Quantum Computing and Shor's Algorithm

Applying Quantum Mechanics to Computation

A Brief Introduction

- What is a **Quantum Computer**?

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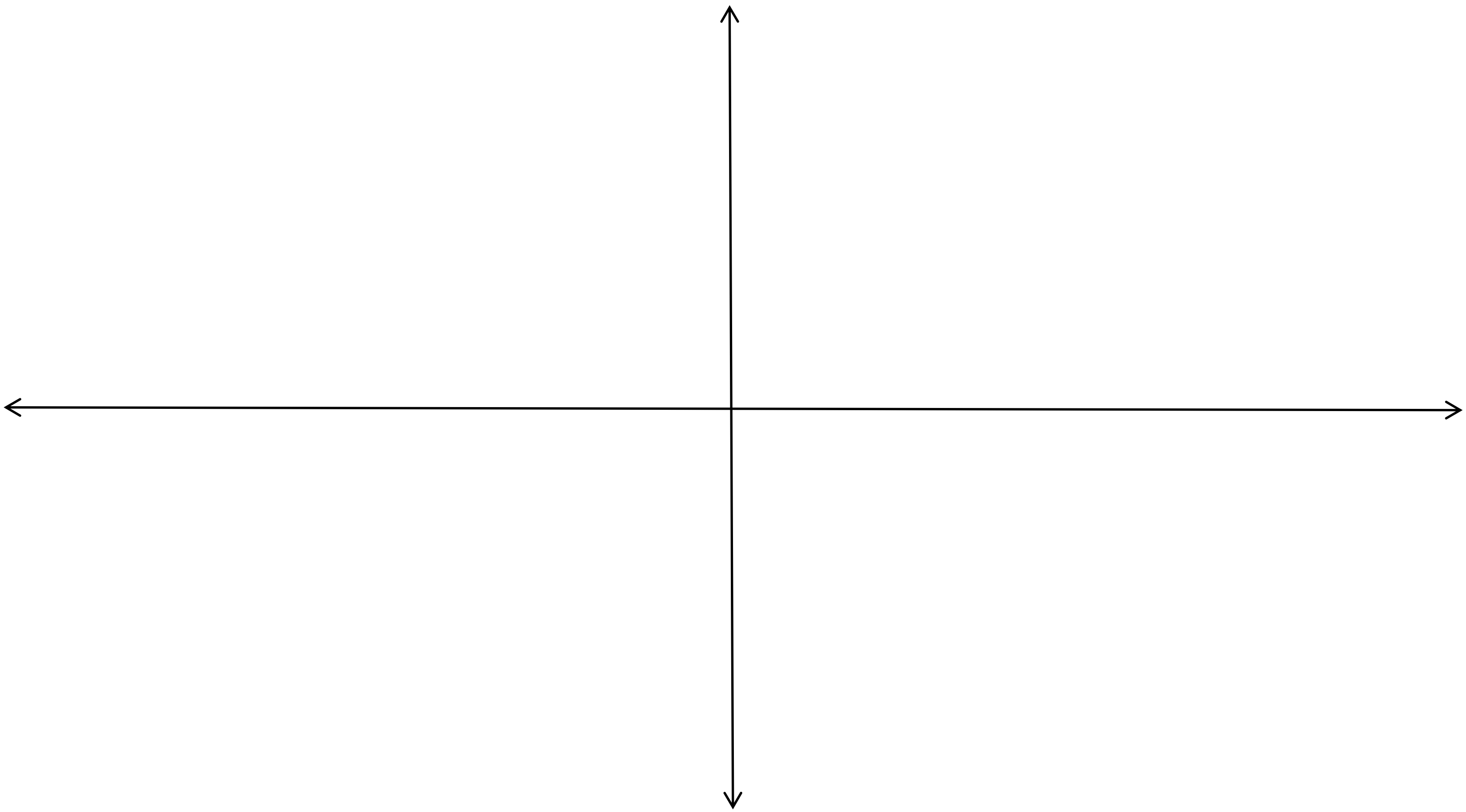
- We don't know where an electron is (until we observe it)
- **Amplitudes** – square to find probabilities

Quantum States and Dirac Notation



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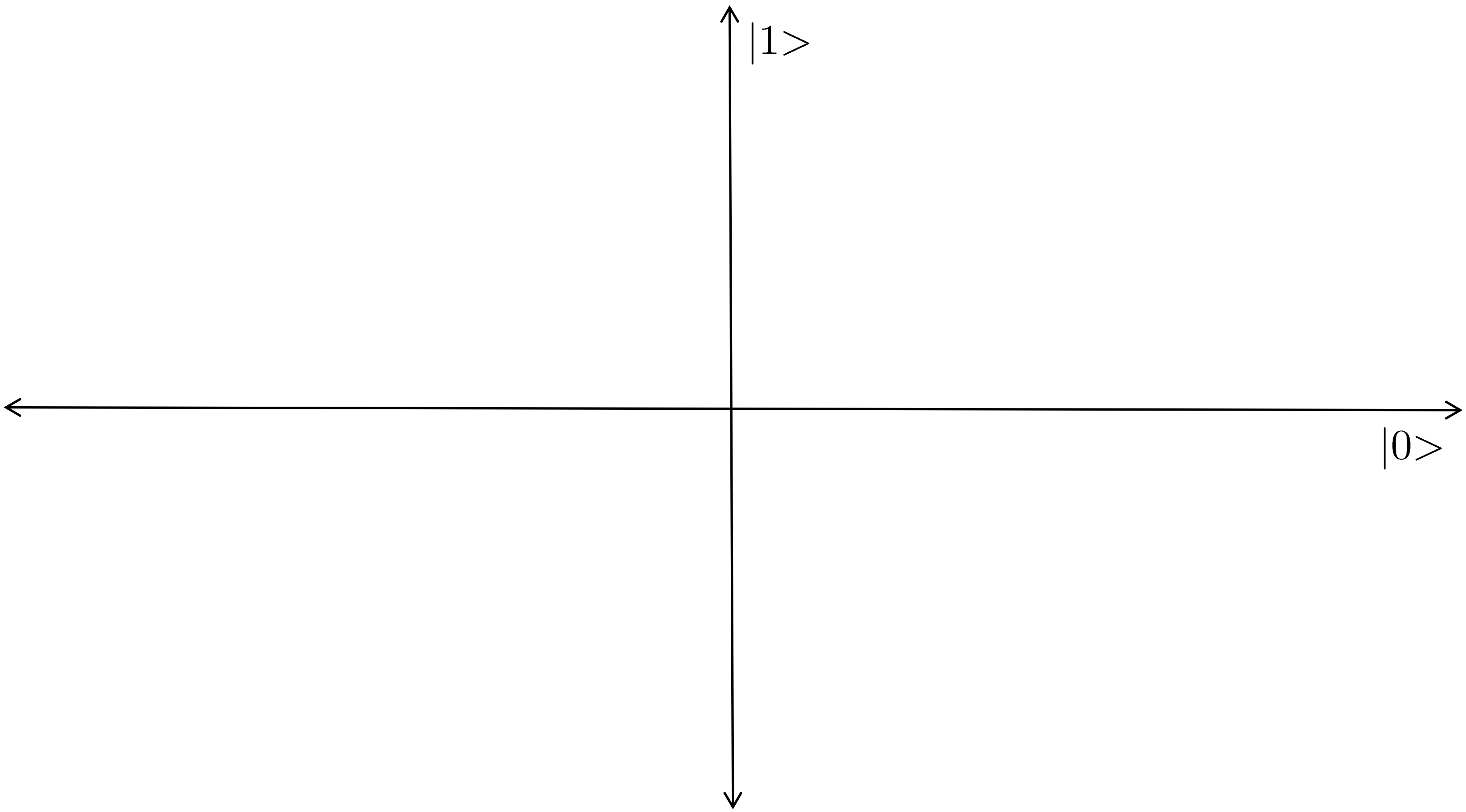

