APA-L5-python

September 7, 2018

1 APA Laboratori 5 - LDA/QDA/NBayes/RegLog

In [1]:
```
# Uncomment to upgrade packages
# !pip install pandas --upgrade
# !pip install numpy --upgrade
# !pip install scipy --upgrade
# !pip install statsmodels --upgrade
# !pip install scikit-learn --upgrade
%load_ext autoreload
```

In [2]:
```
#%matplotlib notebook
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sn
import pandas as pd
pd.set_option('precision', 3)
from IPython.core.interactiveshell import InteractiveShell
InteractiveShell.ast_node_interactivity = "all"
```

In [3]:
```
# Extra imports
from pandas import read_csv
from sklearn.metrics import confusion_matrix, \
                  classification_report, accuracy_score
from pandas.api.types import CategoricalDtype
from pandas.plotting import scatter_matrix
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.model_selection import LeaveOneOut
from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
from sklearn.preprocessing import Imputer
from sklearn.model_selection import train_test_split
from sklearn.naive_bayes import BernoulliNB
from sklearn.datasets import load_iris
from sklearn.neighbors import KNeighborsClassifier
from numpy.random import  normal, binomial
from statsmodels.genmod.generalized_linear_model import GLM
from statsmodels.genmod.families.family import Binomial
from statsmodels.tools.tools import add_constant
```

```
from sklearn.feature_selection import RFECV
from sklearn.linear_model import LogisticRegression
```

```
In [4]: def confusion(true, pred, classes):
            """
            Function for pretty printing confusion matrices
            """
            cm =pd.DataFrame(confusion_matrix(true, pred),
                             index=classes,
                             columns=classes)
            cm.index.name = 'Actual'
            cm.columns.name = 'Predicted'
            return cm
```

1.1 Example 1: Visualizing and classifying wines with LDA and QDA

We have the results of an analysis on wines grown in a region in Italy but derived from three different cultivars. The analysis determined the quantities of 13 chemical constituents found in each of the three types of wines. The goal is to separate the three types of wines:

```
In [5]: wine = read_csv("wine.data", delimiter=',', header=None)
        wine_classes = ['cultivar %d'%(i+1) for i in range(3)]
        wine.shape
        wine.columns = ['Wine_type','Alcohol','Malic_acid','Ash',
                        'Alcalinity_of_ash','Magnesium','Total_phenols',
                        'Flavanoids','Nonflavanoid_phenols',
                        'Proanthocyanins','Color_intensity','Hue',
                        'OD280/OD315','Proline']
```

```
Out[5]: (178, 14)
```

```
In [6]: wine.Wine_type = wine.Wine_type.astype(CategoricalDtype(categories=[1, 2, 3],
                                                                 ordered=True))
        wine.describe(include='all')
```

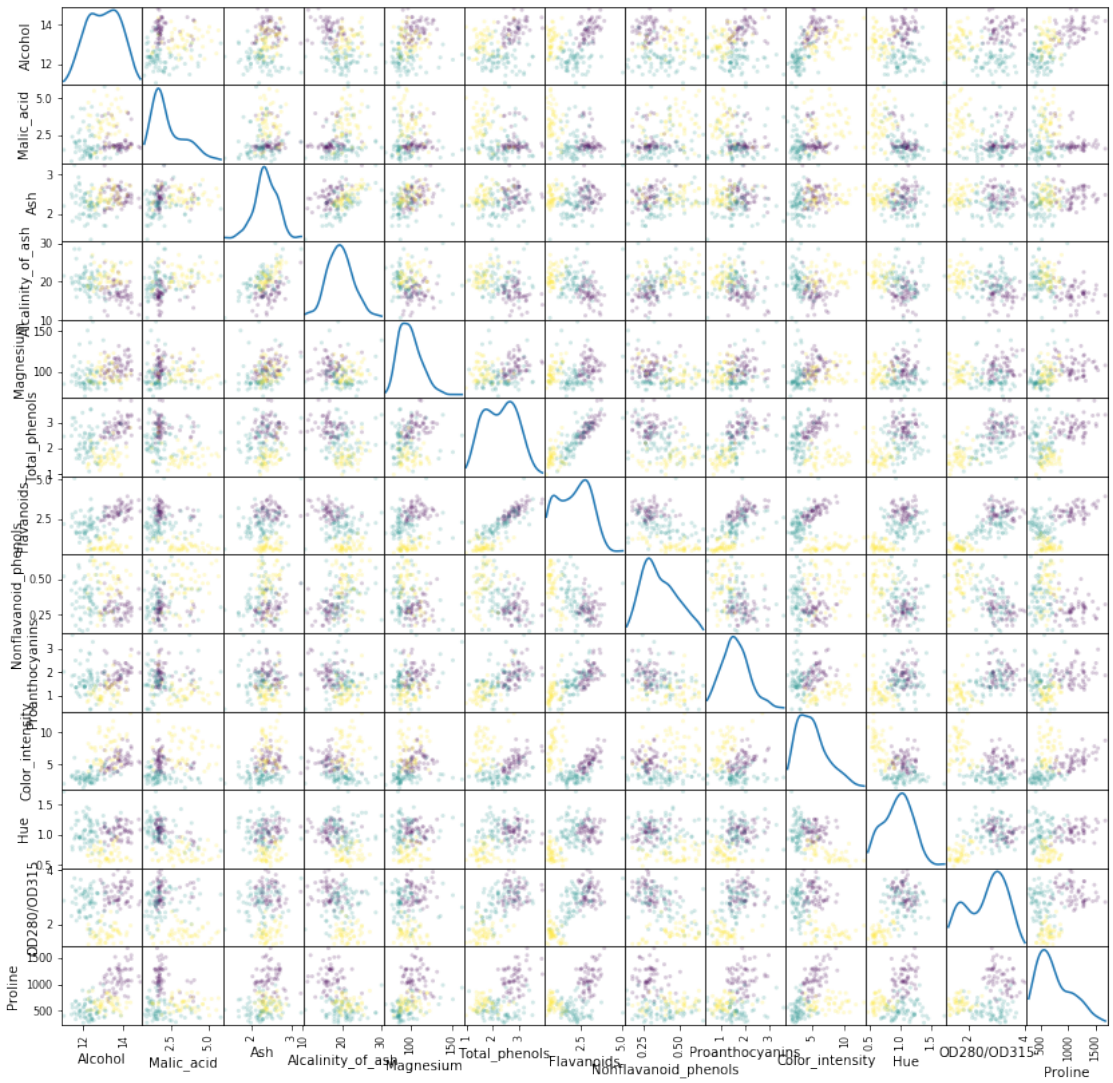
```
Out[6]:         Wine_type  Alcohol  Malic_acid      Ash  Alcalinity_of_ash  Magnesium  \
        count       178.0  178.000     178.000  178.000            178.000    178.000
        unique        3.0      NaN         NaN      NaN                NaN        NaN
        top           2.0      NaN         NaN      NaN                NaN        NaN
        freq         71.0      NaN         NaN      NaN                NaN        NaN
        mean          NaN   13.001       2.336    2.367             19.495     99.742
        std           NaN    0.812       1.117    0.274              3.340     14.282
        min           NaN   11.030       0.740    1.360             10.600     70.000
        25%           NaN   12.362       1.603    2.210             17.200     88.000
        50%           NaN   13.050       1.865    2.360             19.500     98.000
        75%           NaN   13.678       3.083    2.558             21.500    107.000
        max           NaN   14.830       5.800    3.230             30.000    162.000

                Total_phenols  Flavanoids  Nonflavanoid_phenols  Proanthocyanins  \
```

```
count           178.000    178.000                     178.000           178.000
unique              NaN        NaN                         NaN               NaN
top                 NaN        NaN                         NaN               NaN
freq                NaN        NaN                         NaN               NaN
mean              2.295      2.029                       0.362             1.591
std               0.626      0.999                       0.124             0.572
min               0.980      0.340                       0.130             0.410
25%               1.742      1.205                       0.270             1.250
50%               2.355      2.135                       0.340             1.555
75%               2.800      2.875                       0.438             1.950
max               3.880      5.080                       0.660             3.580

        Color_intensity      Hue  OD280/OD315   Proline
count           178.000  178.000      178.000   178.000
unique              NaN      NaN          NaN       NaN
top                 NaN      NaN          NaN       NaN
freq                NaN      NaN          NaN       NaN
mean              5.058    0.957        2.612   746.893
std               2.318    0.229        0.710   314.907
min               1.280    0.480        1.270   278.000
25%               3.220    0.782        1.938   500.500
50%               4.690    0.965        2.780   673.500
75%               6.200    1.120        3.170   985.000
max              13.000    1.710        4.000  1680.000
```

