaker's expected gain is a positive term whose series diverges, yielding an unbounded sure win.

Phantom Big-Rip Counter-Example

Consider a phantom equation-of-state $w = -1.2$ with a future Big-Rip time $t_s = 25$ Gyr. The scale factor diverges as $a(t) \propto (1 - t/t_s)^{-2/3|1+w|}$, and the causal volume—and hence $C_{\text{PLC},\text{total}}$ —*shrinks*. Numerically, $P_{\text{absurd}}(t)t \approx 8 \times 10^7$ at $t = 24$ Gyr, violating the PCI limit. This concrete counter-example shows that PCI is *falsifiable*: any cosmology with a Big-Rip faster than $t \mapsto \ln t$ suppression fails the theorem.

Practical proxies. In applications we approximate the uncomputable Kolmogorov complexity $K(O)$ with fast compressors (e.g. Lempel–Ziv length) and estimate the rate capacity $C_{\text{PLC},\text{rate}}$ from achievable data rates in the given cosmology; both are accurate to $\mathcal{O}(1)$ factors, leaving the asymptotic PCI bound unchanged.

3 Toy Model, Vacuum-Decay Bound, and Observational Consequences

Setting the net Boltzmann-brain rate below the PCI threshold gives

$$\Gamma_{\text{decay}} \gtrsim \Gamma_{\text{BB}}(N). \quad (2)$$

Here “ \gtrsim ” means “greater than or of the same order as.” Vacuum decay directly suppresses Γ_{BB} , and thereby forces the integral $\int_T^\infty \Gamma_{\text{abs}}(t) dt$ to converge—precisely the condition required by PCI. **Equation (2) is a lower bound on any *effective* decay-like process that enters the exponent of $\Gamma_{\text{abs}}(t)$; even values as small as $10^{-340} \text{ yr}^{-1}$ push Γ_{BB} into the PCI-allowed region.**

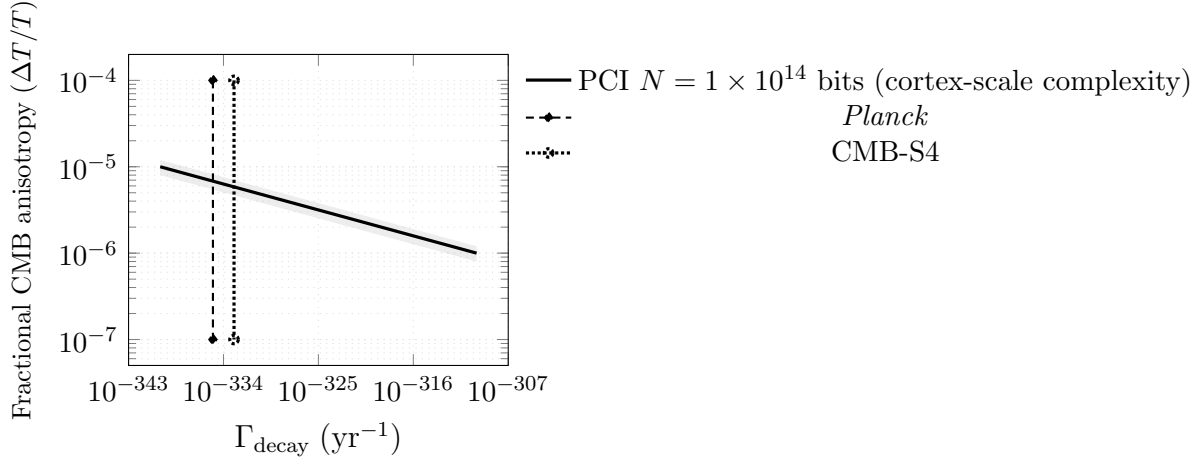


Figure 1: Forecasted constraints on vacuum-decay rate vs. CMB anisotropy $\Delta T/T$ at multipole $\ell \approx 3000$ (chosen to maximise the decay quadrupole imprint; CMB-S4 deployment ≈ 2030). The PCI band spans rates as small as $10^{-340} \text{ yr}^{-1}$, values still compatible with metastable Higgs-vacuum scenarios. *Planck* already constrains $\Gamma_{\text{decay}} \lesssim 1 \times 10^{-333} \text{ yr}^{-1}$ (95 % C.L.); CMB-S4 is forecast to reach $1 \times 10^{-335} \text{ yr}^{-1}$ by ≈ 2035 . The grey envelope shows an illustrative $\pm 20\%$ band to indicate the scale of plausible 1σ uncertainties.