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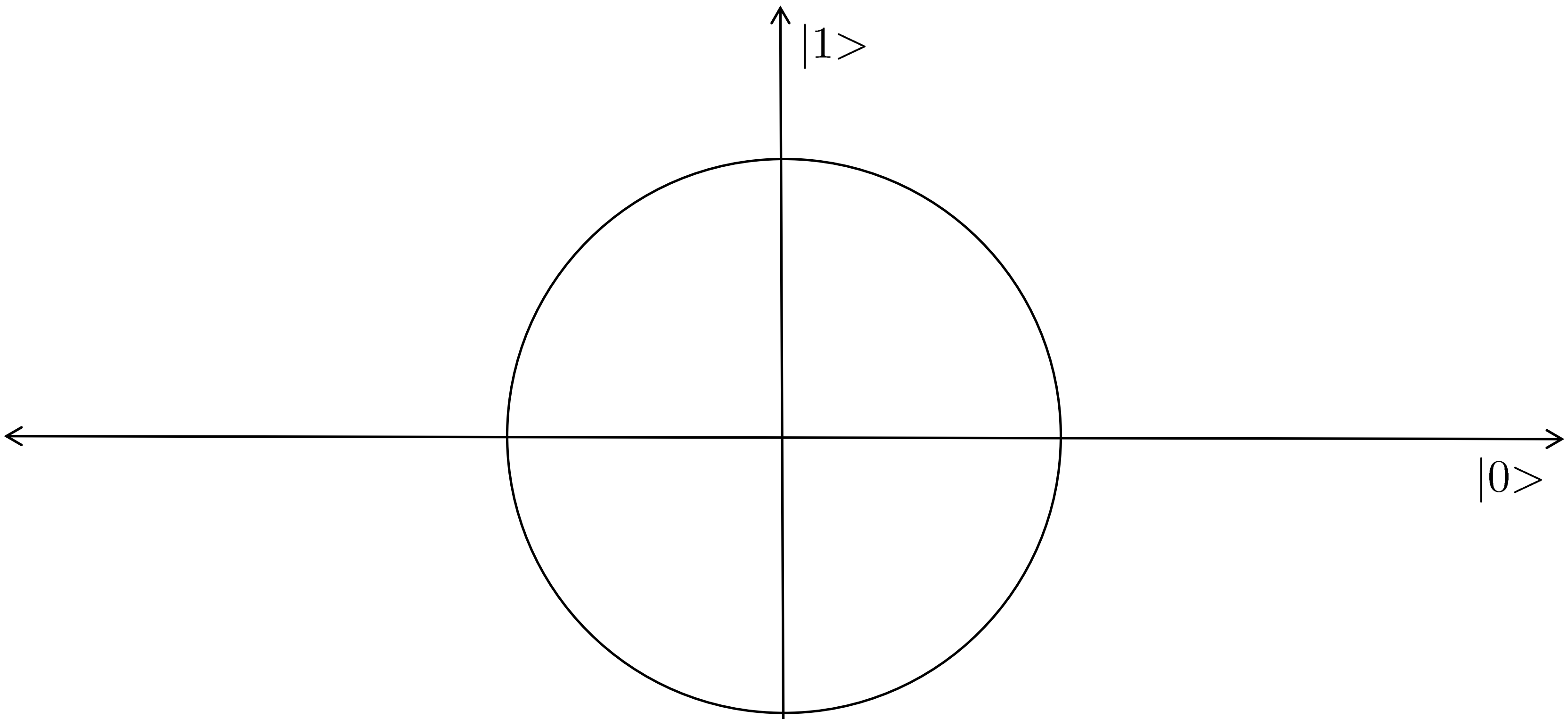

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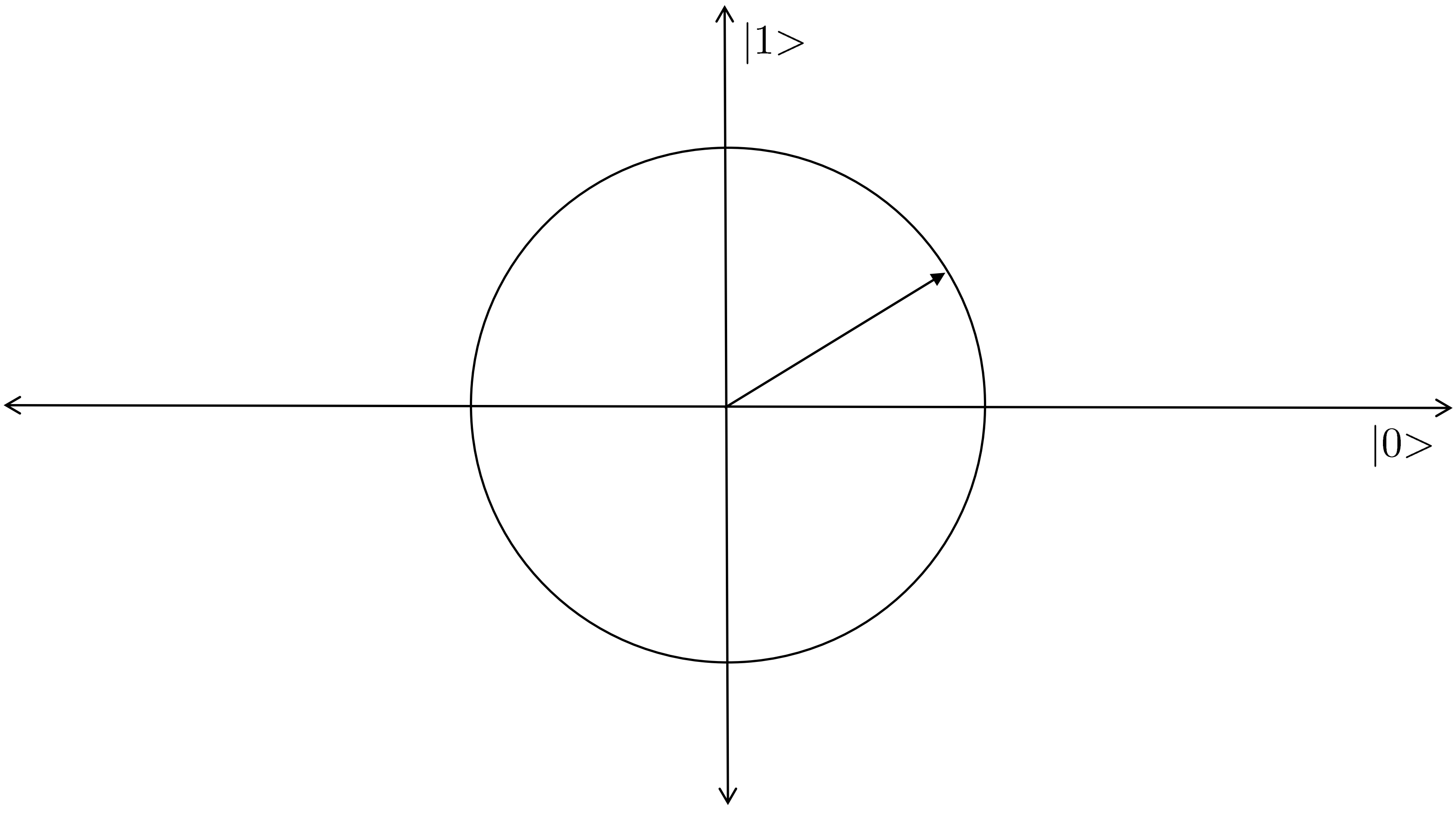

