



# WALKABLE ROBOT DESIGN AND ANALYSIS

GROUP 42

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## Introduction

### Scope

Engineers must understand how to solve many fundamental engineering problems to apply and solve future complex problems of the world. Some of these fundamentals are position, velocity, and acceleration analyses through balance and linkage problems. The project of making a walking robot applies each of these fundamentals and offers a hands-on learning opportunity. Understanding these fundamentals allows engineers to solve future and complex robotics and mechanical problems.

### Aim

The aim of this project is to design and create a stable and efficient walkable robot with a functional geared motor system and 4 bar linkages, that can walk straight for 2 meters in less than 30 seconds.

## Design Process

### Design Parameters

To design a walkable robot, a certain set of criteria is needed. For the linkages, it must include two sets 4bar linkages as the front legs and two sets of slider-crankes as the back legs. All linkages must also be powered by the same input driving link. When designing the linkages, certain lengths must be calculated to ensure that the robot can walk straight for 2 meters in under 30 seconds, while optimising the transmission angle to be as close to 90 degrees. For the chassis, it must be able to house the gearbox and motor system and have minimal interference with the linkage movement. The chassis must also be able to withstand the movements of the linkages.

### Linkage Design

The design and manufacturing of the walkable robot challenge followed the design considerations outlined in the previous section. The lengths of the linkages were chosen to ensure that the 4bar linkage follows Grashof's criterion and maintains the inequality  $S + L \leq P + Q$  that allows for full rotations of the input link (Chang et al., 2005). Additionally, the 4bar linkage aimed to optimise the transmission efficiency of the robot and, using Norton Linkage Software to model the linear velocities of the front and back legs.

### Designing the four-bar linkage:

The 4bar linkage was configured as a crank-rocker type (GCRR) with the input link as the shortest connection (25mm) and the ground link as the longest connection (90mm). The remaining two links were designed to have the coupling length greater than the sum of the shortest and longest links to satisfy the Grashof inequality criteria. The specific lengths of the coupler and output links were simulated using Norton Linkages and compared to lengths calculated using an online optimisation formula:

$$a = \frac{\left( \sin\left(\frac{\psi}{2} * \frac{\pi}{180}\right) * \cos\left(\left(\frac{\phi}{2} + \beta\right) * \frac{\pi}{180}\right) \right)}{\sin\left(\frac{\phi - \psi}{2} * \frac{\pi}{180}\right)} * d \quad \text{eqn (1.1)}$$

$$b = \frac{\left( \sin\left(\psi * \frac{\pi}{360}\right) * \sin\left(\left(\frac{\phi}{2} + \beta\right) * \frac{\pi}{180}\right) \right)}{\cos\left((\phi - \psi) * \frac{\pi}{360}\right)} * d \quad \text{eqn (1.2)}$$

$$c = \left(\frac{a}{d} + \frac{b}{d}\right)^2 + 1 - 2\left(\frac{a}{d} - \frac{b}{d}\right) * \cos\beta * d \quad \text{eqn (1.3)}$$

(*The Classical Transmission Angle Problem, 2023*)

The design considerations for the 4bar linkage required an optimal transmission efficiency, which is the angle between the coupler link and the adjacent output link. The ‘Classical Transmission Angle Problem’ states that “the crank-rocker proportions of a four-bar mechanism [must operate] such that the maximum deviation of the transmission angle from 90° is minimum” (*The Classical Transmission Angle Problem, 2023*). This problem relates the position synthesis of GCRR with a given swing angle ( $\psi$ ) and a corresponding crank-rotation ( $\phi$ ). Using these parameters, the lengths of the remaining linkages for the walkable robot were determined using equations 1.1 to 1.3.

Where  $d = 90\text{mm}$ ,  $\phi = 180^\circ$ ,  $\psi = 42.9^\circ$ ,  $\beta = 45^\circ$ .

$$\text{eqn (1.1)} \quad a = 25.00\text{mm}$$

$$\text{eqn (1.2)} \quad b = 63.64\text{mm}$$

$$\text{eqn (1.3)} \quad c = 84.17\text{mm}$$

As a result, the minimum and maximum transmission angles occur when the input link ( $a$ ) becomes collinear with the ground link ( $d$ ) either extended or folded. This is given by the equation:

$$\mu_{min,max} = \cos^{-1} \left( \frac{c^2 + b^2 - a^2 - d^2}{2cb} \pm \frac{ad}{cb} \right) \quad eqn (2.1)$$

(FOUR-BAR MECHANISM, 2023)

Which gives  $\mu_{min} = 49.84^\circ$  and  $\mu_{max} = 101.3^\circ$ .

However, compared to the Norton Linkages software simulation, it was found that approximating the lengths of the linkages, as shown in Figure 1, where  $b = 59mm$  and  $c = 69mm$ , had improved the critical transmission efficiency. Where  $\mu_{max}$  and  $\mu_{min}$  can be recalculated as  $\mu_{max} = 127.74^\circ$  and  $\mu_{min} = 60.44^\circ$ . Mechanically, optimal transmission efficiency occurs when the critical transmission angle  $\mu_{min} > 45^\circ$  (Ahmadi, 2022).

