# Congressional Redistricting Project Proposal:

## Redistricting the Congressional Districts of Idaho

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# Executive Summary

The goal of the project was to redistrict the state of Idaho, while following all federal and state laws regarding redistricting. Federal guidelines state that redistricting must follow equal population requirements stated in The Apportionment Clause of Article I, Section 2 of the U.S. Constitution and minority representation requirements stated in Section 2 of the Voting Rights Act of 1965. Idaho state law also adds guidelines for compactness, contiguity, preserving communities of interest, and preserving political subdivisions. Due to Idaho's population, as of the 2020 census, they were given two total congressional states. The district needed to be split into two districts. Except for Ada County, which encompasses the city of Boise, all counties must remain whole and can't be split.

We started by researching redistricting criteria, both federal and state. From here we looked at different models and approaches we could take. We decided on the minimum cut edge with contiguity model and the minimum moment of inertia with contiguity model. Next, we created mathematical models for our approaches. These models allowed us to move forward with creating python code, for a program that could redistrict the state.

We chose to take two approaches to redistricting the congressional districts withing Idaho. The first approach was a minimum cut edges with contiguity model. This approach allowed us to split the state into the two districts. This approach resulted in a map which contained a population deviation of 3,112 people which is 0.34%. The second approach was a minimum moment of inertia with contiguity model. This approach again allowed us to split the state into two districts, a northern district and a southern district. This approach resulted a population deviation of 776 people which is 0.08%. With both plans having a population deviation of under 1.00% per district they both passed the equal population representation. The state of Idaho's population is roughly 81.48% white. It is also a republican dominated state, roughly 63.2% of the population are registered as republicans. Idaho would be a complicated state to truly offer a racially equal and politically equal redistricting plan for. Both plans preserved traditional neighborhoods, communities of interest, and voting precinct boundaries. The plans both also scored moderately in terms of compactness using the Polsby-Popper formula. Plan 1 scored a 0.2963, and plan 2 scored a 0.2365. Both plans maintained the connection between each county in the district by state and/or federal highways, and both plans maintained the contiguity of the districts.

While both plans created similar districts. Plan 1 had a better Polsby-Popper score (compactness), its population deviation was much larger than that of plan 2. We finally concluded that plan 2, the minimum moment of inertia with contiguity model, was the better plan due to it satisfing all federal and state criteria and resulted in a lower population deviation.

# Introduction

Jordan Wheeler and Tim Jones formed the consulting group JOTIYA with the sole purpose of redistricting Idaho's congressional districts. Our main goal was to efficiently create the best possible plan for redistricting the state. The methods of minimum cut edges with continuity and minimum moment of inertia with continuity were used. The following report outlines the process we used and the outcomes we obtained.

# Criteria

## Federal Criteria

- Equal population requirements - The Apportionment Clause of Article I, Section 2, of the U.S. Constitution requires that all districts have nearly equal populations.
- Minority representation - Section 2 of the Voting Rights Act of 1965 prohibits the sectioning of districts that intentionally or inadvertently discriminate on the basis of race.

## State Criteria

- Compactness: Districts should not be "oddly shaped."
- Contiguity: County lines must be maintained "to the extent possible." In districts comprising more than one county or a portion thereof, "those constituent pieces must also be connected by a state or federal highway."
- Preservations: Districts must "preserve traditional neighborhoods, communities of interest, and (if possible) voting precinct boundaries."

# Problem Statement

As the US population grows and demographics change, Congressional voting districts must also change in order to maintain compliance with state and federal requirements. Manipulating voting districts in favor of a political party during the redistricting process is called gerrymandering, therefore redistricting proposals are strictly scrutinized before adaptation and implementation. Computer modeling has added clarity to the process and is now the "go to" method for redistricting.