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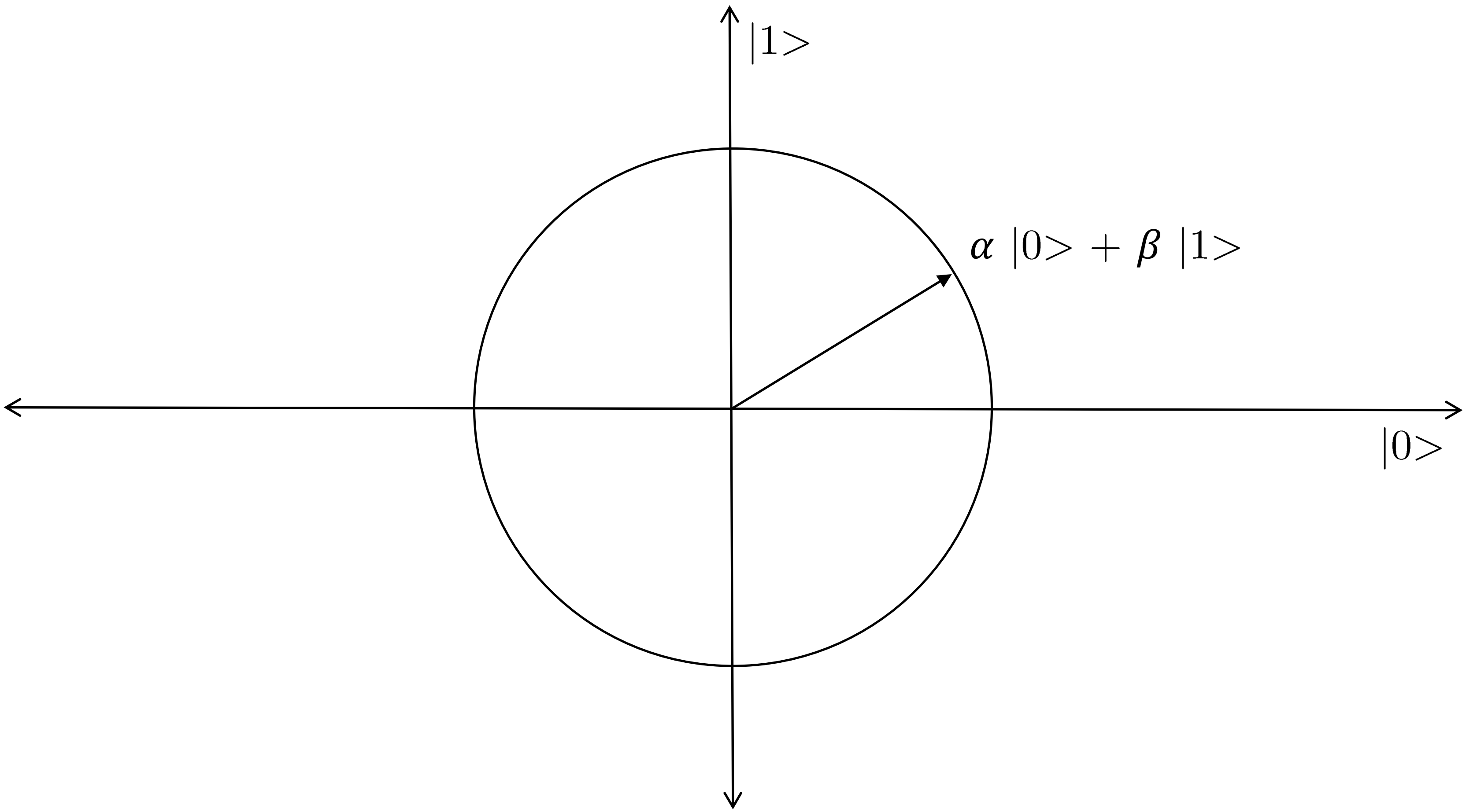