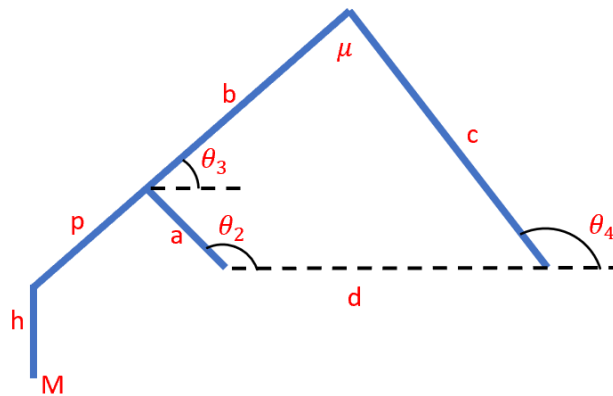


Figure 1 – 4bar linkage schematic

The position of point M was also simulated using Norton Linkage software to determine the optimal lengths of ( $h$ ) and ( $p$ ). This was determined by modelling the time ratio between forward and reverse strokes of the coupler link close to one (Equation 3.1). This would show the time ratio of forward movement compared to the reverse movement of the linkage. In addition, the forward and reverse strokes were then compared to demonstrate a clear line where the front leg would spend equal time on the ground and in the air. This means that while one leg is lowered, the opposite leg is raised for an identical amount of time, which will reduce the amount of time wasted not producing forward motion (Adam, A., 2022).

Time Ratio:  $Q = \frac{\theta_f}{\theta_r} \quad eqn (3.1)$

(Adam, A., 2022)

The forward stroke angle was determined by deriving the stationary points of the velocity of point M in the  $x$  direction. Therefore, when  $h = 50\text{mm}$  and  $p = 71.5\text{mm}$ , this led to the discovery of  $\theta_f = 162.0^\circ$  (forward stroke angle) and  $\theta_r = 152.6^\circ$  (reverse stroke angle). Consequently, the time ratio of the front leg producing forward motion was found to be  $Q = 1.06$ .

### Designing the slider-crank linkage:

The length of the linkages in a slide-crank mechanism is critical to achieving the desired motion and force transmission. The input link ( $a$ ) is shared with the 4bar linkage making the crank length  $25\text{mm}$ . This link determines the angular velocity and acceleration of the slider and affects the force transmission between the input force and the load. Similarly, the connecting rod length ( $d'$ ) affects the stroke of the slider, and the location of the slider ( $c'$ ) affects the angle of the connecting rod relative to the crank.

The connecting rod length was estimated to be  $b' = 199\text{mm}$ , which improved the mechanical advantage due to the distance for the slider to oscillate between minimum and maximum. The elevation of the slider relative to the origin was estimated to be  $c' = -12.5\text{mm}$ . This estimation also increases the displacement of the slider from the crank's centreline, allowing larger steps of the back legs.

The position and velocity of the rear leg at point N was specifically modelled using Norton Linkages to determine synchronous movement with point M of the front legs. This was achieved by graphing the position and velocity in the  $x$  axis for both points M and N (see figures 2.1 and 2.2). As demonstrated, the position and velocity of both points generally follow the same curve suggesting synchronous movement between forward and reverse positions.

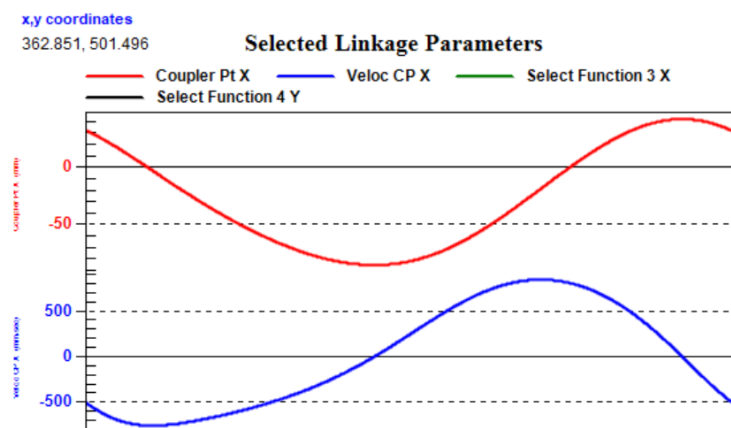


Figure 2.1 Position and Velocity Analysis of four-bar linkage

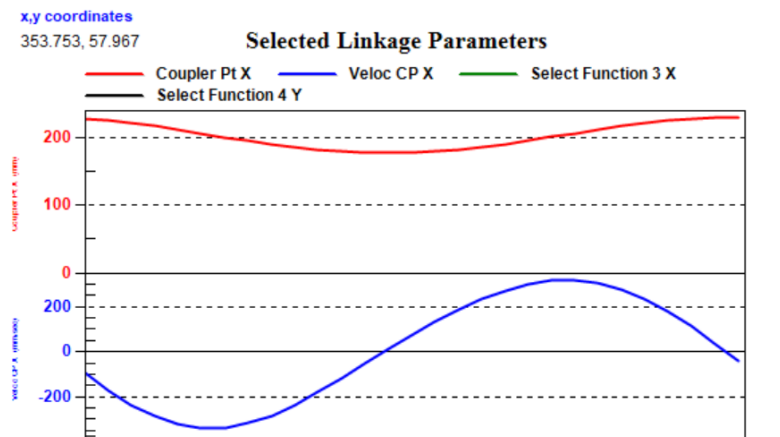


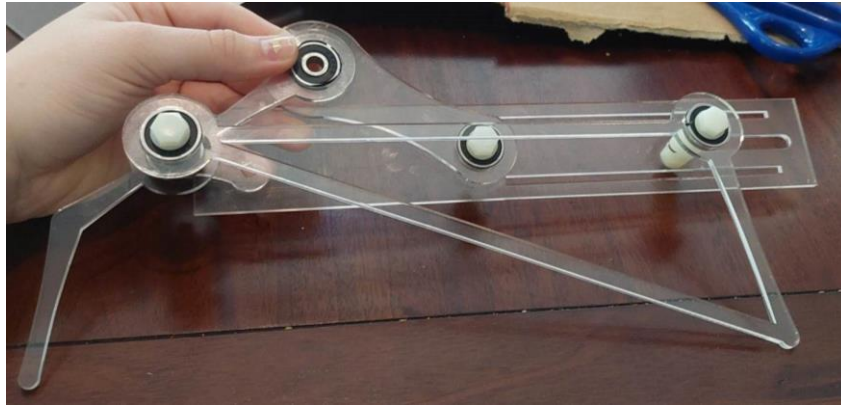
Figure 2.2 Position and Velocity Analysis of slider Crank

Overall, the linkage design considerations of the 4bar linkage and slide-crank have been thoroughly investigated to meet the specifications and requirements of the project.