In [7]:
```
scatter_matrix(wine.loc[:,'Alcohol':'Proline'],
               alpha=0.2, figsize=(14, 14),
               diagonal='kde',marker='.',
               c=wine.Wine_type);
```



For this example let's practice a different call mode to lda(), using a formula; this is most useful when our data is in a dataframe format:

```
In [8]: lda_model = LinearDiscriminantAnalysis().fit(wine.loc[:,'Alcohol':'Proline'],
                                                  wine.Wine_type)


        print('Priors:', lda_model.priors_)
        print('Means:\n')
        means =pd.DataFrame(lda_model.means_)
        means.columns=wine.columns[1:]
        means
        print('Coefs:')
        coefs = pd.DataFrame(lda_model.scalings_)
        coefs.index =wine.columns[1:]
        coefs
```

```python
print('Explained Variance Ratio')
pd.DataFrame(lda_model.explained_variance_ratio_ )
```

```
Priors: [0.33146067 0.3988764  0.26966292]
Means:
```

Out[8]:

	Alcohol	Malic_acid	Ash	Alcalinity_of_ash	Magnesium	Total_phenols
0	13.745	2.011	2.456	17.037	106.339	2.840
1	12.279	1.933	2.245	20.238	94.549	2.259
2	13.154	3.334	2.437	21.417	99.312	1.679

	Flavanoids	Nonflavanoid_phenols	Proanthocyanins	Color_intensity	Hue
0	2.982	0.290	1.899	5.528	1.062
1	2.081	0.364	1.630	3.087	1.056
2	0.781	0.447	1.154	7.396	0.683

	OD280/OD315	Proline
0	3.158	1115.712
1	2.785	519.507
2	1.684	629.896

```
Coefs:
```

Out[8]:

	0	1
Alcohol	-0.403	8.718e-01
Malic_acid	0.165	3.054e-01
Ash	-0.369	2.346e+00
Alcalinity_of_ash	0.155	-1.464e-01
Magnesium	-0.002	-4.628e-04
Total_phenols	0.618	-3.221e-02
Flavanoids	-1.661	-4.920e-01
Nonflavanoid_phenols	-1.496	-1.631e+00
Proanthocyanins	0.134	-3.071e-01
Color_intensity	0.355	2.532e-01
Hue	-0.818	-1.516e+00
OD280/OD315	-1.158	5.118e-02
Proline	-0.003	2.853e-03

```
Explained Variance Ratio
```

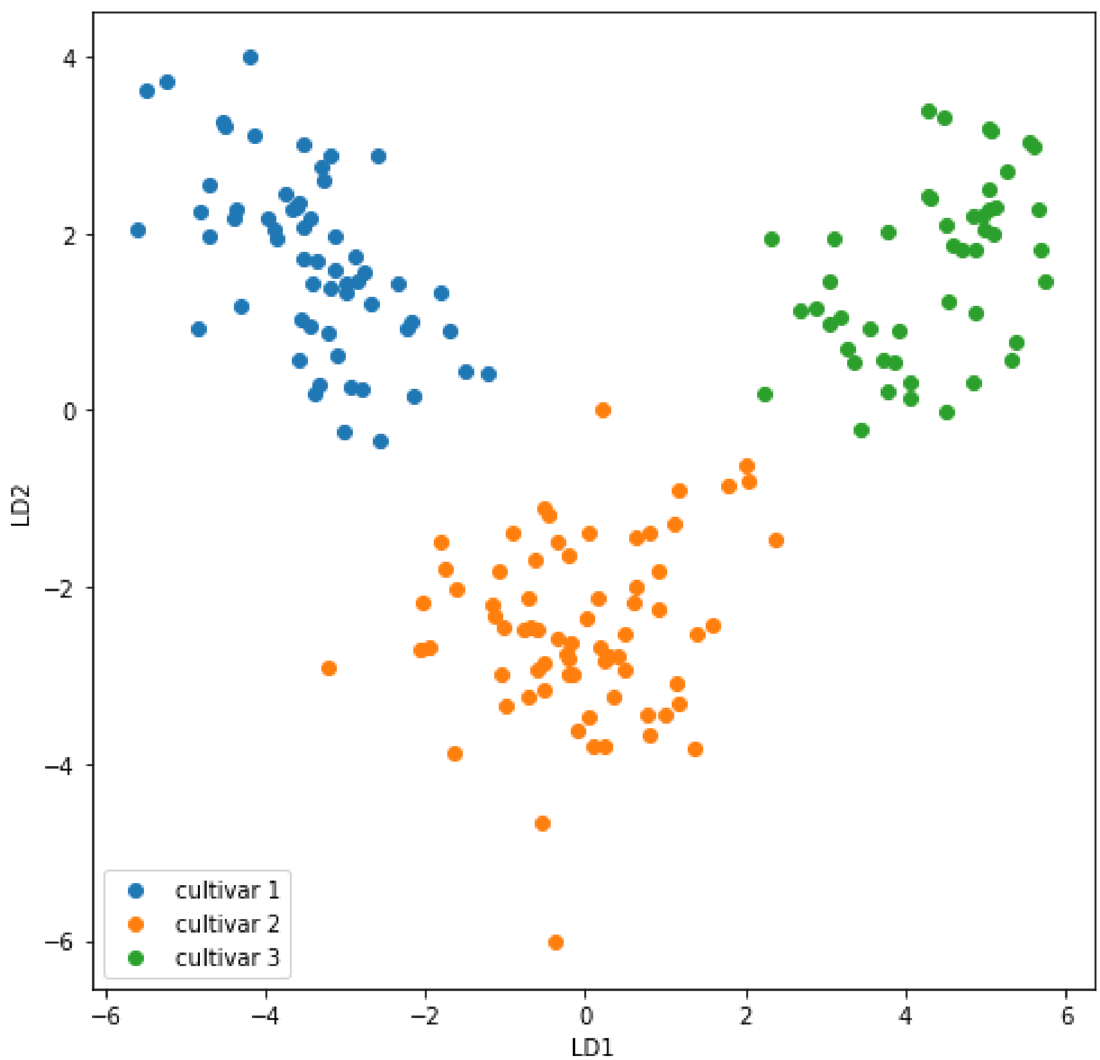
Out[8]:

	0
0	0.687
1	0.313

We can see that neither Magnesium or Proline seem useful to separate the wines; while Flavanoids and Nonflavanoid.phenols do. Ash is mainly used in the LD2.

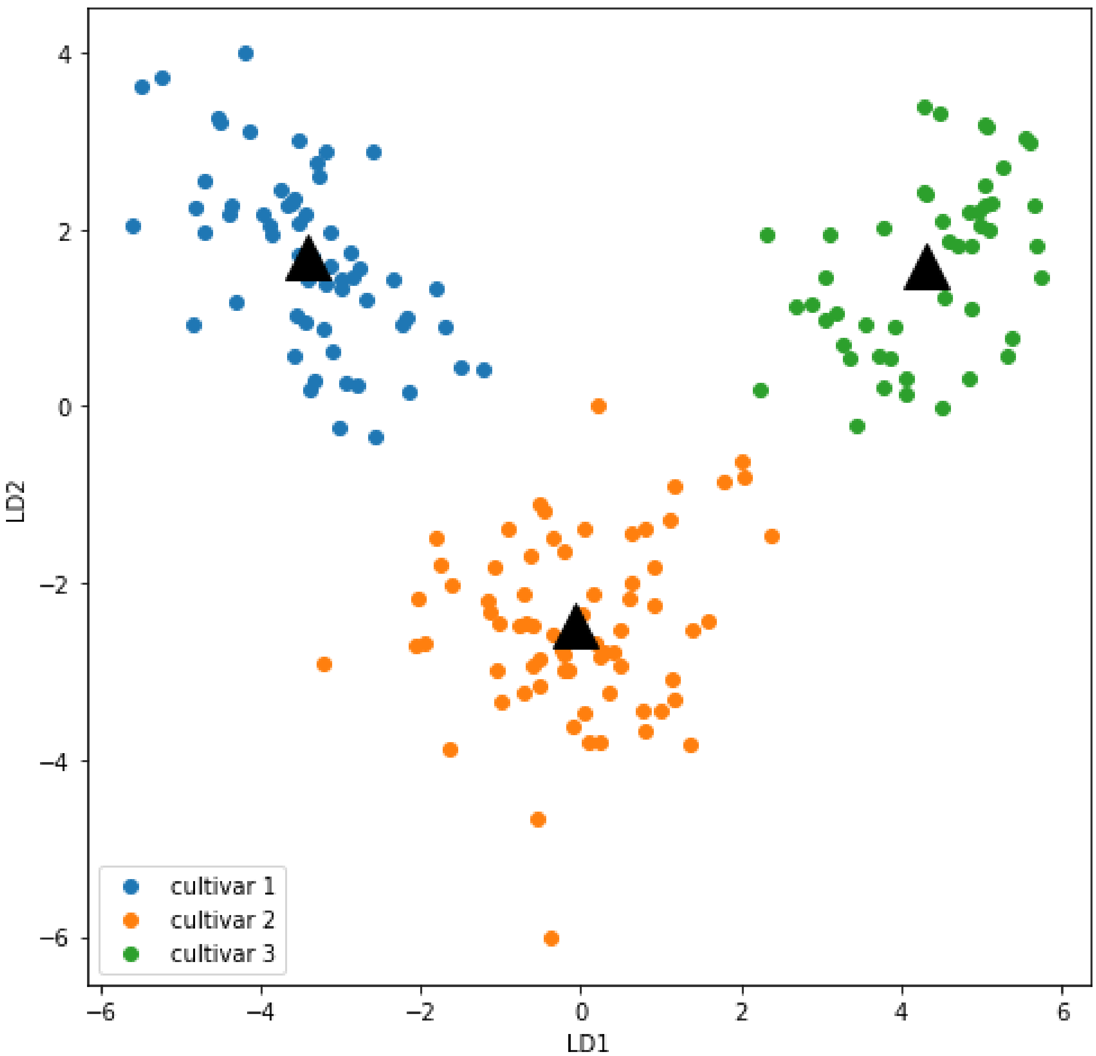
Plot the projected data in the first two LDs
We can see that the discrimination is very good

```
In [9]: wine_trans = lda_model.transform(wine.loc[:,'Alcohol':'Proline'])
        fig, ax = plt.subplots(figsize=(8,8))
        for i in wine.Wine_type.unique():
            plt.scatter(wine_trans[:,0][wine.Wine_type==i],
                        wine_trans[:,1][wine.Wine_type==i],
                        label='cultivar %d'%i)
        ax.set_xlabel('LD1')
        ax.set_ylabel('LD2')
        plt.legend();
```



If need be, we can add the (projected) means to the plot

```
In [10]: fig, ax = plt.subplots(figsize=(8,8))
         for i in wine.Wine_type.unique():
             plt.scatter(wine_trans[:,0][wine.Wine_type==i],
                         wine_trans[:,1][wine.Wine_type==i],
                         label='cultivar %d'%i)
             plt.plot(wine_trans[:,0][wine.Wine_type==i].mean(),
                      wine_trans[:,1][wine.Wine_type==i].mean(),
                      'k^',markersize=20)
         ax.set_xlabel('LD1')
         ax.set_ylabel('LD2')
         plt.legend();
```



indeed classification is perfect

```
In [11]: confusion(wine.Wine_type, lda_model.predict(wine.loc[:,'Alcohol':'Proline']),
                   classes=wine_classes)
```

```
Out[11]: Predicted   cultivar 1  cultivar 2  cultivar 3
         Actual
         cultivar 1          59           0           0
         cultivar 2           0          71           0
         cultivar 3           0           0          48
```