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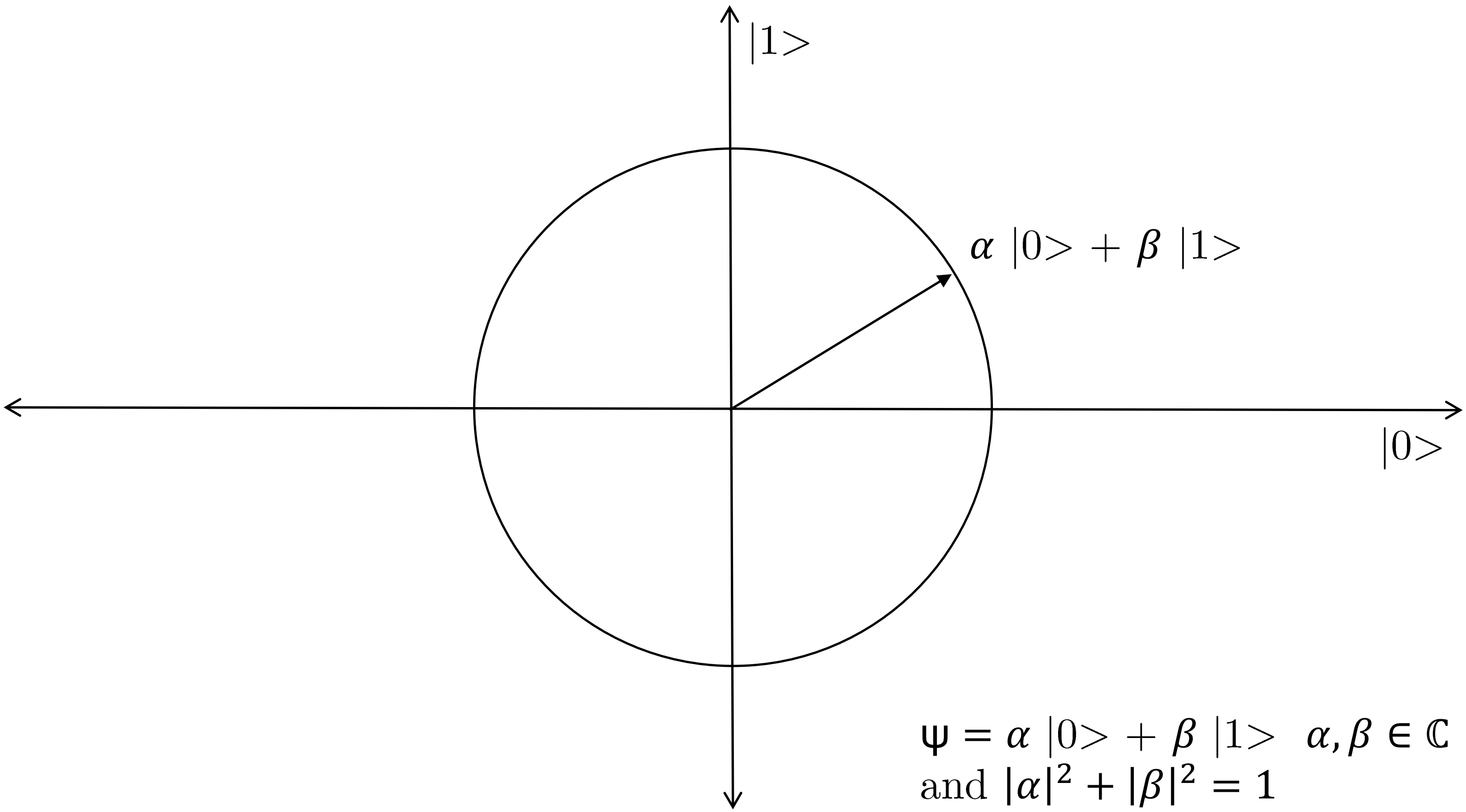

$\psi = \alpha\ |0> + \beta\ |1>\quad \alpha, \beta \in \mathbb{C}$
and $|\alpha|^2 + |\beta|^2 = 1$

Intrinsic Randomness

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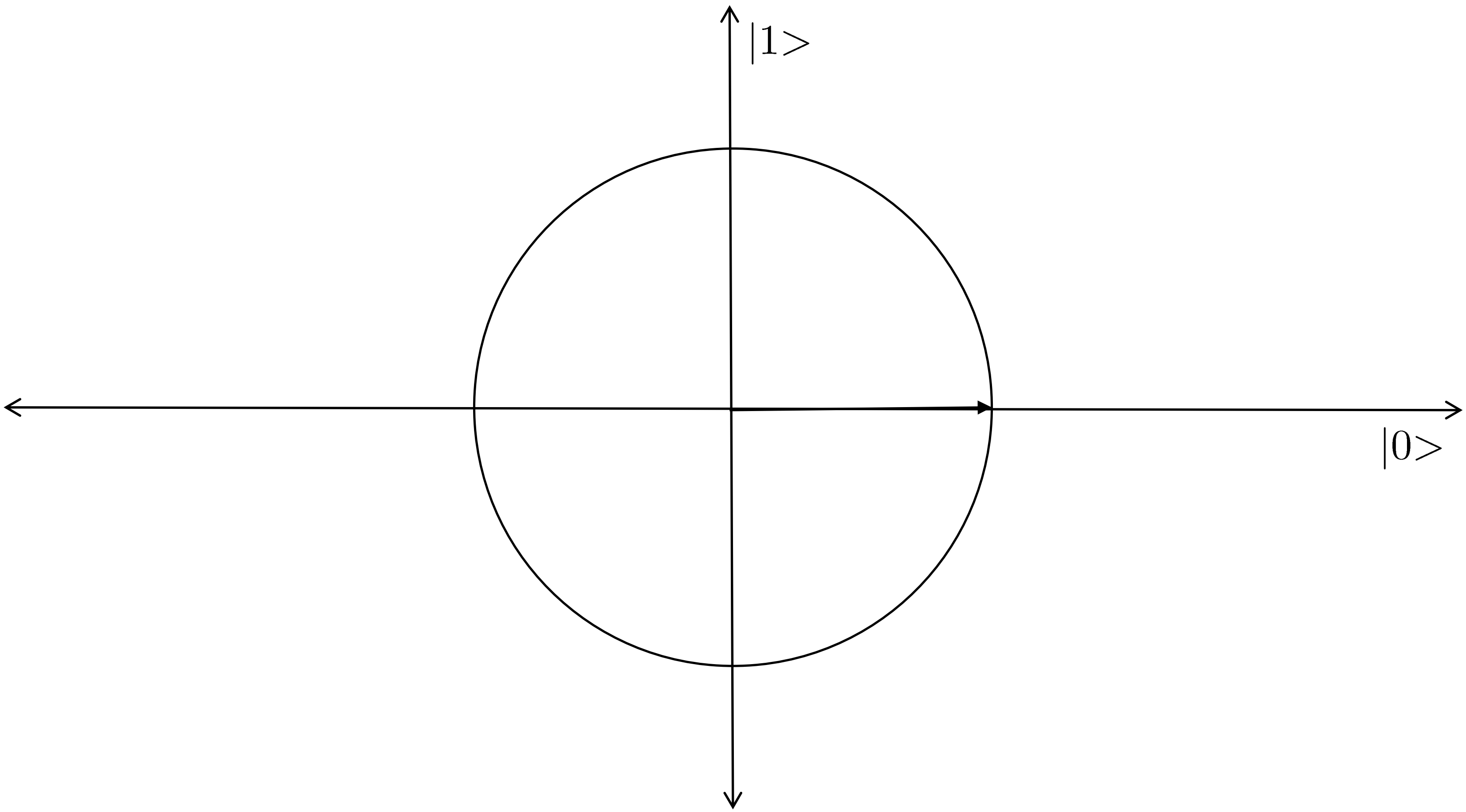