## Fabrication Process

The design of the details, of the robot, required experimentation for the robot to work the most efficiently. The lengths for the initial design were calculated using Norton Linkages to ensure the robot would travel 2 metres in under 30 seconds with the velocities of the front and back legs close to synchronous. The linkages were modelled using AutoCAD and SolidWorks and laser cut out of acrylic. This material is light and durable, and the method allowed for precise measurements using the least fabrication time.

Through the stages of fabrication, the following issues were fixed before the final version. Initially the holes for the bearings fit too exact, so the bearing would slip through instead of gripping the acrylic. For following versions, the diameter of the holes was reduced by 0.3mm and ground out with a dremel to the correct size. The joint between the coupler and output link on the 4bar linkage shared the outer edge of a bearing. The design of the output link was changed so a bolt would go through the linkage and the inner hole of bearing. The initial designs overlooked the slider ( $y$ ) position needing to be lower than the input shaft and that the length ( $h$ ) was intended to be equal for the front and back legs. These were fixed by adjusting the height of ( $c'$ ) in the body and grinding the legs to have an equal ( $h$ ) value. Once these issues were resolved, the robot would fulfil all design requirements.



*Figure 3 – Initial Robot Linkage Design*

For the motor, a kit was used that included all the components needed to build a functioning motor and gearbox. This kit was designed for a spider-looking robot design, which included the gearbox, a switch, a motor, legs, wires, and gears. However, the legs provided weren't necessary for the design. The motor is powered by one AA battery, which turns a 3mm shaft on either side of the motor, with both shafts rotating at the same rate and time.

The design of the body was important to controlling the stability of the robot. Once the motor was assembled and the linkages were calculated, the body was modelled on SolidWorks and 3D printed using PLA. It was found that the joints were weak and prone to breaking, so a soldering iron was used to melt and reinforce the plastic. Once the components had been assembled, the two sides of the body were permanently connected using hot plate plastic welding.

## Schematic Diagrams

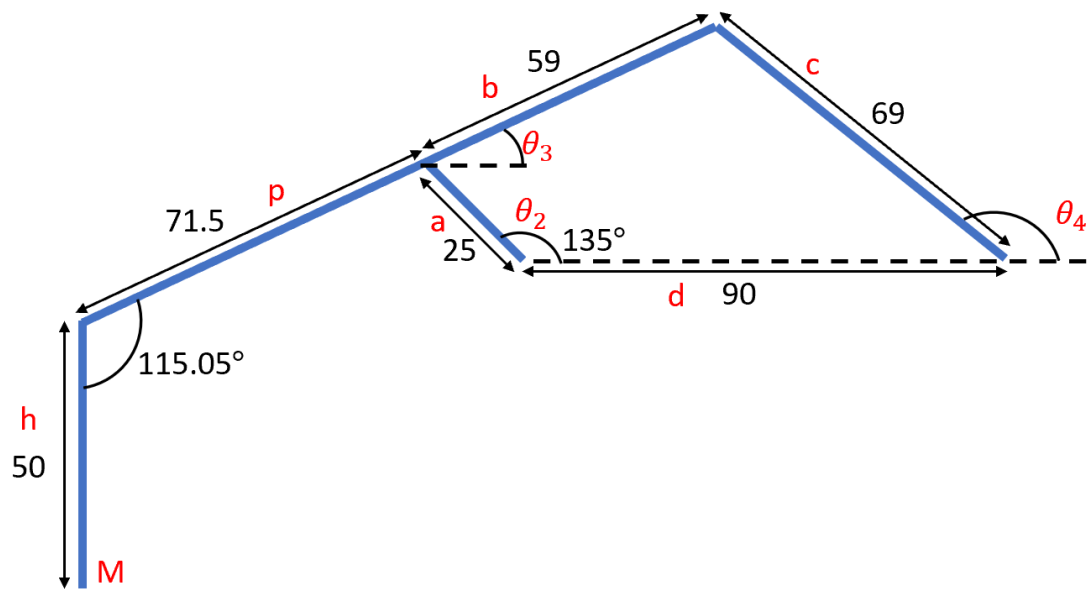


Figure 4.1 – Four-bar linkage diagram after fabrication (in mm)

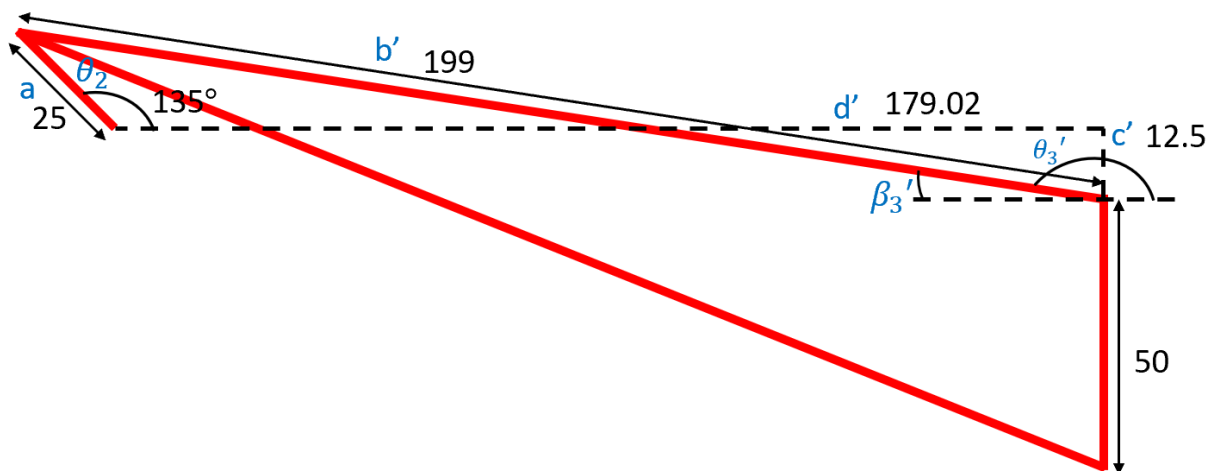


Figure 4.2 – Slider-Crank linkage diagram after fabrication (in mm)