# OR Model

## Minimum Cut Edges with Contiguity

Sets:

N is the set of nodes

E is the set of edges

B is the set of borders (neighbors) of county i

Indices:

i is a county (from 1 to n)

j is a district (from 1 to k)

e is an edge connecting two counties

Variables:

$X_{ij}$ = {0,1} $X_{ij}$ is 1 if all counties i (in set N) is assigned to a district j (from 1 to k), and 0 otherwise

$Y_e$ = {0,1} $Y_e$ is 1 if edge e (in set E) is cut, and 0 otherwise

$R_{ij}$ = {0,1) $R_{ij}$ is 1 if county i (in set V) is the root of district j (from 1 to k), and 0 otherwise

$F_{ij}$ is the flow on the edge of county I (in set N) in district j (from 1 to k)

$F_{ji}$ = is the flow on the edge of district j (from 1 to k) in county I (in set N)

Parameters:

u is the upper population bound

l is the lower population bound

$p_i$ is the population of county I (in set N)

k is the number of districts

n is the number of counties

m is the number of counties minus the number of districts plus one

$$Minimize \sum_{e \in E} Y_e \quad \textbf{(1)}$$

$$Subject\ to \sum_{j=1}^{k} X_{ij} = 1, \qquad \forall j \in V \quad \textbf{(2)}$$

$$L \leq \sum_{i \in V} p_i X_{ij} \leq U, \qquad \forall j \in \{1,2,\dots,k\} \tag{3}$$

$$X_{uj} - X_{vj} \leq Y_e, \qquad \forall e = \{u,v\} \in E, \forall j \in \{1,2,\dots,k\} \tag{4}$$

$$X_{ij} \in \{1,0\}, \qquad \forall i \in V, \forall j \in \{1,2,\dots,k\} \tag{5}$$

$$Y_e \in \{1,0\}, \qquad \forall e \in E \tag{6}$$

$$\sum_{i \in V} R_{ij} = 1\,, \qquad \forall j \in \{1,2,\dots,k\} \tag{7}$$

$$R_{ij} \leq X_{ij}, \qquad \forall i \in V, \forall j \in \{1,2,\dots,k\} \tag{8}$$

$$\sum_{U \in B} (F_{ui} - F_{iu}) \geq 1 - M \sum_{j=1}^{k} R_{ij}, \qquad \forall i \in V \tag{9}$$

$$F_{ij} + F_{ji} \leq m(1 - Y_e), \qquad \forall e \in \{i,j\} \in E \tag{10}$$

$$F_{ij}, F_{ji} \geq 0, \qquad \forall \{i,j\} \in E \tag{11}$$

$$R_{ij} \in \{1,0\}, \ \forall i \in V, \forall j \in \{1,2,\dots,k\}. \tag{12}$$

(1) The objective function is to minimize the number of cut edges between the counties i in district j of the state of Idaho.
(2) Each county i is assigned to only one district j.
(3) The population of each district lies between the lower bounds (L) and upper bounds (U).
(4) Edge e is considered cut if node u is assigned to district j and node v is not assigned to district j.
(5) Binary constraint for variable $X_{ij}$.
(6) Binary constraint for variable $Y_e$.
(7) Each district j has only one root node.
(8) A county i cannot be a county for a district to which it is not assigned.
(9) County i must consume flow if it is not the root of its district j.
(10) Flow is not allowed to be present along cut edges e.
(11) All flow variables $F_{ij}$ and $F_{ji}$ are non-negative.
(12) Binary constraint for variable $R_{ij}$.

## Minimum Moment of Inertia with Contiguity

Indices:

i is a county (from 1 to n)

j is a district (from 1 to k)

Variables:

$X_{ij}$ = {0,1} $X_{ij}$ is 1 if all counties i (in set V) is assigned to a district j (from 1 to k), and 0 otherwise

$F_{ij}^{V}$ is the flow across arc [i,j,v] that is sent from the source node to root n

$W_{ij}$ = $P_i$ * $F_{ij}^{V}$ which is the population of district i multiplied by the flow across arc [i,j,v] that is sent from the source node to root n

Parameters:

u is the upper population bound

l is the lower population bound

$p_i$ is the population of county i

k is the number of districts

n is the number of counties

v is the center of district i

$$Minimize \sum_{i \in V} \sum_{j \in V} W_{ij} X_{ij} \quad \textbf{(1)}$$

$$Subject\ to \sum_{j \in V} X_{ij} = 1, \qquad \forall j \in V \quad \textbf{(2)}$$

$$\sum_{j \in V} X_{ij} = k, \qquad \forall j \in V \quad \textbf{(3)}$$

$$LX_{jj} \leq \sum_{i \in V} P_i X_{ij} \leq UX_{jj}, \qquad \forall j \in V \quad \textbf{(4)}$$

$$X_{ij} \leq X_{jj}, \qquad \forall i \in V, \forall j \in V \quad \textbf{(5)}$$

$$X_{ij} \in \{0,1\}, \qquad \forall i \in V, \forall j \in V \quad \textbf{(6)}$$

$$\sum_{U \in B} \left(F_{ui}^{j} - F_{iu}^{j}\right) = X_{ij}, \qquad \forall i \in V, \forall j \in V \quad \textbf{(7)}$$

$$\sum_{U \in B} F_{ui}^{j} \leq (n-1) X_{ij}, \qquad \forall i \in V, \forall j \in V \quad \textbf{(8)}$$

$$\sum_{U \in B} F_{ui}^{j} = 0, \qquad \forall j \in V \tag{9}$$

$$F_{ij}^{v} \geq 0, \qquad \forall \{i,j\} \in E, \forall v \in V \tag{10}$$

$$F_{ji}^{v} \geq 0, \ \ \forall \{i,j\} \in E, \forall v \in V. \tag{11}$$

(1) The objective function is to minimize the moment of inertia for county i in district j.
(2) Each vertex of county i is assigned to only one district.
(3) A k number of districts is chosen.
(4) The population of each district lies between the lower bounds (L) and upper bounds (U).
(5) If vertex i is assigned to center j, then j is the center of the district.
(6) Binary variable for the vertex of each district.
(7) Vertex i can receive flow from only center j, if i is assigned to center j.
(8) Ensures that vertex i can receive the flow of type j, then vertex i consumes one unit of flow of type j, otherwise it consumes zero (0).
(9) Preventing flow circulations.
(10) The flow variable $F_{ji}$ is non-negative.
(11) The flow variable $F_{ij}$ is non-negative.

# Experiments

## Minimum Cut Edges with Contiguity

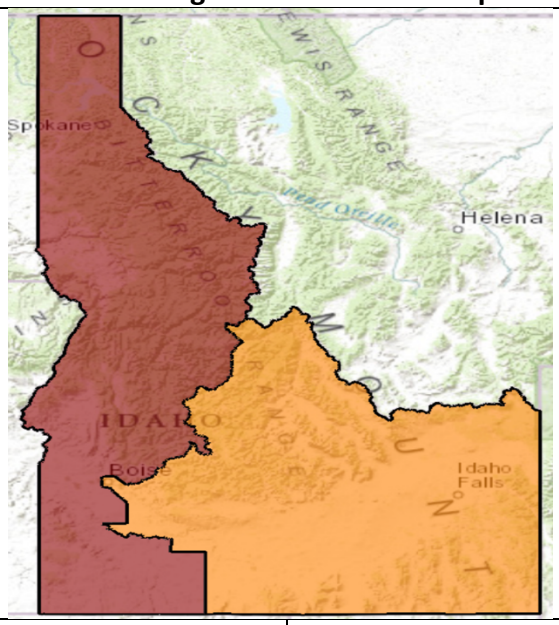
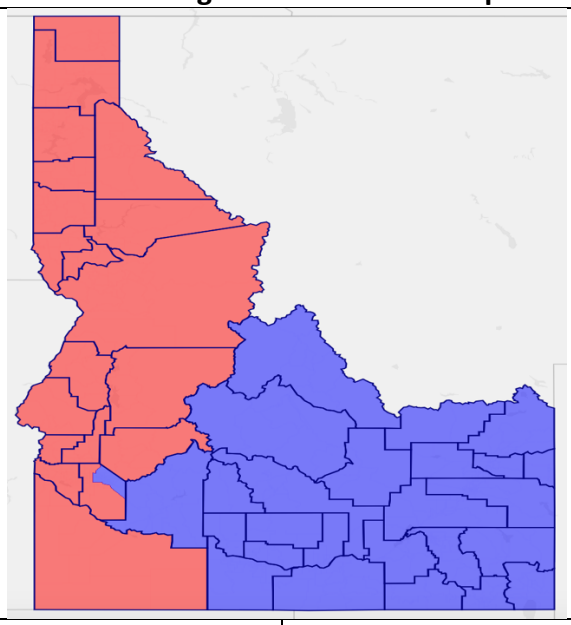
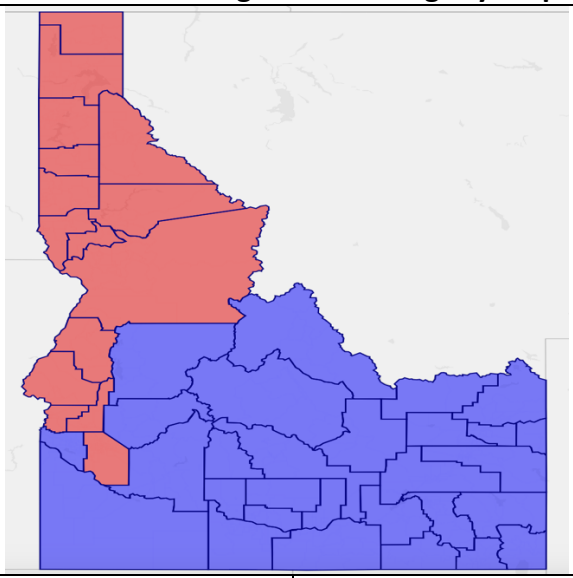
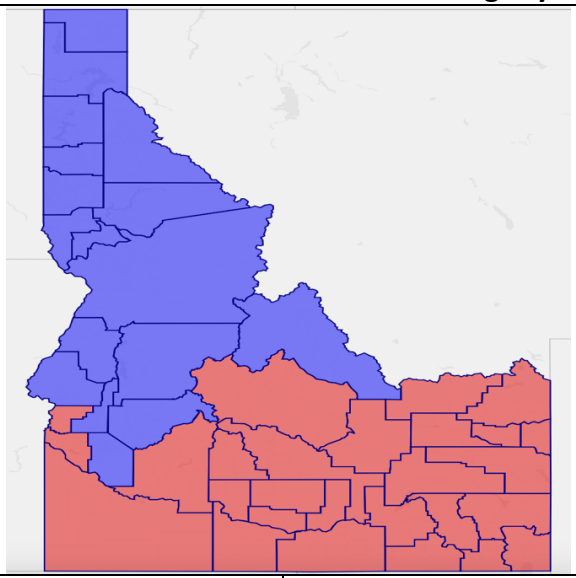
The minimum cut edges with contiguity model was run on a 2017 MacBook Pro, running macOS Ventura 13.3.1. The processor used was a 3.1 GHz Dual-Core Intel Core i5. The graphics card used was an Intel Iris Plus Graphics 650 1536 MB. The computer has 8 GB 2133 MHz LPDDR3. Version 10.0.1 of the Gurobi Optimizer software was used to optimize the model in 0.88 seconds, with 9657 simplex iterations being completed, resulting in an objective value of 1.1e+01.

## Minimum Moment of Inertia with Contiguity

The minimum moment of inertia with contiguity model was run on a 2017 MacBook Pro, running macOS Ventura 13.3.1. The processor used was a 3.1 GHz Dual-Core Intel Core i5. The graphics card used was an Intel Iris Plus Graphics 650 1536 MB. The computer has 8 GB 2133 MHz LPDDR3. Version 10.0.1 of the Gurobi Optimizer software was used to optimize the model in 4.88 seconds, with 5185 simplex iterations being completed, resulting in an objective value of 2.84e+10.

# Plans, Maps and Evaluation

## Maps

<table>
<tr><th colspan="2">2010 Congressional District Map</th><th colspan="2">2022 Congressional District Map</th></tr>
<tr><td colspan="2"> 
</td><td colspan="2">  </td></tr>
<tr><th>District 1</th><th>District 2</th><th>District 1</th><th>District 2</th></tr>
<tr><td>784,132</td><td>783,450</td><td>919,553</td><td>919,553</td></tr>
<tr><th colspan="2">Population Difference</th><th colspan="2">Population Difference</th></tr>
<tr><td colspan="2">682</td><td colspan="2">0</td></tr>
<tr><th colspan="2">Minimum Cut Edges with Contiguity Map</th><th colspan="2">Minimum Moment of Inertia with Contiguity Map</th></tr>
<tr><td colspan="2">  </td><td colspan="2">  </td></tr>
<tr><th>District 1</th><th>District 2</th><th>District 1</th><th>District 2</th></tr>
<tr><td>921,109</td><td>917,997</td><td>919,941</td><td>919,165</td></tr>
<tr><th colspan="2">Population Difference</th><th colspan="2">Population Difference</th></tr>
<tr><td colspan="2">3,112</td><td colspan="2">776</td></tr>
</table>

## Plan and Evaluation

We used Dave's Redistricting to gather general information about our two redistricting plans. The state of Idaho proposed a change to the congressional districts, which was accepted in 2022. Ada county which contains the city of Boise was split up between the two districts. While Idaho requires all counties in a district to be whole, they have an exception for Ada County. With our redistricting model, we wanted to maintain the completeness of the counties within each district.

Plan 1 was the minimum cut edges with contiguity. This plan, which is shown above, has district 1 in red with a population of 921,109 people, and district 2 in blue with a population of 917,997 people. The difference in population between the two districts is 3,112 people, and the population deviations is 0.34%. The state of Idaho has a low percentage of minorities and a low percentage of registered democratic voters. There isn't a way to create the districts where the minorities in the state could win.

| | | | |
|---|---|---|---|
| Equal Population Requirement | **0.34%** | Compactness | **0.2963** |
| Minority Representation | **Yes** | Preservation of Precincts | **Yes** |
| Maintained County Lines | **Yes** | Highway Connectivity | **Yes** |

Plan 2 was the minimum moment of inertia with contiguity. This plan, which is shown above, has district 1 in blue with a population of 919,941 people, and district 2 in red with a population of 919,165 people. The difference in population between the two counties is 776 people, and the population deviation is 0.08%.

| | | | |
|---|---|---|---|
| Equal Population Requirement | **0.08%** | Compactness | **0.2365** |
| Minority Representation | **Yes** | Preservation of Precincts | **Yes** |
| Maintained County Lines | **Yes** | Highway Connectivity | **Yes** |

With the given information from above we decided to propose plan 2. While the compactness and competitiveness scores decrease the overall appeal of this plan, the population deviation and the preservation of whole counties boost the appeal.

# Conclusion

In conclusion plan 2, the minimum moment of inertia with contiguity, meets all state and federal criteria for congressional redistricting. The plan: maintains an equal population between the two districts within 1.0%, minority representation is maintained, the districts are compact, county lines and district lines are contiguous, the counties within each district are connected by either a state or federal highway, and the preservation of the traditional neighborhoods and communities of interest.

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