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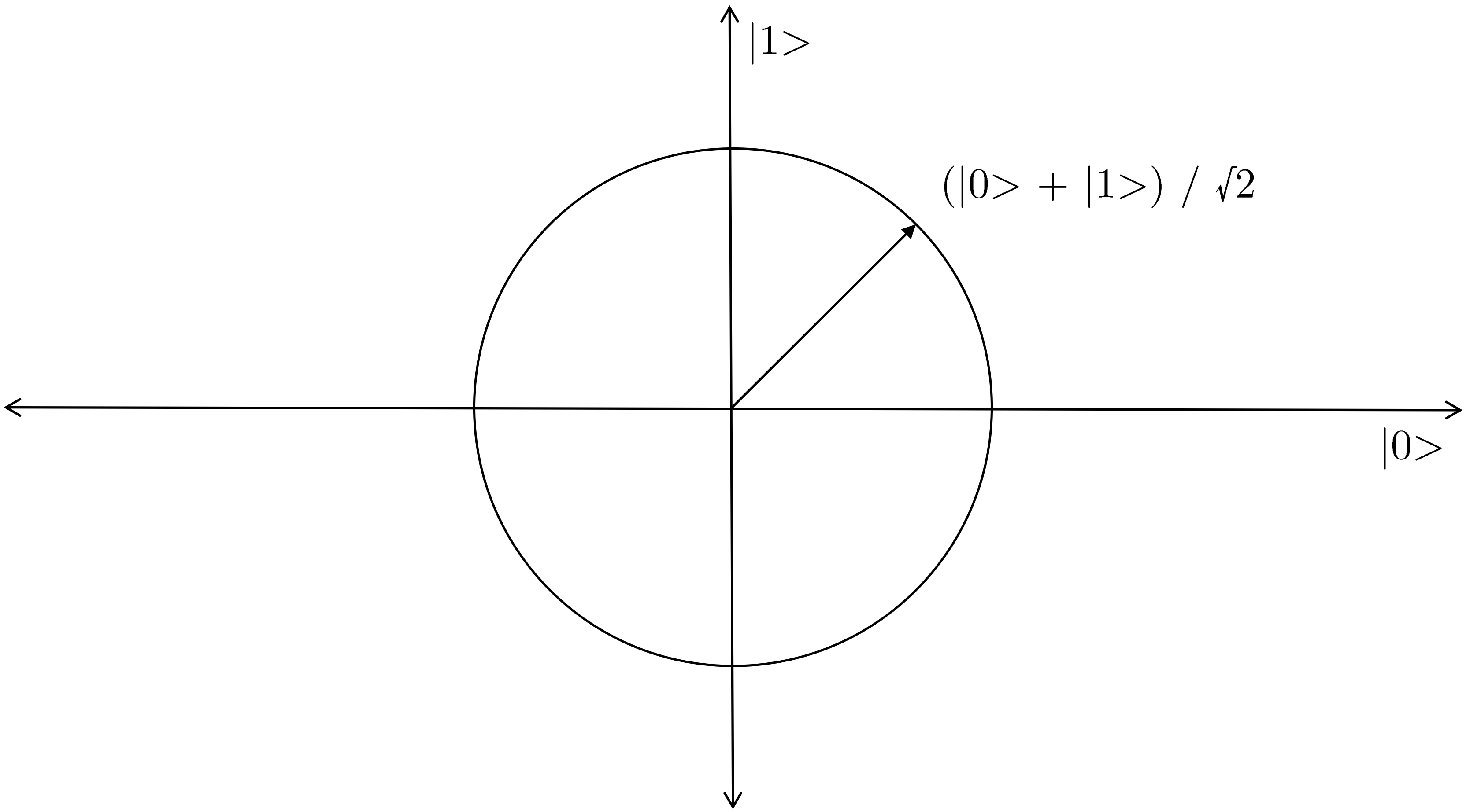

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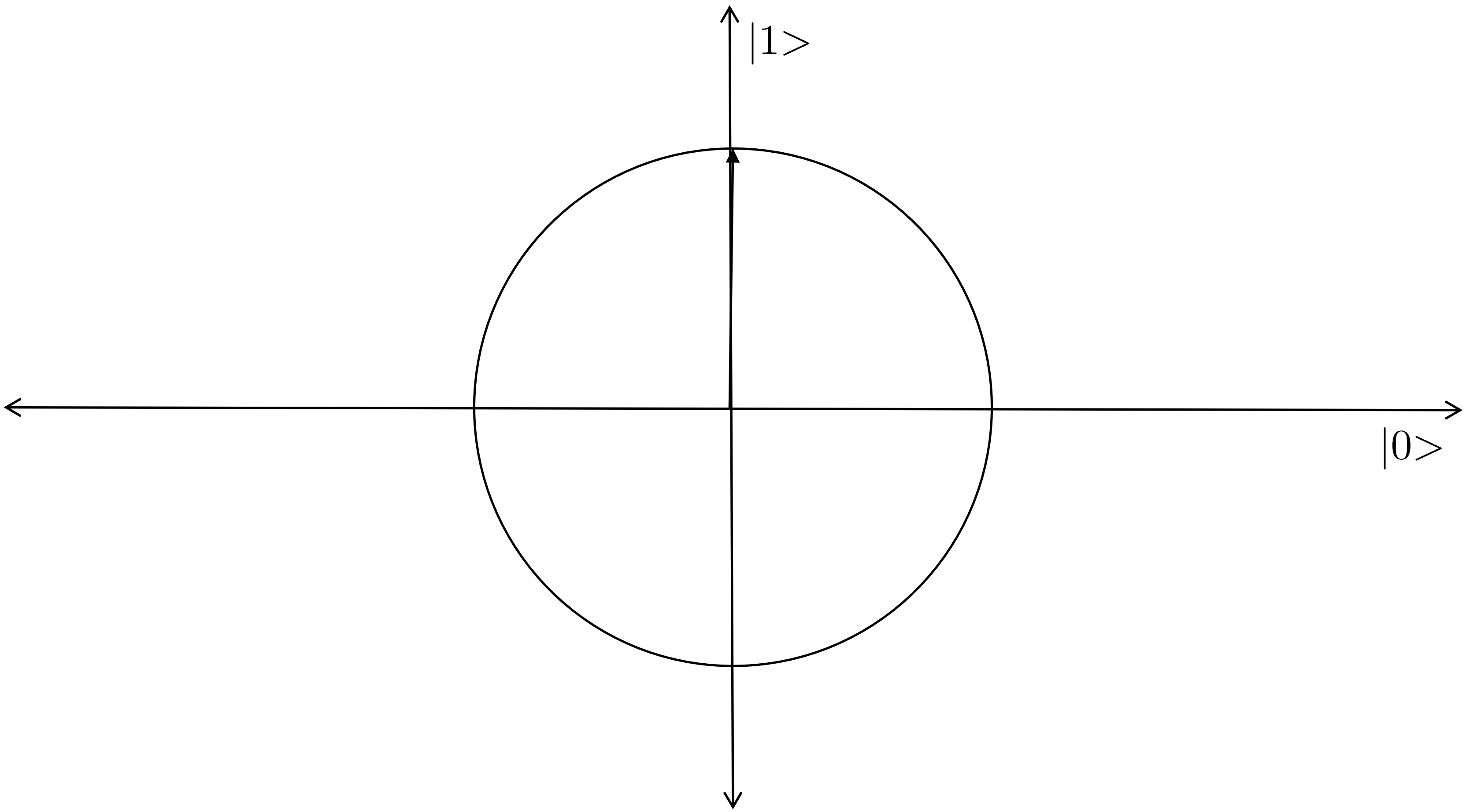