Let us switch to leave-one-out cross-validation

```
In [12]: def loocv(X,y,model,classes):
             loo = LeaveOneOut()
             pred=[]
             for train_index, test_index in loo.split(X):
                 X_tr, X_ts = X[train_index], X[test_index]
                 y_tr, _ = y[train_index], y[test_index]
                 model.fit(X_tr,y_tr)
                 pred.append(model.predict(X_ts)[0])
             return confusion(y,pred,classes), 1-accuracy_score(y,pred)
```

```
In [13]: cm, err = loocv(wine.loc[:,'Alcohol':'Proline'].values,
                         wine.Wine_type,
                         LinearDiscriminantAnalysis(),
                         wine_classes)

         cm

         err*100
```

```
Out[13]: Predicted   cultivar 1  cultivar 2  cultivar 3
         Actual
         cultivar 1          59           0           0
         cultivar 2           1          69           1
         cultivar 3           0           0          48
```

```
Out[13]: 1.1235955056179803
```

2 mistakes (on 178 observations): 1.12% error
Quadratic Discriminant Analysis is the same
problems may arise if for some class there are less (or equal) observations than dimensions (is not the case for the wine data)

```
In [14]: qda_model = QuadraticDiscriminantAnalysis().fit(wine.loc[:,'Alcohol':'Proline'],
                                                         wine.Wine_type)

         print('Priors:\n')
         pd.DataFrame(qda_model.priors_)
         print('Means:\n')
         means =pd.DataFrame(qda_model.means_)
         means.columns=wine.columns[1:]
         means
```

Priors:

Out[14]:

	0
0	0.331
1	0.399
2	0.270

Means:

Out[14]:

	Alcohol	Malic_acid	Ash	Alcalinity_of_ash	Magnesium	Total_phenols	\
0	13.745	2.011	2.456	17.037	106.339	2.840	
1	12.279	1.933	2.245	20.238	94.549	2.259	
2	13.154	3.334	2.437	21.417	99.312	1.679	

	Flavanoids	Nonflavanoid_phenols	Proanthocyanins	Color_intensity	Hue	\
0	2.982	0.290	1.899	5.528	1.062	
1	2.081	0.364	1.630	3.087	1.056	
2	0.781	0.447	1.154	7.396	0.683	

	OD280/OD315	Proline
0	3.158	1115.712
1	2.785	519.507
2	1.684	629.896

There is no projection this time (because projection is a linear operator and the QDA boundaries are quadratic ones)

but let's have a look at classification:

In [15]:
```
confusion(wine.Wine_type, qda_model.predict(wine.loc[:,'Alcohol':'Proline']),
          classes=wine_classes)
```

Out[15]:

Predicted Actual	cultivar 1	cultivar 2	cultivar 3
cultivar 1	59	0	0
cultivar 2	1	70	0
cultivar 3	0	0	48

Let us switch to leave-one-out cross-validation

In [16]:
```
cm, err = loocv(wine.loc[:,'Alcohol':'Proline'].values,
                wine.Wine_type,
                QuadraticDiscriminantAnalysis(),
                wine_classes)


cm

err*100
```

```
Out[16]: Predicted   cultivar 1  cultivar 2  cultivar 3
         Actual
         cultivar 1          59           0           0
         cultivar 2           1          70           0
         cultivar 3           0           0          48
```

```
Out[16]: 0.5617977528089901
```

1 mistake (on 178 observations): 0.56% error

it would be nice to ascertain which wine is the "stubborn" one: it is a wine of type '2' classified as class '1'. Maybe there is something special with this wine ...

In the event of numerical errors (insufficient number of observations per class), we can use regularization.

in this case the regularization parameter (0..1) is applied to the covariance matrix (Sigma) so it is not ill conditioned in this fashion

`(1-reg_param)*Sigma + reg_param*np.eye(n_features)`

```
In [17]: qda_model = QuadraticDiscriminantAnalysis(reg_param=0.1).\
                        fit(wine.loc[:,'Alcohol':'Proline'],
                            wine.Wine_type)


         print('Priors:', qda_model.priors_)
         print('Means:\n')
         means =pd.DataFrame(qda_model.means_)
         means.columns=wine.columns[1:]
         means
```

```
Priors: [0.33146067 0.3988764  0.26966292]
Means:
```

```
Out[17]:    Alcohol  Malic_acid    Ash  Alcalinity_of_ash  Magnesium  Total_phenols  \
         0   13.745       2.011  2.456             17.037    106.339          2.840
         1   12.279       1.933  2.245             20.238     94.549          2.259
         2   13.154       3.334  2.437             21.417     99.312          1.679

            Flavanoids  Nonflavanoid_phenols  Proanthocyanins  Color_intensity    Hue  \
         0       2.982                 0.290            1.899            5.528  1.062
         1       2.081                 0.364            1.630            3.087  1.056
         2       0.781                 0.447            1.154            7.396  0.683

            OD280/OD315   Proline
         0        3.158  1115.712
         1        2.785   519.507
         2        1.684   629.896
```

```
In [18]: confusion(wine.Wine_type, qda_model.predict(wine.loc[:,'Alcohol':'Proline']),
                   classes=wine_classes)
```

```
Out[18]: Predicted   cultivar 1  cultivar 2  cultivar 3
         Actual
         cultivar 1          59           0           0
         cultivar 2           0          69           2
         cultivar 3           0           0          48
```

1.2 Example 2: The Naïve Bayes classifier

Naive Bayes Classifier for Discrete Predictors: we use the 1984 United States Congressional Voting Records;

This data set includes votes for each of the U.S. House of Representatives Congressmen on 16 key votes In origin they were nine different types of votes:

- voted for, paired for, and announced for (these three simplified to yea or 'y'),
- voted against, paired against, and announced against (these three simplified to nay or 'n'),
- voted present, voted present to avoid conflict of interest, and did not vote or otherwise make a position known (these three simplified to an 'unknown' disposition)

The goal is to classify Congressmen as Republican or Democrat as a function of their voting profiles, which is not immediate because in the US Congressmen have a large freedom of vote (obviously linked to their party but also to their own feelings, interests and compromises with voters)

```
In [19]: HouseVotes84 = read_csv("house-votes-84.data",
                                  delimiter=',',
                                  header=None,na_values='?')
         house_classes = ['n','y']
```

add meaningful names to the votes

```
In [20]: HouseVotes84.columns=["Class","handicapped_infants","water_project_sharing",
                              "budget_resolution","physician_fee_freeze",
                              "el_salvador_aid","religious_groups_in_schools",
                              "anti_satellite_ban", "aid_to_nicaraguan_contras",
                              "mx_missile","immigration","synfuels_cutback",
                              "education_spending","superfund","crime",
                              "duty_free_exports","export_South_Africa"]
         HouseVotes84.describe()
```

```
Out[20]:            Class handicapped_infants water_project_sharing budget_resolution  \
         count        435                 423                   387               424
         unique         2                   2                     2                 2
         top     democrat                   n                     y                 y
         freq         267                 236                   195               253

                physician_fee_freeze el_salvador_aid religious_groups_in_schools  \
         count                   424             420                         424
```

```
unique                     2             2                         2
top                        n             y                         y
freq                     247           212                       272

       anti_satellite_ban aid_to_nicaraguan_contras mx_missile immigration  \
count                 421                       420        413         428
unique                  2                         2          2           2
top                     y                         y          y           y
freq                  239                       242        207         216

       synfuels_cutback education_spending superfund crime duty_free_exports  \
count               414                404       410   418               407
unique                2                  2         2     2                 2
top                   n                  n         y     y                 n
freq                264                233       209   248               233

       export_South_Africa
count                  331
unique                   2
top                      y
freq                   269
```