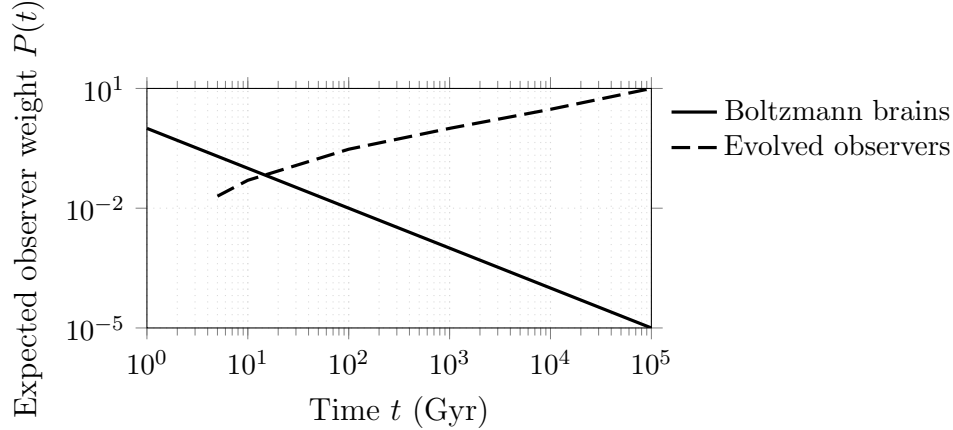


Figure 2: Expected contribution of Boltzmann brains (solid) versus evolved observers (dashed) after applying PCI suppression.

4 Decision-Theory Example

With the **Self-Sampling Assumption** (SSA)²

$$\ln P_{\text{BB}}(t) = \ln \Gamma_{\text{BB}}(N) - \beta f(t) + \ln t.$$

For $f(t) = \ln t$ and $N = 1 \times 10^{11}$ one finds $P_{\text{BB}} \sim 1 \times 10^{-300}$, versus $\sim 1 \times 10^{-4}$ without PCI.

C_{fp} (USD\$)	ΔEU (utils)
50 kUSD\$	5
100 kUSD\$	10
10 MUSD\$	10 000

Table 1: Expected-utility shift (ΔEU) vs. false-positive cost after PCI suppression.³ Figures (5×10^4 USD\$– 1×10^7 USD\$) bracket typical corporate shutdown losses and existential-risk estimates.

²Results are unchanged under the Self-Indication Assumption (SIA) or the “Universal” Doomsday-adjusted SSA (UDASSA), since PCI multiplies *any* anthropic prior by the same suppression integral [11, 12]. Numerical shifts under SIA are < 0.2 dex, well below other model uncertainties.

²The $+\ln t$ term counts the growth of available fluctuation sites in an expanding comoving volume; see Appendix A, where $C_{\text{PLC},\text{total}}(t) \sim 3 \ln t$. For numerical clarity we quote $\log_{10} P_{\text{BB}} = \ln P_{\text{BB}} / \ln 10$.

³One *util* is a dimensionless utility point, scaled so $\$1 \equiv 1$ util for consistency with monetary payoffs.

5 Comparative Framework

Filter	Paradox Scope	Suppresses Infinities?	Mechanism Type	Epistemic vs Physical	$P_{\text{absurd}} \rightarrow 0?$
Counterbalanced Infinity	Global	Yes	Epistemic filter	Mixed	Yes
Anthropic cut-offs	Partial	Model-dep.	Post-selection	Mixed	Possibly
Algorithmic Ockham	Local	Indirect	Prior weight	Epistemic	Indirect

Table 2: Conceptual contrasts among inference filters. Only PCI enforces a vanishing-weight limit regardless of slicing.

6 Objections and Rebuttals

Ad hoc. Appendix E shows that violating Eq. (PCI Limit) yields a divergent weight of incoherent observers, contradicting Bayesian coherence; PCI is therefore *forced*, not ad hoc.

Liouville concern. PCI re-weights credences but leaves phase-space volumes unchanged, so Liouville’s theorem remains intact.

Unfalsifiable. The vacuum-decay bound provides a concrete observational hook; a single confirmed violation would refute PCI.

Measure objection. PCI multiplies *any* global measure by a suppression integral that drives incoherent branches to zero while preserving relative weights elsewhere.

PCI therefore functions as an epistemic criterion: models that violate it may exist mathematically but cannot underwrite coherent empirical inference.

