UK Bebras 2015

Computational Thinking Challenge

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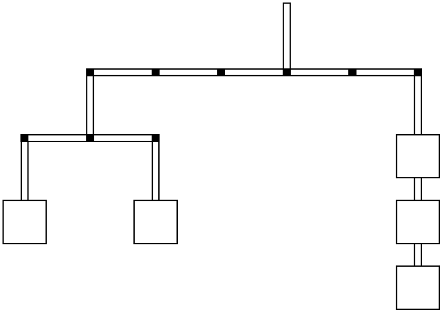
Mobiles

Recursive Structures



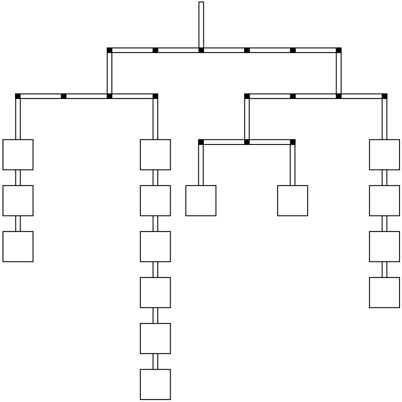
(-3 (-1 1) (1 1)) (2 3)

A mobile is either:

- A single square, or
- A Stick with one or more mobiles attached to it.

A possible mobile:

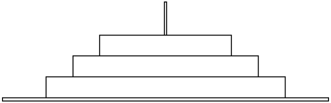
(-3 (-1 -4) (2 (-1 1) (1 1))) (2 (-1 6) (2 3)))



Using recursion to find minimum number of steps for Tower of Hanoi:

$$T_0 = 0,\ T_1 = 1,\ T_2 = 3$$

$$T_n = T_{n-1} + 1 + T_{n-1}$$



$$T_3 = 2 \times 3 + 1 = 7$$

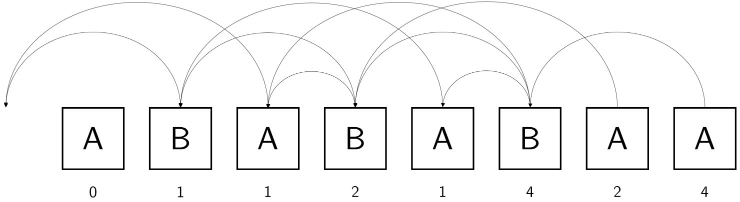
$$T_4 = 2 \times 7 + 1 = 15$$

We can prove by induction that:

$$T_n = 2^n - 1$$

$$\implies T_8 = 2^8 - 1 = 255 \text{ steps required.}$$

The words AB, BAB, ABA, AA, and B have meaning. How many different meanings are possible from the sequence ABABABAA?

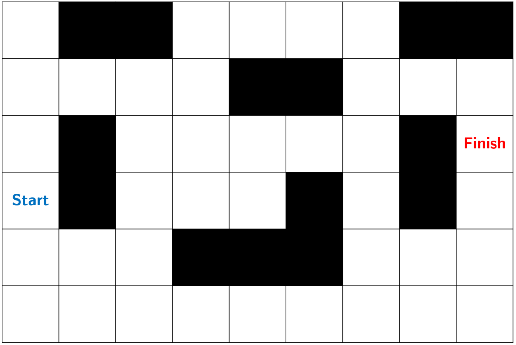


How can we compress data without introducing ambiguity?

Building a Chip

Breadth First Search

How many routes from start to finish of shortest length?



Building a Chip

Breadth First Search

3			6	7	8	9		
2	3	4	5			10	11	12
1		5	6	7	8	9		**Finish**
Start		4	5	6		10		12
1	2	3				9	10	11
2	3	4	5	6	7	8	9	10

This is the BFS algorithm. Has applications like:

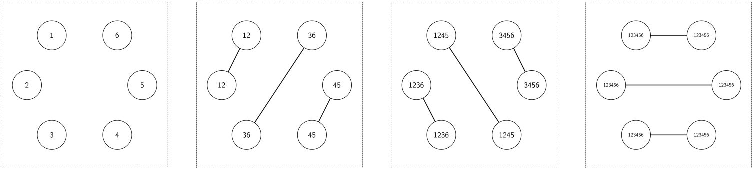
- Web crawling
- Social networks
- Diameter of configuration space of $n \times n \times n$ Rubik's cube

Building a Chip

Breadth First Search

1			1	1	1	1		
1	1	1	1			6	6	6
1		2	4	5	5	5		**15**
Start		1	1	1		8		9
1	1	1				3	6	9
1	2	3	3	3	3	3	3	3

We have 6 spies, each with one piece of information. Can only communicate with one spy at a time - minimum of three rounds required:



How many are required for n spies?

$$f(1) = 0, F(1) = 0$$
$$f(2) = 1, F(2) = 1$$
$$f(3) = 3, F(3) = 3$$
$$f(4) = 4, F(4) = 2$$
$$f(5) = 6, F(5) = 4$$
$$f(6) = 8, F(6) = 3$$

$f(n) = 2n - 4$ for $n \geq 4$ so $F(n) = \lceil \frac{2n-4}{\lfloor n/2 \rfloor} \rceil$ (need to check!)