## Mathematical Analysis of 4bar and Slide-Crank Linkages

### Position Analysis

4bar Linkage - $\theta_3$ & $\theta_4$		
$k_1 = \frac{d}{a}$ $= \frac{90}{25}$ $= 3.6$	$k_2 = \frac{d}{c}$ $= \frac{90}{69}$ $= 1.30$	$k_3 = \frac{a^2 + c^2 + d^2 - b^2}{2ac}$ $= \frac{25^2 + 69^2 + 90^2 - 59^2}{2 \times 25 \times 69}$ $= 2.9$
$A = \cos \theta_2 - k_1 - k_2 \cos \theta_2 + k_3$ $= \cos(135^\circ) - 3.6$ $\quad - 1.3 \cos(135^\circ) + 2.9$ $= -0.49$	$B = -2 \sin \theta_2$ $= -2 \sin(135^\circ)$ $= -1.41$	$C = k_1 - (k_2 + 1) \cos \theta_2 + k_3$ $= 3.6 - (1.3 + 1) \cos(135^\circ)$ $\quad + 2.9$ $= 8.13$
$\theta_4 = 2 \tan^{-1} \left( \frac{-B - \sqrt{B^2 - 4AC}}{2A} \right)$ $= 2 \tan^{-1} \left( \frac{-(-1.41) - \sqrt{(-1.41)^2 - 4(-0.49)(8.13)}}{2(-0.49)} \right)$ $= 141.81^\circ$ $\theta_3 = \sin^{-1} \left( \frac{-a \sin \theta_2 + c \sin \theta_4}{b} \right)$ $= \sin^{-1} \left( \frac{-25 \sin(135^\circ) + 69 \sin(141.73^\circ)}{59} \right)$ $= 25.06^\circ$		
Slider Crank Linkage - $\theta_3'$		
$\beta_3' = \sin^{-1} \left( \frac{a \sin \theta_2 - c'}{b'} \right)$ $= \sin^{-1} \left( \frac{25 \sin(135^\circ) - (-12.5)}{199} \right)$ $= 8.72^\circ$	$\theta_3' = 180 - \beta_3$ $= 180 - 8.72$ $= 171.28^\circ$	

## Angular Velocity Analysis

### Angular Velocity of Front Leg

Finding the angular velocity of  $\omega_3$  and  $\omega_4$ , when  $\omega_2 = 12.56 \text{ rad/s}$ :

$$\omega_3 = \frac{a\omega_2 \sin(\theta_4 - \theta_2)}{b \sin(\theta_3 - \theta_4)}$$

$$\omega_3 = \frac{25 \times 12.56 \sin(141.81 - 135)}{59 \sin(25.06 - 141.81)}$$

$$\omega_3 = -0.706423422$$

$$\omega_3 \approx -0.71 \text{ rad/s}$$

$$\omega_4 = \frac{a\omega_2 \sin(\theta_2 - \theta_3)}{c \sin(\theta_4 - \theta_3)}$$

$$\omega_4 = \frac{25 \times 12.56 \sin(135 - 25.06)}{69 \sin(141.81 - 25.06)}$$

$$\omega_4 = 4.790544972$$

$$\omega_4 \approx 4.79 \text{ rad/s}$$

### Angular Velocity of Back Leg

Finding the angular velocity of  $\omega'_3$  and  $\dot{d}$ , when  $\omega_2 = 12.56 \text{ rad/s}$ :

$$\omega'_3 = \frac{a\omega_2 \sin(\theta_2)}{b' \sin(\theta'_3)}$$

$$\omega'_3 = \frac{25 \times 12.56 \sin(135)}{199 \sin(171.28)}$$

$$\omega'_3 = 1.128791045$$

$$\omega'_3 \approx 1.13 \text{ rad/s}$$

$$\dot{d} = -a\omega_2 \sin(\theta_2) + b' \omega_3' \sin(\theta_3')$$

$$\dot{d} = -25 \times 12.56 \times \sin(135) + 199 \times 1.13 \times \sin(171.28)$$

$$\dot{d} = -187.9672461$$

$$\dot{d} \approx -187.97 \text{ rad/s}$$

## Line Velocity Analysis

### Front Leg Line Velocity of M

Finding the angular velocity of  $V_A$ , when  $\omega_2 = 12.56 \text{ rad/s}$ :

$$V_A = ja\omega_2 e^{j\theta_2}$$

$$V_A = ja\omega_2 (\cos(\theta_2) - jsin(\theta_2))$$

$$V_A = j \times 25 \times 12.56 (\cos(135) - jsin(135))$$

$$V_A = -222.032 - j222.032$$

$$V_A \approx -222.03 - j222.03 \text{ mm/s}$$

Finding the length of  $l$ , using the cosine rule, when  $\gamma_2 = 115.05$  (see Figure 4 for dimensions):

$$b^2 = c^2 + a^2 - 2ac \times \cos(B)$$

$$l^2 = h^2 + p^2 - 2hpcos(\gamma_2)$$

$$l^2 = 50^2 + 71.5^2 - 2 \times 50 \times 71.5 \times \cos(115.05)$$

$$l = 103.14855488457$$

$$l \approx 103.15 \text{ mm}$$

Finding the angle of  $\gamma_1$ , using the cosine rule, when  $\gamma_2 = 115.05$  (see Figure 4 for dimensions):