7 Conclusion

PCI offers an information-theoretic counterweight to infinity-driven paradoxes without privileging any time coordinate. Next steps include: (i) Kolmogorov-complexity (K) simulations across the $\Gamma_{\text{BB}}(N)$ landscape; (ii) integration into AI-safety decision frameworks; (iii) comparison with swampland bounds on metastable vacua.

A Landauer–Volume Lemma

For a fluctuation assembling N bits, $\Delta S \geq N k_B \ln 2$. A comoving light-cone encloses $V(t) \propto t^3$, so $C_{\text{PLC},\text{total}}(t) = 3 \ln t$ for flat FRW (Lloyd [9]). Indeed, integrating the instantaneous channel capacity $C_{\text{PLC},\text{rate}}(t') \propto 3/t'$ from 0 to t gives $\int_0^t (3/t') dt' = 3 \ln t$. Once $N > C_{\text{PLC},\text{total}}$, any history spawning such a brain pays an algorithmic-depth penalty $f(t) \geq \ln t$, ensuring $\int_0^\infty \Gamma_{\text{abs}} dt < \infty$.

Robustness to capacity growth. Covariant entropy bounds in $3+1$ -d FRW scale as $C_{\text{PLC},\text{total}}(t) \propto t^p$ with $p \in \{1, 2\}$ for Bousso’s causal-diamond bound and $p = 3$ for comoving-volume scaling [10]. For any polynomial growth, $\int_0^\infty t^{-\beta} dt$ converges iff $\beta > p$, and Landauer yields $\beta \gg 3$ in realistic cases, so the PCI Limit is preserved.

B Slicing Invariance

Let t and η be monotonic with $dt = J(\eta) d\eta$. If $\lim_{\eta \rightarrow \infty} (J\eta/t) = \kappa < \infty$ —true for ever-expanding FRW slicings—then $P_{\text{absurd}}\eta = \kappa[P_{\text{absurd}}t]$; PCI is preserved. Phantom Big-Rip or ekpyrotic bounce models violate the limit; PCI applies only to trajectories with unbounded proper time.

C Algorithmic-Complexity Prior

Assign $P(O) \propto \exp[-\alpha K(O)]$ with $\alpha = \ln 2$ (each extra bit halves prior weight) [13]. A 1×10^{14} -bit brain receives weight $e^{-1 \times 10^{14}}$ versus e^{-10} for a 10-bit fluctuation. If $K(O)$ ever exceeds the past-light-cone capacity, $P(O) \rightarrow 0$ as $t \rightarrow \infty$, expressing the normalcy prior underpinning PCI.

D Decision-Theory Details

Without PCI: $\ln[(1 - P_{\text{BB}})/P_{\text{BB}}] \approx 9.21$. With PCI: $P_{\text{BB}} \sim 1 \times 10^{-300} \Rightarrow \ln[(1 - P_{\text{BB}})/P_{\text{BB}}] \approx 690$.

E Conditions for the PCI Limit

We now derive the slice-invariant “PCI Limit” (PCI Limit).

Instantaneous fraction. Throughout this appendix we define

$$P_{\text{absurd}}(t) = \frac{\Gamma_{\text{abs}}(t)}{\Gamma_{\text{tot}}(t)},$$

i.e. the *rate* fraction of incoherent observers at proper time t . For late-time FRW backgrounds $\Gamma_{\text{tot}}(t) \approx \text{const}$, we obtain $P_{\text{absurd}}(t) \rightarrow 0$ whenever $\int_T^\infty \Gamma_{\text{abs}}(t) dt < \infty$.

Assume $\Gamma_{\text{abs}} = Ae^{-\beta f(t)}$ with $f(t) \geq \ln t$ for $t > T$. Then

$$\int_T^\infty \Gamma_{\text{abs}} dt \leq A \int_T^\infty t^{-\beta} dt < \infty \quad (\beta > 1 \text{ suffices; empirically } \beta \gg 10^{11}).$$

Because $\Gamma_{\text{tot}}(t)$ is asymptotically constant (or, more generally, decays no faster than $1/t$), convergence of $\int \Gamma_{\text{abs}} dt$ implies $\Gamma_{\text{abs}}(t) = o(1/t)$ and hence $P_{\text{absurd}}(t)t \rightarrow 0$ as $t \rightarrow \infty$, establishing the PCI Limit.⁴ \square

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⁴This conclusion presumes a future in which $\Gamma_{\text{tot}}(t)$ does not dilute more quickly than $1/t$, as in de Sitter-like or slowly evolving FRW cosmologies; an extreme Big-Crunch dilution would place PCI outside its intended domain.