Interference

Interference

- $|0> \rightarrow (|0> + |1>) \,/\, \sqrt{2}$
- $|1> \rightarrow (-\,|0> + |1>) \,/\, \sqrt{2}$

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E.g. Test on $(|0> + |1>) / \sqrt{2}$

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E.g. Test on $(|0> + |1>) / \surd 2$

$((|0> + |1> - |0> + |1>) / \surd 2) / \surd 2$

$= ((2|1>) / \surd 2) / \surd 2 = |1>$ as required

Interference

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$$+|0>$$

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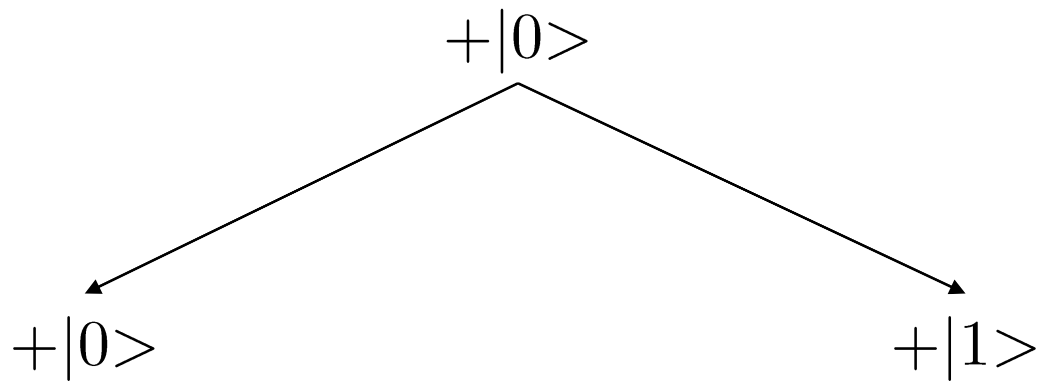

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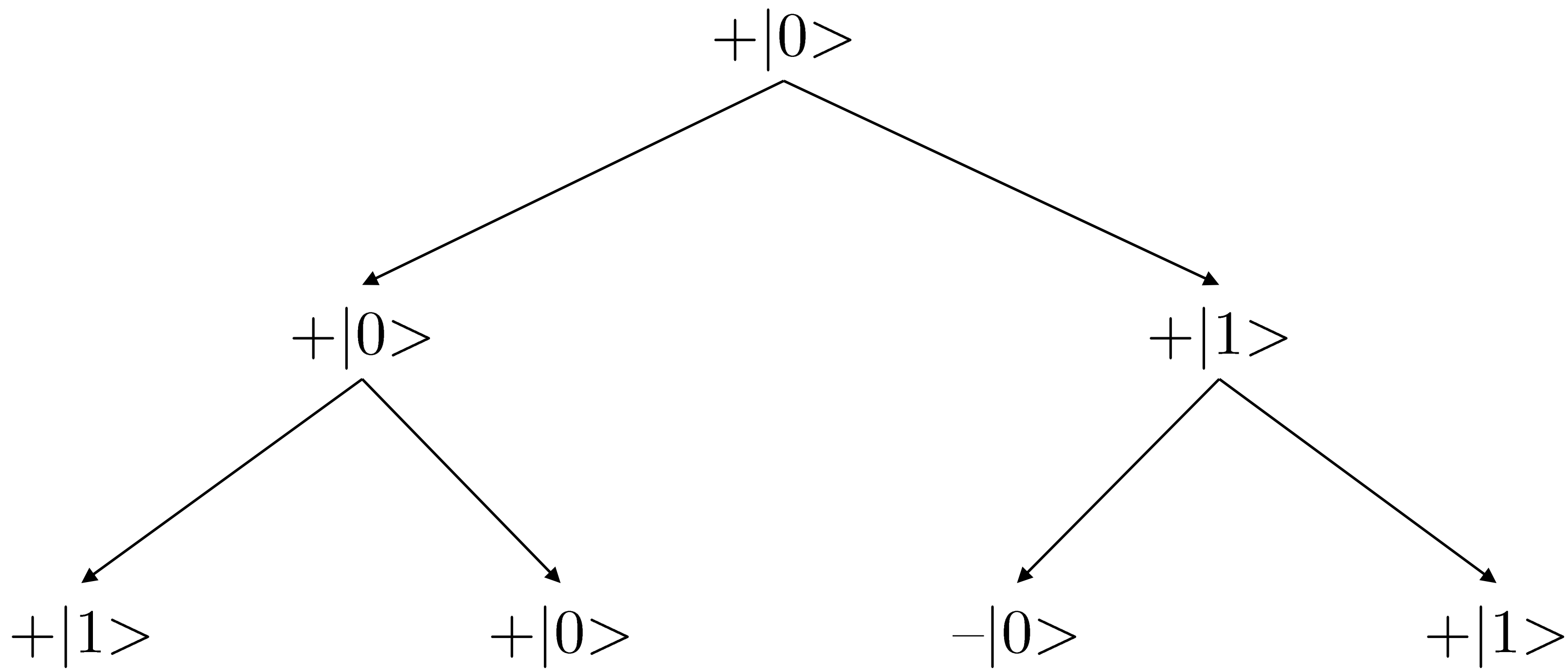

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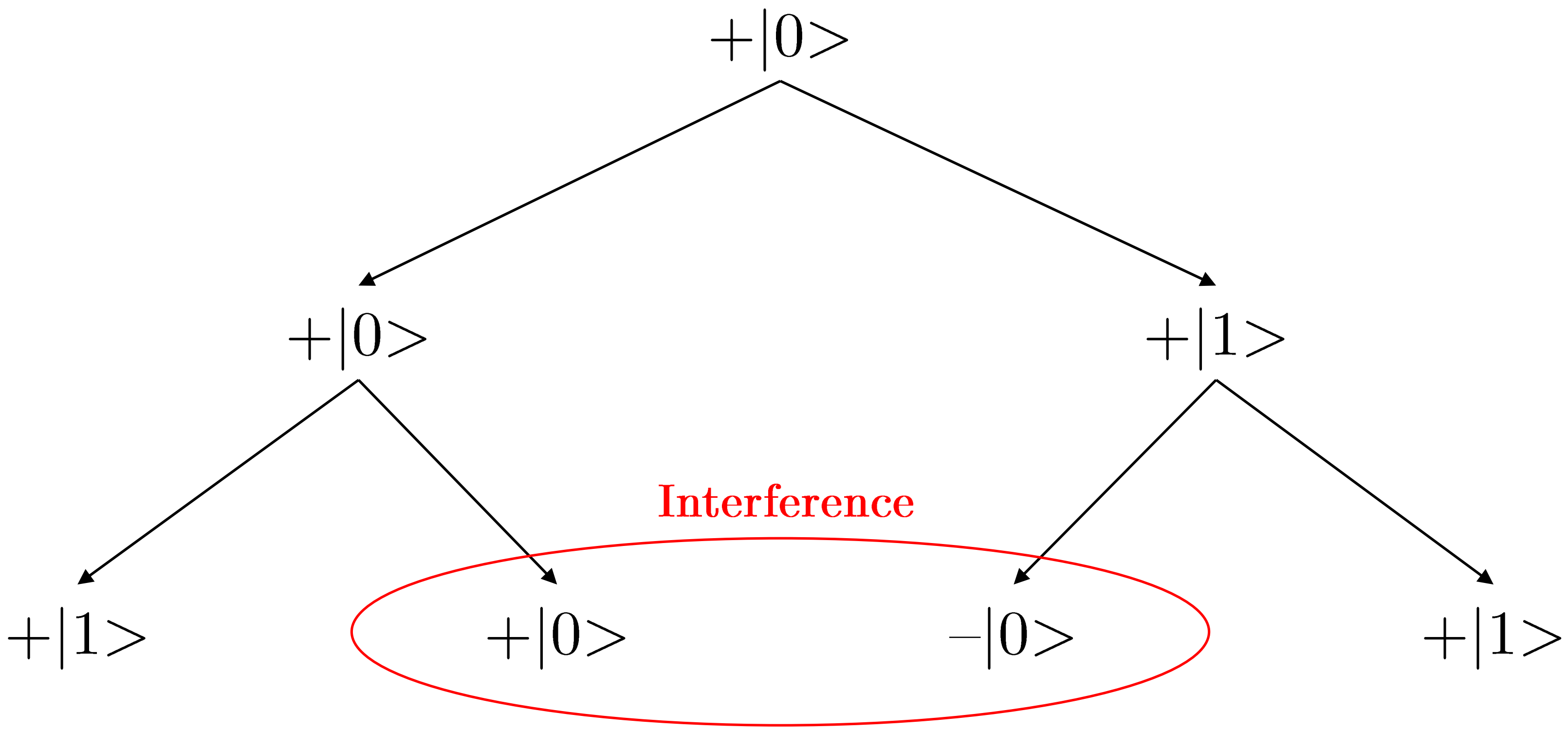

Applying to Computation

Taking 2 bits / qubits:

<table>
<tr><th>CLASSICAL COMPUTER
(Bits)</th><th>QUANTUM COMPUTER
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<tr><td>00</td><td>|0, 0></td></tr>
<tr><td>01</td><td>|s> = |0, 1 – 1, 0> / √2</td></tr>
<tr><td>10</td><td>|T_0> = |0, 1 + 1, 0> / √2</td></tr>
<tr><td>11</td><td>|1, 1></td></tr>
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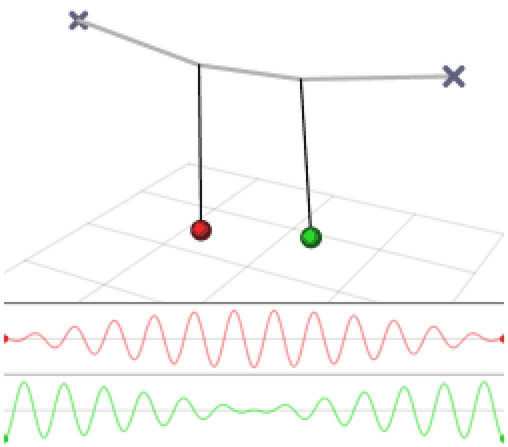
Entangled States

Explaining the Entangled States



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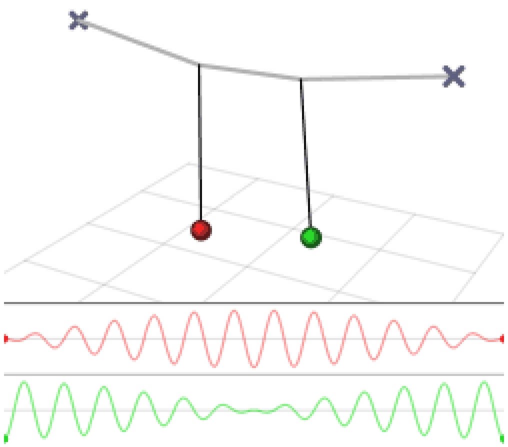

$|0,1>$

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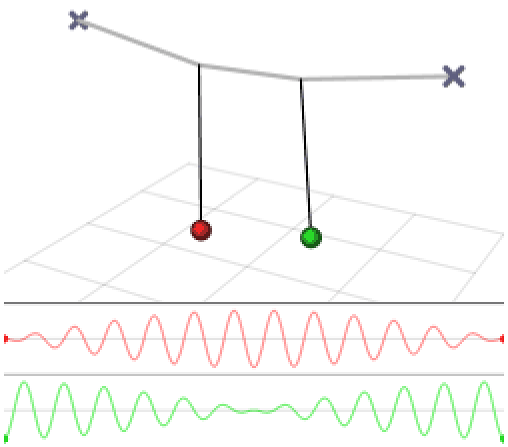

$|0,1>$
$|1,0>$

$$\frac{1}{\sqrt{2}}|0,1> - \frac{1}{\sqrt{2}}|1,0>$$ (Singlet State)

(Qubits are not independent of each other!)

$$\frac{1}{\sqrt{2}}|0,1> + \frac{1}{\sqrt{2}}|1,0>$$ (T_0 State)

Superposition of States

α $|0,\ 0>$

β $|\text{T\_0}> = |0,\ 1 - 1,\ 0> /\ \sqrt{2}$

γ $|\text{s}> = |0,\ 1 + 1,\ 0> /\ \sqrt{2}$

δ $|1,\ 1>$

Superposition of States

$\alpha\ |0,\ 0>$

$\beta\ |T\_0> = |0,\ 1 - 1,\ 0>\ /\ \sqrt{2}$

$\gamma\ |s> = |0,\ 1 + 1,\ 0>\ /\ \sqrt{2}$

$\delta\ |1,\ 1>$

No. of bits	No. of values held by classical bits	No. of values held by qubits
1	1	2
2	2	4
3	3	8
n	n	2^n

Problems

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- Classical Algorithms

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- Superpositions

The Prime Factorisation Problem

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$O(e^{(\log n \log\log n)^{\frac{1}{2}}})$ problem using classical algorithms

The Period Finding Problem

2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, ...