$$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

$$C = \cos^{-1}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$\gamma_1 = \cos^{-1}\left(\frac{p^2 + l^2 - h^2}{2pl}\right)$$

$$\gamma_1 = \cos^{-1}\left(\frac{71.5^2 + 103.15^2 - 50^2}{2 \times 71.5 \times 103.15}\right)$$

$$\gamma_1 = 26.049192963248$$

$$\gamma_1 \approx 26.05^\circ$$

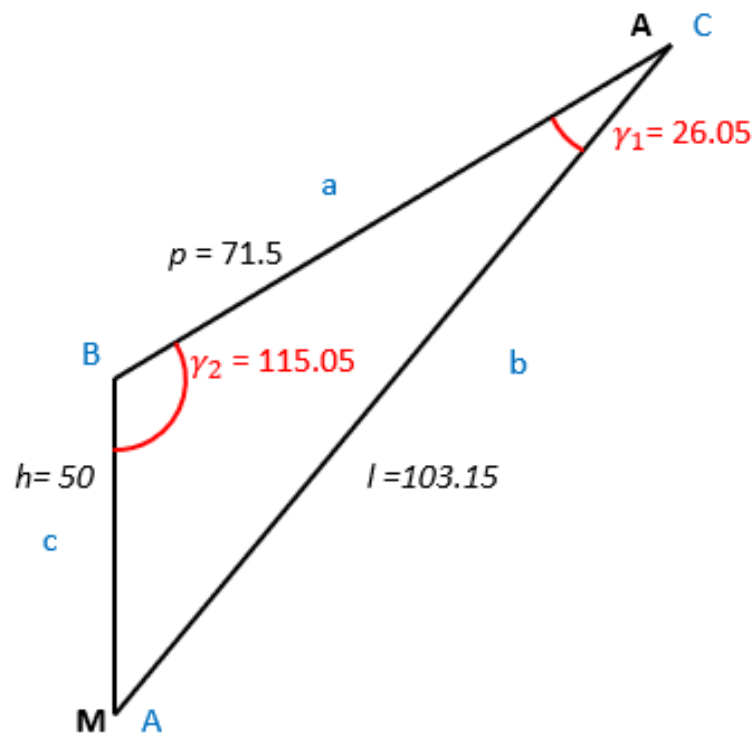


Figure 5 - Diagram of point M including cosine rule dimensions.

Finding the angular velocity of  $V_{MA}$  and  $V_M$ , when  $\omega_2 = 12.56 \text{ rad/s}$ :

$$V_{MA} = jl\omega_3 e^{\theta_3 + pi + \gamma_1}$$

$$V_{MA} = jl\omega_3 (\cos(\theta_3 + pi + \gamma_1) - jsin(\theta_3 + pi + \gamma_1))$$

$$V_{MA} = j \times 103.15 \times -0.71 (\cos(25.05 + 180 + 26.05) - jsin(25.05 + 180 + 26.05))$$

$$V_{MA} = -56.71120366 + j45.75340682$$

$$V_{MA} \approx -56.71 + j45.75 \text{ mm/s}$$

$$V_M = V_{MA} + V_A$$

$$V_M = -56.71 + j45.75 + (-222.03 - j222.03)$$

$$V_M = -278.742733 - j176.2781225$$

$$V_M \approx -278.74 - j176.28 \text{ mm/s}$$

Back Leg Line Velocity of N

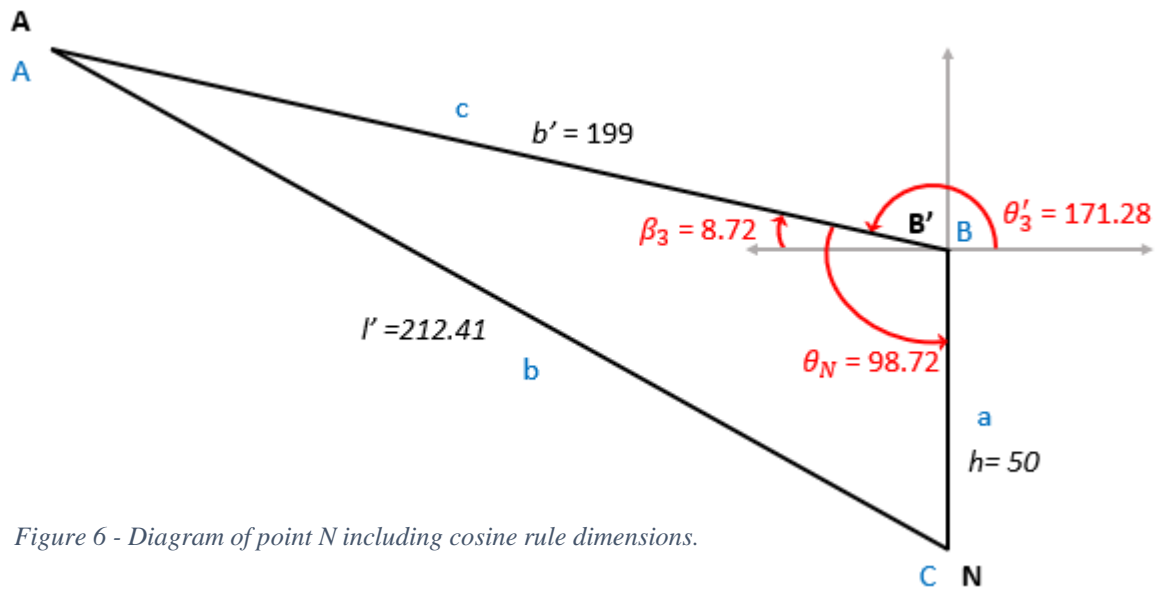


Figure 6 - Diagram of point N including cosine rule dimensions.