In [21]:
```
for v in HouseVotes84.columns:
    HouseVotes84[v].value_counts(dropna=False)
```

Out[21]:
```
democrat      267
republican    168
Name: Class, dtype: int64
```

Out[21]:
```
n      236
y      187
NaN     12
Name: handicapped_infants, dtype: int64
```

Out[21]:
```
y      195
n      192
NaN     48
Name: water_project_sharing, dtype: int64
```

Out[21]:
```
y      253
n      171
NaN     11
Name: budget_resolution, dtype: int64
```

Out[21]:
```
n      247
y      177
NaN     11
Name: physician_fee_freeze, dtype: int64
```

```
Out[21]: y      212
         n      208
         NaN     15
         Name: el_salvador_aid, dtype: int64
```

```
Out[21]: y      272
         n      152
         NaN     11
         Name: religious_groups_in_schools, dtype: int64
```

```
Out[21]: y      239
         n      182
         NaN     14
         Name: anti_satellite_ban, dtype: int64
```

```
Out[21]: y      242
         n      178
         NaN     15
         Name: aid_to_nicaraguan_contras, dtype: int64
```

```
Out[21]: y      207
         n      206
         NaN     22
         Name: mx_missile, dtype: int64
```

```
Out[21]: y      216
         n      212
         NaN      7
         Name: immigration, dtype: int64
```

```
Out[21]: n      264
         y      150
         NaN     21
         Name: synfuels_cutback, dtype: int64
```

```
Out[21]: n      233
         y      171
         NaN     31
         Name: education_spending, dtype: int64
```

```
Out[21]: y      209
         n      201
         NaN     25
         Name: superfund, dtype: int64
```

```
Out[21]: y      248
         n      170
         NaN     17
         Name: crime, dtype: int64
```

```
Out[21]: n      233
         y      174
         NaN     28
         Name: duty_free_exports, dtype: int64
```

```
Out[21]: y      269
         NaN    104
         n       62
         Name: export_South_Africa, dtype: int64
```

1 = democrat, 0 = republican Note "unknown dispositions" have been treated as missing values!

The naive bayes implementations of scikit-learn do not allow missing values and also need binary data, so we will preprocess first changing y for 1 and n for 0 and then we perform missing data imputation. Another option would be to eliminate all rows with missing, but that will discard half of the data

```
In [22]: HouseVotes84.replace({'y':1, 'n':0},inplace=True)
         HouseVotes84.head()
```

```
Out[22]:        Class  handicapped_infants  water_project_sharing  budget_resolution  \
         0  republican                  0.0                    1.0                0.0
         1  republican                  0.0                    1.0                0.0
         2    democrat                  NaN                    1.0                1.0
         3    democrat                  0.0                    1.0                1.0
         4    democrat                  1.0                    1.0                1.0

            physician_fee_freeze  el_salvador_aid  religious_groups_in_schools  \
         0                   1.0              1.0                          1.0
         1                   1.0              1.0                          1.0
         2                   NaN              1.0                          1.0
         3                   0.0              NaN                          1.0
         4                   0.0              1.0                          1.0

            anti_satellite_ban  aid_to_nicaraguan_contras  mx_missile  immigration  \
         0                 0.0                        0.0         0.0          1.0
         1                 0.0                        0.0         0.0          0.0
         2                 0.0                        0.0         0.0          0.0
         3                 0.0                        0.0         0.0          0.0
         4                 0.0                        0.0         0.0          0.0

            synfuels_cutback  education_spending  superfund  crime  duty_free_exports  \
         0               NaN                 1.0        1.0    1.0                0.0
         1               0.0                 1.0        1.0    1.0                0.0
         2               1.0                 0.0        1.0    1.0                0.0
         3               1.0                 0.0        1.0    0.0                0.0
         4               1.0                 NaN        1.0    1.0                1.0

            export_South_Africa
```

```
0                    1.0
1                    NaN
2                    0.0
3                    1.0
4                    1.0
```

We use the most frequent value from each column for imputation

```
In [23]: HouseVotes84.loc[:,'handicapped_infants':] =  Imputer(strategy='most_frequent').\
                         fit_transform(HouseVotes84.loc[:,'handicapped_infants':])
         HouseVotes84.head()
```

```
Out[23]:         Class  handicapped_infants  water_project_sharing  budget_resolution  \
         0  republican                  0.0                    1.0                0.0
         1  republican                  0.0                    1.0                0.0
         2    democrat                  0.0                    1.0                1.0
         3    democrat                  0.0                    1.0                1.0
         4    democrat                  1.0                    1.0                1.0

            physician_fee_freeze  el_salvador_aid  religious_groups_in_schools  \
         0                   1.0              1.0                          1.0
         1                   1.0              1.0                          1.0
         2                   0.0              1.0                          1.0
         3                   0.0              1.0                          1.0
         4                   0.0              1.0                          1.0

            anti_satellite_ban  aid_to_nicaraguan_contras  mx_missile  immigration  \
         0                 0.0                        0.0         0.0          1.0
         1                 0.0                        0.0         0.0          0.0
         2                 0.0                        0.0         0.0          0.0
         3                 0.0                        0.0         0.0          0.0
         4                 0.0                        0.0         0.0          0.0

            synfuels_cutback  education_spending  superfund  crime  duty_free_exports  \
         0               0.0                 1.0        1.0    1.0                0.0
         1               0.0                 1.0        1.0    1.0                0.0
         2               1.0                 0.0        1.0    1.0                0.0
         3               1.0                 0.0        1.0    0.0                0.0
         4               1.0                 0.0        1.0    1.0                1.0

            export_South_Africa
         0                  1.0
         1                  1.0
         2                  0.0
         3                  1.0
         4                  1.0
```

```
In [24]: np.random.seed(1111)
         N = HouseVotes84.shape[0]
```

We first split the available data into learning and test sets, selecting randomly 2/3 and 1/3 of the data.