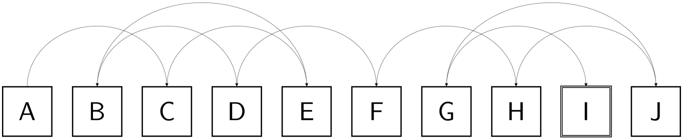
Kangaroo

NP-Completeness

A	B	C	D	E	F	G	H	I	J

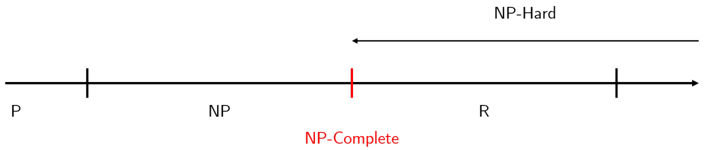
- Start at A.
- Can jump forward 2 or back 3.

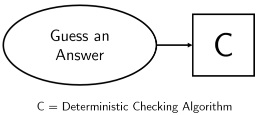
Which letter do you end on after visiting every box?



This is a Hamiltonian path problem, which is ‘NP-complete’.

Kangaroo
NP-Completeness





P = {Set of all problems solvable in polynomial time}

NP = {Set of all problems solvable in polynomial time by a non-deterministic algorithm / verifiable in polynomial time}

R = {Set of all problems solvable in finite time}

NP-Hard = {Set of all problems at least as hard as the hardest problems in NP}

NP-Complete = {Set of all problems that are both in NP and NP-Hard}

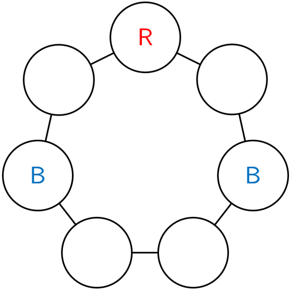
Reaching the Target

Binary Search

- 0.5m long target can be at any value within 10m range.
- Choose a value between 0 and 10m - you will be told whether the value is before, beyond, or on the target.

How many values need to be tested to be sure to hit the target?

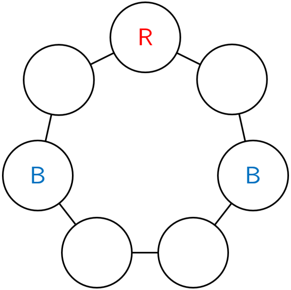
- We should use the binary search algorithm. Each time our search space is divided in half so we have an accuracy of $\frac{1}{2^n} \times 10$.
- Let $\frac{1}{2^n} \times 10 = 0.5 \implies 2^n = 20 \implies n = \log_2 20 = 4.32\ldots$
- So 5 values required. (Binary search is said to have time complexity $O(\log n)$)



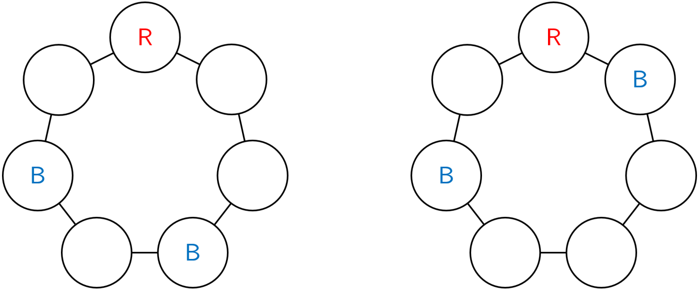
- Police to move first. One policeman moves one circle to left or right.
- Pirate moves to two circles to left or right.
- Repeat until pirate is forced to move to a circle occupied by a policeman.

If the pirate plays perfectly, how many moves does it take to capture them?

Thm: It is not possible for the police to win.
Proof: (by cont.) Assume it is possible to catch the pirate. Hence the police can force this state before a pirate's move:

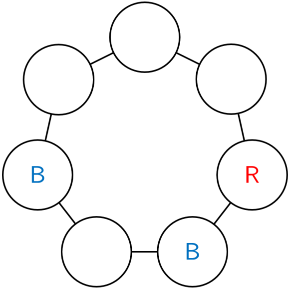


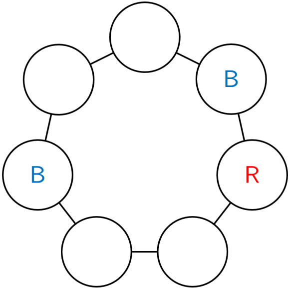
By symmetry, there are only two possible states we can be in, one move before the final state:



We must be able to force the pirate into at least one of these states.

Again, there are only two possible states we can be in, two moves before the final state:





The pirate can always move to not enter the second-to-last state. Contradiction: the final state cannot be forced. Hence the police cannot win. □

Proving that the 'halting problem' is unsolvable by a computer: