Independent Events

Probability Theory: Foundation for Data Science

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Learning Goals for Module 2

In this module, we'll learn about conditional probability and Bayes formula. At the end of this module, learners should be able to:

- Explain the concept of conditional probability.
- Calculate probabilities using conditioning and Bayes Theorem.
- **Explain the concepts of independence and mutually exclusive events and provide examples.**

Independence

Two events are **independent** if knowing the outcome of one event does not change the probability of the other.

Examples:

- Flip a two-sided coin repeatedly. Knowing the outcome of one flip does not change the probability of the next.
- Roll a dice repeatedly.
- What about polling? What if you ask two randomly selected people about their political affiliation? What if the two people are friends?

Definition

Two events, A and B, are **independent** if $P(A|B) = P(A)$, or equivalently, if $P(B|A) = P(B)$.

Recall:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

then, if A and B are independent, we get the multiplication rule for independent events:

$$P(A \cap B) = P(A)P(B)$$

.

Definition Events $A_1, \cdots, A_n$ are **mutually independent** if for every k ($k = 2, 3, \ldots n$) and every subset of indices $i_1, i_2, \ldots, i_k$:

$$P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2}) \cdots P(A_{i_k})$$

Use the definition of independence in two ways:

- We can use the definition to show two events A and B are (or are not) independent. To do this, we calculate $P(A)$, $P(B)$, and $P(A \cap B)$ to check if $P(A \cap B) = P(A)P(B)$.
- If we know two events are independent, we can find the probability of their intersection.

Example 1

Example: Roll a six-sided dice twice. Recall, $S = \{(i,j) \mid i,j \in \{1,2,3,4,5,6\}\}$, $|S| = 36$ and each of the 36 outcomes of S is equally likely.

Let E be the event that the sum is 7.
Let F be the event that the first roll is a 4.
Let G be the event that the second roll is a 3.
What can you say about the independence of E, F and G?

Example 2

In a school of 1200 students, 250 are juniors, 150 students are taking a statistics course, and 40 students are juniors and also taking statistics. One student is selected at random from the entire school. Let J be the event the selected student is a junior. Let S be the event that the selected student is taking statistics.

If the randomly chosen student is a junior, then what is the probability that they are also taking stats? Are J and S independent?

Example 3

Suppose you have a system of components as in the diagram. Let A_i be the event that the i^{th} component works and assume $P(A_i) = .9$ for $i = 1, 2, 3, 4, 5$. Assume the components work independently of each other. For the system to work, you need a path of working components from the start to the finish. Find the probability that the system works.

Example 3 - continued

One final question: Suppose you know two events A and B are mutually exclusive, that is, $A \cap B = \emptyset$. Are A and B independent?