Finding the length of  $l'$ , using the cosine rule, when  $\beta'_3 = 8.72$  (see Figure 5 for dimensions):

$$b^2 = c^2 + a^2 - 2ac \times \cos(B)$$

$$l'^2 = h^2 + b'^2 - 2b'h \cos(\beta'_3 + 90)$$

$$l'^2 = 50^2 + 199^2 - 2 \times 50 \times 199 \times \cos(8.72 + 90)$$

$$l' = 212.4117863$$

$$l' \approx 212.41 \text{ mm}$$

Finding the angular velocity of  $V_{NA}$  and  $V_M$ , when  $\omega_2 = 12.56 \text{ rad/s}$ :

$$V_{NA} = jl'\omega'_3 e^{(\theta'_3 + \beta'_3 + 90)}$$

$$V_{NA} = jl'\omega'_3 (\cos(\theta'_3 + \beta'_3 + 90) - j\sin(\theta'_3 + \beta'_3 + 90))$$

$$V_{NA} = j \times 212.41 \times 1.13 (\cos(171.28 + 8.72 + 90) - j\sin(171.28 + 8.72 + 90))$$

$$V_{NA} = -239.7685222 + j0$$

$$V_{NA} \approx -239.77 \text{ mm/s}$$

Note: The calculation for  $V_A$  is the same used in the Line Velocity of M (front legs)

$$V_N = V_{NA} + V_A$$

$$V_N = -239.77 + (-222.03 - j222.03)$$

$$V_N = -461.8000515 - j222.032$$

$$V_N \approx -461.80 - j222.03 \text{ mm/s}$$

## Angular Acceleration Analysis

### Angular Acceleration of Front Leg

Finding the values A to F, when  $\omega_2 = 12.56 \text{ rad/s}$  and  $\alpha_2 = 0 \text{ rad/s}$ :

$$A = c \sin(\theta_4)$$

$$B = b \sin(\theta_3)$$

$$A = 69 \times \sin(141.81)$$

$$B = 59 \times \sin(25.05)$$

$$A = 42.66341595$$

$$B = 24.98574642$$

$$A \approx 42.66 \text{ rad/s}^2$$

$$B \approx 25 \text{ rad/s}^2$$

$$C = a\alpha_2 \sin(\theta_2) + a\omega_2^2 \cos(\theta_2) + b\omega_3^2 \cos(\theta_3) - c(\omega_4)^2 \cos(\theta_4)$$

$$C = 0 + 25 \times 12.56^2 \times \cos(125) + 59 \times -0.71^2 \cos(25.05) \\ - 69 \times 4.79^2 \times \cos(141.81)$$

$$C = -1517.514527$$

$$C \approx -1517.51 \text{ rad/s}^2$$

$$D = c \cos(\theta_4)$$

$$E = b \cos(\theta_3)$$

$$D = 69 \times \cos(141.81)$$

$$E = 59 \times \cos(25.05)$$

$$D = -54.22944716$$

$$E = 53.44822238$$

$$D \approx -54.23 \text{ rad/s}^2$$

$$E \approx 53.45 \text{ rad/s}^2$$

$$F = a\alpha_2 \cos(\theta_2) - a\omega_2^2 \sin(\theta_2) - b\omega_3^2 \sin(\theta_3) + c\omega_4^2 \sin(\theta_4)$$

$$F = 0 - 25 \times 12.56^2 \times \sin(135) + 59 \times -0.71^2 \sin(25.05) \\ + 69 \times 4.79^2 \times \sin(141.81)$$

$$F = -1822.088313$$

$$F \approx -1822.09 \text{ rad/s}^2$$

Finding the angular acceleration of  $\alpha_3$  and  $\alpha_4$ :

$$\alpha_3 = \frac{CD - AF}{AE - BD}$$

$$\alpha_3 = \frac{(-1517.51 \times -54.23) - (42.66 \times -1822.09)}{(42.66 \times 53.45) - (25 \times -54.23)}$$

$$\alpha_3 = 44.02190203$$

$$\alpha_3 \approx \mathbf{44.02 \text{ rad/s}^2}$$

$$\alpha_4 = \frac{CE - BF}{AE - BD}$$

$$\alpha_4 = \frac{(-1517.51 \times 53.45) - (25 \times -1822.09)}{(42.66 \times 53.45) - (25 \times -54.23)}$$

$$\alpha_4 = -9.788115572$$

$$\alpha_4 \approx \mathbf{-9.79 \text{ rad/s}^2}$$

### Angular Acceleration of Back Leg

Finding the angular acceleration of  $\alpha'_3$ , when  $\omega_2 = 12.56 \text{ rad/s}$  and  $\alpha_2 = 0 \text{ rad/s}$ :

$$\alpha'_3 = \frac{a\alpha_2 \cos(\theta_2) - a\omega_2^2 \sin(\theta_2) + b\omega_3^2 \sin(\theta_3)}{b \cos(\theta_3)}$$

$$\alpha'_3 = \frac{0 - 25 \times 12.56^2 \times \sin(135) + 59 \times -0.71^2 \times \sin(\theta_3)}{59 \times \cos(25.05)}$$

$$\alpha'_3 = -51.94274283$$

$$\alpha'_3 \approx \mathbf{-51.947 \text{ rad/s}^2}$$



## Computational Analysis

See Appendix 1 for the MATLAB for the following graphs.

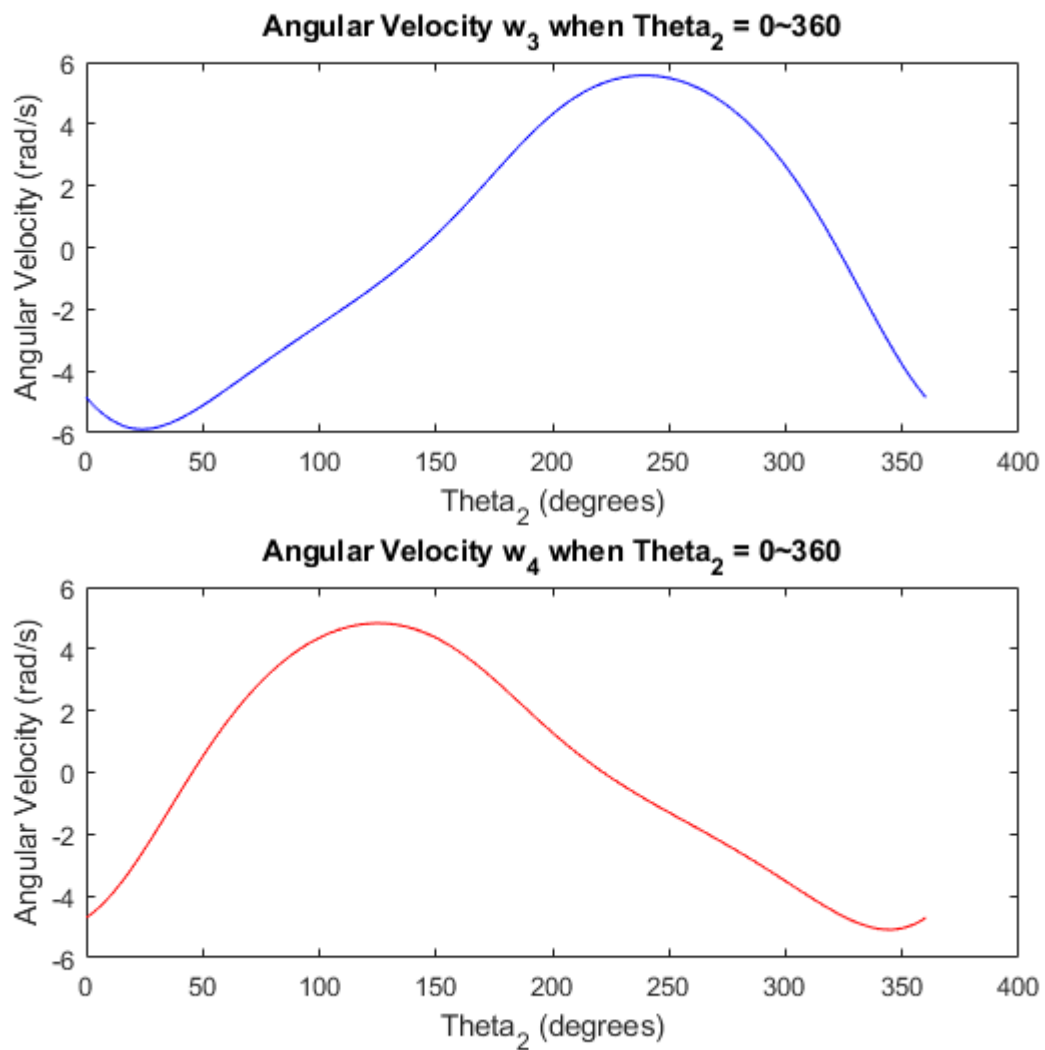


Figure 7 – 4bar linkage angular velocities

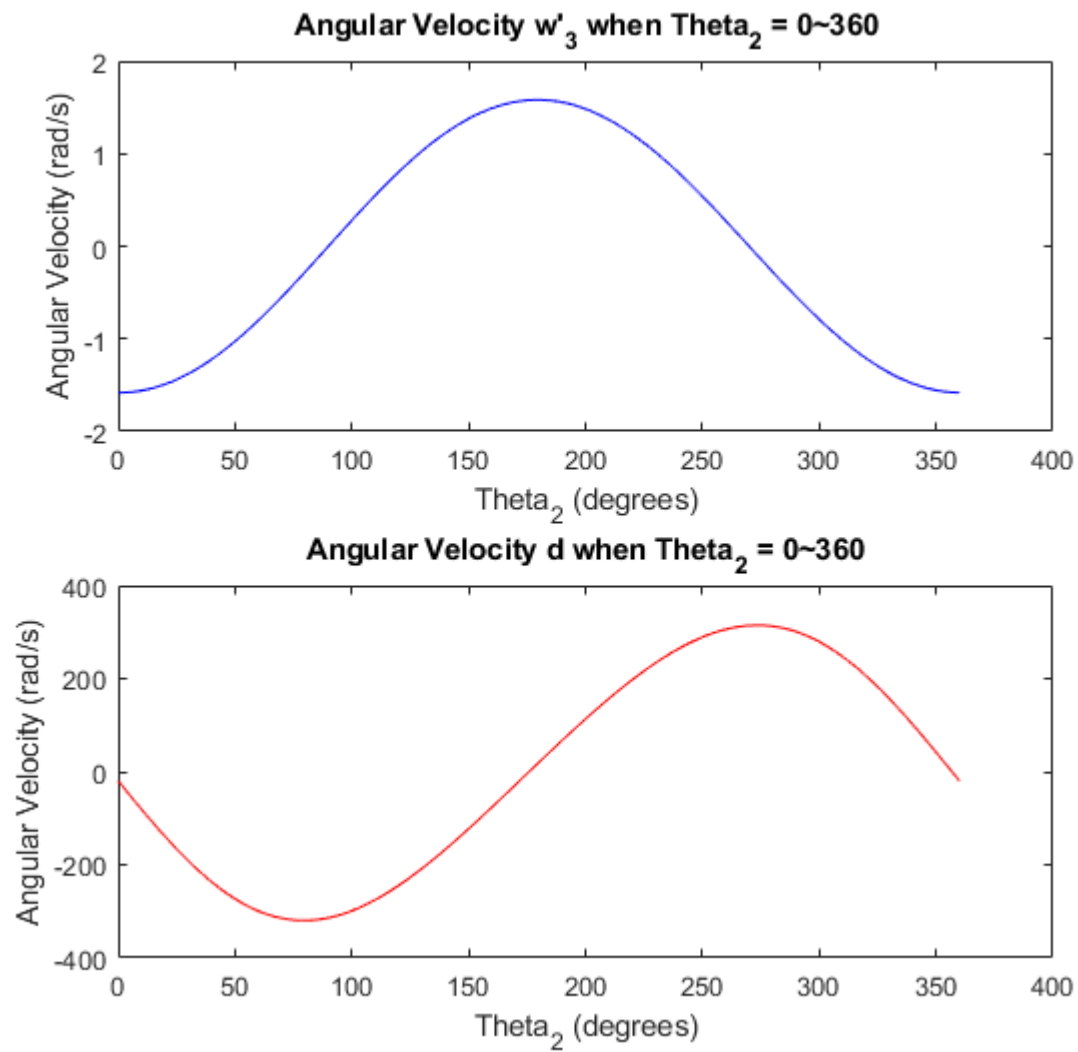


Figure 8 – Slide-crank angular velocities

## Conclusion

In conclusion, this project was able to achieve part of the aim. Functional 4bar and slide-crank linkages were powered through the same input driving link and designed through theoretical testing using Norton Linkages. Prototype versions were designed in AutoCAD and SolidWorks, and the final version was created using laser cut acrylic and bearings. The chassis was designed through SolidWorks and made using 3D printing from PLA. It was able to meet all requirements through securely housing the geared motor, with minimal interference on the linkage moment through design choices. Once everything was combined, the motor was able to power the input driving link to correctly move the linkages. However, once the robot was positioned on the floor, it was found that the motor did not have enough torque to power the linkages with the weight of the chassis, bearings and legs. Therefore, it could not move the robot forwards. A math analysis of the position, angular velocity, line velocity of endpoints (N and M), and angular acceleration was completed. Along with modelling the angular velocity, of the front and back legs throughout a whole rotation ( $\theta_2 = 0 - 360^\circ$ ), using MATLAB was done. Through testing, it was found that the robot could move forward when part of the robot's weight was supported by an external factor. When suspended in mid-air, it took 0.55 seconds for a full cycle of the leg movement, and simulation showed a displacement of 127mm per cycle. Allowing friction from walking, the robot would have theoretically walked the 2 metres within 15 seconds. This shows that the linkages have been designed correctly but further testing and redesigning would be needed to get the robot to move, by itself for 2 meters in 30 seconds.

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## Appendices

### Appendix 1 – MATLAB code for ‘Computational Analysis’

```
% Variables
a = 25;
b = 59;
c = 69;
d = 90;
h = 50;
p = 71.5;
theta_2 = [0:360];
g_1 = 115.05;
db = 199;
dc = -12.5;
h = 50;
w_2 = 12.56;

% Creating matrices to store values for both linkages
A = zeros(1,361);
B = zeros(1,361);
C = zeros(1,361);
theta_3 = zeros(1,361);
theta_4 = zeros(1,361);
w_3 = zeros(1,361);
w_4 = zeros(1,361);
dbeta_3 = zeros(1,361);
dtheta_3 = zeros(1,361);
dw_3 = zeros(1,361);
dd = zeros(1,361);

% Finding K values of 4-bar linkages
K_1 = d/a;
K_2 = d/c;
K_3 = (a^2 - b^2 + c^2 + d^2)/(2*a*c);

for x = 1:length(theta_2)

    % Finding point values of 4-bar linkages
    A(x) = cosd(x-1) - K_1 - K_2*cosd(x-1) + K_3;
    B(x) = -2*sind(x-1);
    C(x) = K_1 - (K_2 + 1)*cosd(x-1) + K_3;

    % Finding the theta values of 4-bar linkages
    theta_4(x) = 2*atand((-B(x) - sqrt(B(x)^2 - 4*A(x)*C(x)))/(2*A(x)));
    theta_3(x) = asind((-a*sind(x) + c*sind(theta_4(x)))/(b));

    % Finding angles of slide-crank linkages
    dbeta_3(x) = asind((a*sind(theta_2(x)) - dc)/db);
    dtheta_3(x) = 180 - dbeta_3(x);

    % Finding the angular velocity of the 4-bar
    w_3(x) = (a*w_2*sind(theta_4(x) - x))/(b*sind(theta_3(x) - theta_4(x)));
    w_4(x) = (a*w_2*sind(x - theta_3(x)))/(c*sind(theta_4(x) - theta_3(x)));
end
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% Finding the angular velocities of the slide-crank
dw_3(x) = (a*w_2*cosd(theta_2(x)))/(db*cosd(dtheta_3(x)));
dd(x) = -a*w_2*sind(theta_2(x)) + db*dw_3(x)*sind(dtheta_3(x));
end

figure(1)
subplot(2,1,1)
plot(theta_2, w_3, 'b')
title("Angular Velocity w_3 when Theta_2 = 0~360")
xlabel('Theta_2 (degrees)')
ylabel('Angular Velocity (rad/s)')

subplot(2,1,2)
plot(theta_2, w_4, 'r')
title("Angular Velocity w_4 when Theta_2 = 0~360")
xlabel('Theta_2 (degrees)')
ylabel('Angular Velocity (rad/s)')

figure(2)
subplot(2,1,1)
plot(theta_2, dw_3, 'b')
title("Angular Velocity w'_3 when Theta_2 = 0~360")
xlabel('Theta_2 (degrees)')
ylabel('Angular Velocity (rad/s)')

subplot(2,1,2)
plot(theta_2, dd, 'r')
title("Angular Velocity d when Theta_2 = 0~360")
xlabel('Theta_2 (degrees)')
ylabel('Angular Velocity (rad/s)')

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