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2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, ...

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2, 4, 8, 1, 2, 4, 8, 1, 2, 4, ...

2, 4, 8, 1, 16, 11, 2, 4, 8, 16, ...

Euler’s Totient Function

x mod N, x^2 mod N, x^3 mod N, x^4 mod N, ...

Period evenly divides (p-1)(q-1)

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x=2, N=21; p=3, q=7

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Hence p=3, q=5

(p-1)(q-1) = 8, and 8|4 as required

x=2, N=21; p=3, q=7

(p-1)(q-1) = 12 and 12|6 as required

Shor's Algorithm: The Classical Part

x^r mod N (r may be very large)

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Let N=17, x=3, r=14

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$$x^r = 3^{14} = 3^{2^3+2^2+2^1} = 3^{2^3} \cdot 3^{2^2} \cdot 3^{2^1} = ((3^2)^2)^2 \cdot (3^2)^2 \cdot 3^2$$

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6561 mod 17 * 81 mod 17 * 9 mod 17 = 16 * 13 * 9

and (16 * 13 * 9) mod 17 = 2

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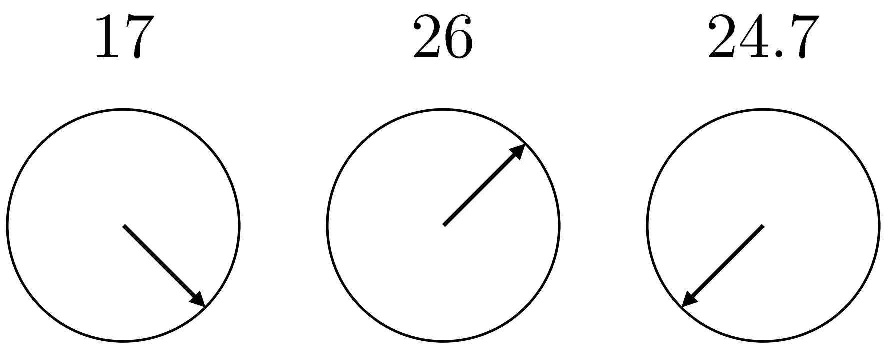

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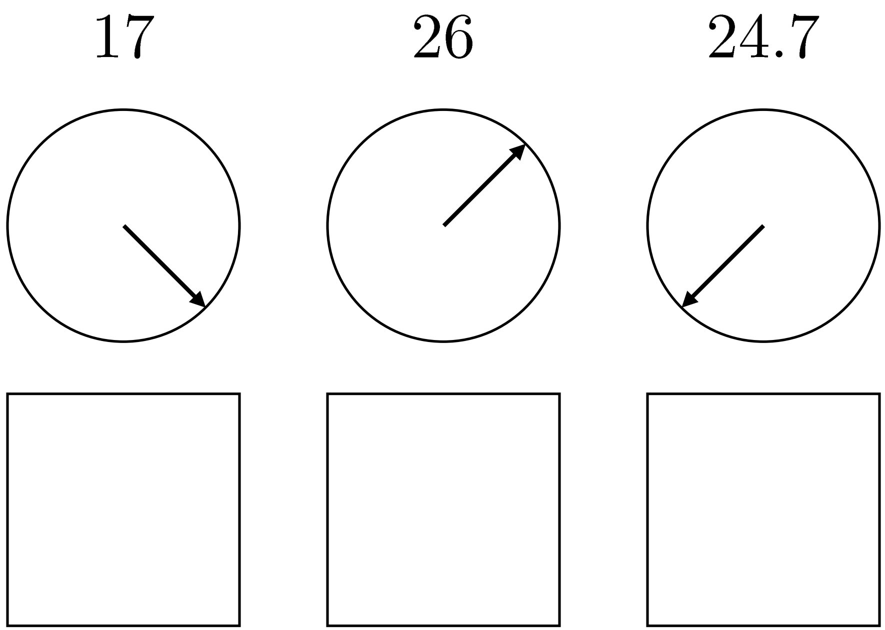

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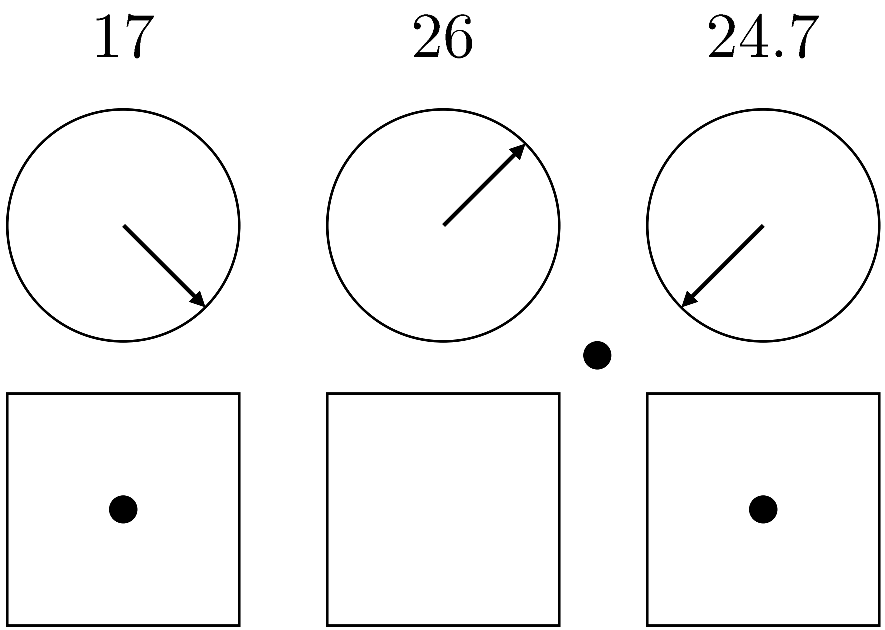

The Quantum Fourier Transform

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$$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x_n e^{i2\pi k \frac{n}{N}}$$

To find the energy at a particular frequency, spin your signal around a circle at that frequency, and average a bunch of points along that path.

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http://betterexplained.com/articles/an-interactive-guide-to-the-fourier-transform/

Back to Interference