We do this for a honest estimation of prediction performance

```
In [25]: train, test = train_test_split(HouseVotes84, test_size=N//3)
```

We use the BernoulliNB estimator because we have binary data

```
In [26]: model = BernoulliNB().fit(train.loc[:,'handicapped_infants':],
                                   train.Class)
```

To obtain the probabiblities from the model is a little bit tricky.

The attribute `class_log_prior_` stores the priot log probabilities for the classes, so we can compute the probabilities doing:

```
In [27]: np.e**model.class_log_prior_

Out[27]: array([0.62068966, 0.37931034])
```

For the attributes/class probabilities is trickier because ony one of the probabilities is stored (the othe is the complement) and also are the log probabilities

```
In [28]: probs=pd.DataFrame({'Democrat Y':np.e**model.feature_log_prob_.T[:,0],
                             'Democrat N':1-np.e**model.feature_log_prob_.T[:,0],
                             'Republican Y':np.e**model.feature_log_prob_.T[:,1],
                             'Republican N':1-np.e**model.feature_log_prob_.T[:,1]},
                             index=HouseVotes84.columns[1:])

         probs

Out[28]:                               Democrat Y  Democrat N  Republican Y  \
         handicapped_infants                0.533       0.467         0.205
         water_project_sharing              0.549       0.451         0.518
         budget_resolution                  0.896       0.104         0.196
         physician_fee_freeze               0.055       0.945         0.955
         el_salvador_aid                    0.236       0.764         0.929
         religious_groups_in_schools        0.511       0.489         0.875
         anti_satellite_ban                 0.775       0.225         0.313
         aid_to_nicaraguan_contras          0.835       0.165         0.259
         mx_missile                         0.797       0.203         0.170
         immigration                        0.505       0.495         0.545
         synfuels_cutback                   0.473       0.527         0.116
         education_spending                 0.143       0.857         0.750
         superfund                          0.352       0.648         0.839
         crime                              0.363       0.637         0.982
         duty_free_exports                  0.610       0.390         0.125
         export_South_Africa                0.962       0.038         0.679

                                      Republican N
```

```
handicapped_infants                    0.795
water_project_sharing                  0.482
budget_resolution                      0.804
physician_fee_freeze                   0.045
el_salvador_aid                        0.071
religious_groups_in_schools            0.125
anti_satellite_ban                     0.687
aid_to_nicaraguan_contras              0.741
mx_missile                             0.830
immigration                            0.455
synfuels_cutback                       0.884
education_spending                     0.250
superfund                              0.161
crime                                  0.018
duty_free_exports                      0.875
export_South_Africa                    0.321
```

predict the outcome of the first 20 Congressmen

```
In [29]: model.predict(HouseVotes84.loc[0:20,'handicapped_infants':])
```

```
Out[29]: array(['republican', 'republican', 'republican', 'democrat', 'democrat',
                'republican', 'republican', 'republican', 'republican', 'democrat',
                'republican', 'republican', 'democrat', 'democrat', 'republican',
                'republican', 'democrat', 'democrat', 'republican', 'democrat',
                'democrat'], dtype='<U10')
```

```
In [30]: pred=pd.DataFrame(model.predict_proba(HouseVotes84.loc[0:20,'handicapped_infants':]))

         pred.columns=['democrat', 'republican']
         pred
```

Out[30]:

	democrat	republican
0	2.084e-07	1.000e+00
1	2.439e-07	1.000e+00
2	3.163e-02	9.684e-01
3	9.740e-01	2.604e-02
4	9.492e-01	5.077e-02
5	3.829e-01	6.171e-01
6	4.804e-05	1.000e+00
7	4.390e-06	1.000e+00
8	2.439e-07	1.000e+00
9	1.000e+00	7.242e-09
10	2.483e-06	1.000e+00
11	2.995e-05	1.000e+00
12	1.000e+00	3.079e-07
13	1.000e+00	7.603e-10
14	2.439e-07	1.000e+00
15	2.084e-07	1.000e+00

```
16  1.000e+00   1.239e-05
17  1.000e+00   6.372e-09
18  3.707e-07   1.000e+00
19  1.000e+00   9.700e-11
20  1.000e+00   1.246e-08
```

form and display confusion matrix & overall error

```
In [31]: confusion(train.Class, model.predict(train.loc[:,'handicapped_infants':]),
                   classes=house_classes)

         (1-accuracy_score(train.Class,
                           model.predict(train.loc[:,'handicapped_infants':])))*100
```

```
Out[31]: Predicted    n    y
         Actual
         n          159   21
         y            9  101
```

```
Out[31]: 10.344827586206895
```

compute the test (prediction) error

```
In [32]: confusion(test.Class,
                   model.predict(test.loc[:,'handicapped_infants':]),
                   classes=house_classes)

         (1-accuracy_score(test.Class,
                           model.predict(test.loc[:,'handicapped_infants':])))*100
```

```
Out[32]: Predicted   n   y
         Actual
         n          77  10
         y           2  56
```

```
Out[32]: 8.275862068965523
```

note how most errors (10/12) correspond to democrats wrongly predicted as republicans

in the event of **empty empirical probabilities**, there is an alpha parameter (0-1) that can be use for performing Laplace correction (aka smoothing) (0 = no smoothing)

```
In [33]: model = BernoulliNB(alpha=0.9).fit(train.loc[:,'handicapped_infants':],
                                            train.Class)
```

1.3 Example 3: The kNN classifier

We are going to use the famous (Fisher's or Anderson's) Iris data set, which gives the measurements in centimeters of the sepal length and width and petal length and width, respectively, for 50 flowers from each of 3 species of Iris. The species are Iris setosa, versicolor, and virginica.

In [34]:
```
np.random.seed(2)

iris_data, iris_labels = load_iris(return_X_y=True)
iris_names = ['setosa', 'versicolor', 'virginica']
```

first we split a separate test set of relative size 30%

In [35]:
```
X_train, X_test, y_train, y_test = train_test_split(iris_data,
                                                    iris_labels,
                                                    test_size=0.3)
```

setup a kNN model with 3 neighbours Notice there is no "learning" ... the data is the model (just test!)

In [36]:
```
myknn = KNeighborsClassifier(n_neighbors=3).fit(X_train, y_train)
confusion(y_test, myknn.predict(X_test), classes=iris_names)
```

Out[36]:
```
Predicted   setosa  versicolor  virginica
Actual
setosa          17           0          0
versicolor       0          15          0
virginica        0           0         13
```

rows are predictions, columns are true test targets
How do we optimize k? One way is by using LOOCV

In [37]:
```
cm, err = loocv(X_train,y_train,myknn, classes=iris_names)

cm

err*100
```

Out[37]:
```
Predicted   setosa  versicolor  virginica
Actual
setosa          33           0          0
versicolor       0          32          3
virginica        0           3         34
```

Out[37]:
```
5.714285714285716
```

aha! now you see that previous training error (0%) was a little bit optimistic
Let's loop over k

In [38]:
```
np.random.seed(4321)

errors = []
for i in range(1,int(np.sqrt(X_train.shape[0]))+1):
    myknn_cv = KNeighborsClassifier(n_neighbors=i)
    _, error = loocv(X_train,y_train,myknn_cv, classes=iris_names)
    errors.append(error)

pd.DataFrame({'K':range(1,int(np.sqrt(X_train.shape[0]))+1),
              'LOOCV error':errors})
```

Out[38]:

	K	LOOCV error
0	1	0.067
1	2	0.086
2	3	0.057
3	4	0.076
4	5	0.076
5	6	0.057
6	7	0.048
7	8	0.048
8	9	0.029
9	10	0.029

It seems that k=9 is the best value.
Now we *refit* with k=9 and predict the test set

In [39]:
```
myknn = KNeighborsClassifier(n_neighbors=9).fit(X_train, y_train)
confusion(y_test, myknn.predict(X_test), classes=iris_names)
(1-accuracy_score(y_test, myknn.predict(X_test)))*100
```

Out[39]:

Predicted	setosa	versicolor	virginica
Actual			
setosa	17	0	0
versicolor	0	14	1
virginica	0	0	13

Out[39]: 2.2222222222222254

so our error is 2.2%

1.4 Example 4: Logistic Regression using artificial data

The goal of this example is to get acquainted with the call to glm() glm() is used to fit generalized linear models (of which both linear and logistic regression are particular cases)

You may need to recall at this point the logistic regression model ...

Let x represent a single continuous predictor

Let y represent a class ('0' or '1'), with a probability of being 1 that is related linearly to the predictor via the logit funtion, that is $logit(p) = a * x + b$ (or $beta_1 * x + beta_0$ if you prefer)

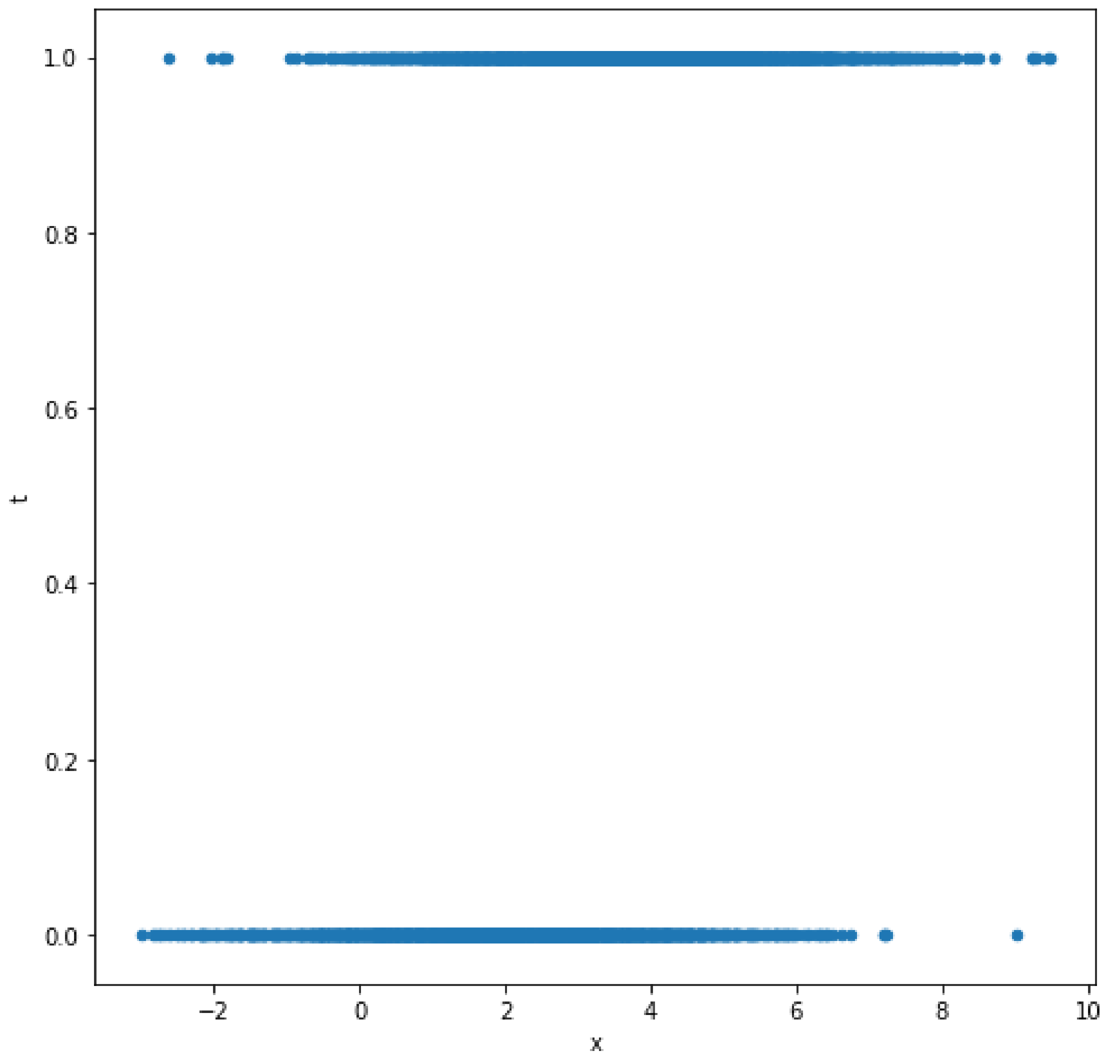
In [40]:
```
np.random.seed(1968)

N = 4000
x = normal (3,2,N)  # generate the x_n

a = 0.6
b = -1.5 # this is the ground truth, which is unknown

p = 1/(1+np.exp( -(a*x + b) )) # generate the p_n
t = binomial(1,p, N)  # generate the targets according to p
data = pd.DataFrame({'x':x, 't':t})

data.plot.scatter('x','t',figsize=(8,8));
```



In [41]:
```
model = GLM.from_formula('t ~ x', data, family=Binomial())
```

```
result = model.fit()
result.summary()
```

Out[41]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                 Generalized Linear Model Regression Results
==============================================================================
Dep. Variable:                      t   No. Observations:                 4000
Model:                            GLM   Df Residuals:                     3998
Model Family:                Binomial   Df Model:                            1
Link Function:                  logit   Scale:                          1.0000
Method:                          IRLS   Log-Likelihood:                -2248.1
Date:                Thu, 06 Sep 2018   Deviance:                       4496.1
Time:                        14:32:49   Pearson chi2:                 3.99e+03
No. Iterations:                     4   Covariance Type:             nonrobust
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept     -1.5217      0.075    -20.278      0.000      -1.669      -1.375
x              0.6040      0.023     26.463      0.000       0.559       0.649
==============================================================================
"""
```

Obviously x is very significant (and the Intercept is always significant)

Therefore, our estimated model is $logit(p_n) ={{result.params[1]}}*x_n {{result.params[0]}} quite close to the ground truth

In general you get this as:

result.params

Interpretation of the coefficients:

- For a 1 unit increase in x, there is an increase in the odds for t by a factor of ...

In [42]:
```
result.params
```

Out[42]:
```
Intercept   -1.522
x            0.604
dtype: float64
```

In [43]:
```
np.exp(result.params[1])
```

Out[43]:
```
1.8293462220130607
```

that is almost doubling the odds (~82% more)

1.5 Example 5: Logistic regression for classifying spam mail

This example will also illustrate how to change the 'cut point' for prediction, when there is an interest in minimizing a particular source of errors

In [44]:
```
spam = read_csv("spambase.data", delimiter=',', header=None)
file = open('spambase.names', 'r')
spam.columns = [n.strip() for n in file.readlines()]

spam.head()
```

Out[44]:
```
   word_freq_make  word_freq_address  word_freq_all  word_freq_3d  \
0            0.00               0.64           0.64           0.0
1            0.21               0.28           0.50           0.0
2            0.06               0.00           0.71           0.0
3            0.00               0.00           0.00           0.0
4            0.00               0.00           0.00           0.0

   word_freq_our  word_freq_over  word_freq_remove  word_freq_internet  \
0           0.32            0.00              0.00                0.00
1           0.14            0.28              0.21                0.07
2           1.23            0.19              0.19                0.12
3           0.63            0.00              0.31                0.63
4           0.63            0.00              0.31                0.63

   word_freq_order  word_freq_mail  ...    char_freq_;  char_freq_(  \
0             0.00            0.00  ...           0.00        0.000
1             0.00            0.94  ...           0.00        0.132
2             0.64            0.25  ...           0.01        0.143
3             0.31            0.63  ...           0.00        0.137
4             0.31            0.63  ...           0.00        0.135

   char_freq_[  char_freq_!  char_freq_$  char_freq_#  \
0          0.0        0.778        0.000        0.000
1          0.0        0.372        0.180        0.048
2          0.0        0.276        0.184        0.010
3          0.0        0.137        0.000        0.000
4          0.0        0.135        0.000        0.000

   capital_run_length_average  capital_run_length_longest  \
0                       3.756                          61
1                       5.114                         101
2                       9.821                         485
3                       3.537                          40
4                       3.537                          40

   capital_run_length_total  Class
0                       278      1
1                      1028      1
```

```
2                          2259      1
3                           191      1
4                           191      1

[5 rows x 58 columns]
```

We do some basic pre-processing

```
In [45]: spam.loc[:,'capital_run_length_average':'capital_run_length_total'] =\
                 spam.loc[:,'capital_run_length_average':'capital_run_length_total'].\
                         apply(lambda x: np.log10(x+1))
         spam = spam[spam.word_freq_george==0]
         spam = spam[spam.word_freq_650==0]
         spam = spam[spam.word_freq_hp==0]
         spam = spam[spam.word_freq_hpl==0]
         spam =spam.drop(columns=['word_freq_george','word_freq_650',
                                  'word_freq_hp','word_freq_hpl'])
         spam['about_money']=spam.word_freq_free+spam.word_freq_business+\
         spam.word_freq_credit+spam.word_freq_money
         spam=spam.drop(columns=['word_freq_free','word_freq_business',
                                 'word_freq_credit','word_freq_money'])
         Class = spam.Class   # move the Class column to the last position
         spam=spam.drop(columns=['Class'])
         spam['Class'] = Class

         spam.shape

Out[45]: (2999, 51)

In [46]: np.random.seed(4321)
         train, test = train_test_split(spam, test_size=0.33)
```

Fit a GLM in the learning data

```
In [47]: spamM1 = GLM(train.Class,
                      add_constant(train.loc[:,:'about_money']),
                      family=Binomial())
         resultM1 = spamM1.fit()
         resultM1.summary()

/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:880: RuntimeWarni
  n_endog_mu = self._clean((1. - endog) / (1. - mu))
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/links.py:167: RuntimeWarnin
  t = np.exp(-z)
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:879: RuntimeWarni
  endog_mu = self._clean(endog / mu)
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:932: RuntimeWarni
  special.gammaln(n - y + 1) + y * np.log(mu / (1 - mu)) +
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:932: RuntimeWarni
```

```
  special.gammaln(n - y + 1) + y * np.log(mu / (1 - mu)) +
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:932: RuntimeWarni
  special.gammaln(n - y + 1) + y * np.log(mu / (1 - mu)) +
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:933: RuntimeWarni
  n * np.log(1 - mu)) * var_weights
/usr/local/lib64/python3.6/site-packages/statsmodels/genmod/families/family.py:933: RuntimeWarni
  n * np.log(1 - mu)) * var_weights
```

```
Out[47]: <class 'statsmodels.iolib.summary.Summary'>
         """
                          Generalized Linear Model Regression Results
         ==============================================================================
         Dep. Variable:                  Class   No. Observations:                 2009
         Model:                            GLM   Df Residuals:                     1958
         Model Family:                Binomial   Df Model:                           50
         Link Function:                  logit   Scale:                          1.0000
         Method:                          IRLS   Log-Likelihood:                    nan
         Date:                Thu, 06 Sep 2018   Deviance:                          nan
         Time:                        14:32:54   Pearson chi2:                 2.24e+04
         No. Iterations:                   100   Covariance Type:             nonrobust
         =================================================================================
                                      coef    std err          z      P>|z|      [0.025
         ---------------------------------------------------------------------------------
         const                     -5.0624      0.464    -10.904      0.000      -5.972
         word_freq_make            -0.1299      0.293     -0.443      0.658      -0.705
         word_freq_address         -0.1619      0.131     -1.239      0.216      -0.418
         word_freq_all             -0.0884      0.167     -0.531      0.595      -0.415
         word_freq_3d             350.6953    8.8e+05      0.000      1.000   -1.73e+06    1
         word_freq_our              0.5344      0.120      4.464      0.000       0.300
         word_freq_over             0.6943      0.353      1.965      0.049       0.002
         word_freq_remove           1.6118      0.329      4.895      0.000       0.966
         word_freq_internet         0.5349      0.148      3.621      0.000       0.245
         word_freq_order           -0.1082      0.304     -0.356      0.722      -0.703
         word_freq_mail             0.0561      0.090      0.625      0.532      -0.120
         word_freq_receive         -0.5076      0.380     -1.334      0.182      -1.253
         word_freq_will            -0.1398      0.107     -1.308      0.191      -0.349
         word_freq_people          -0.0531      0.334     -0.159      0.874      -0.708
         word_freq_report           0.0120      0.307      0.039      0.969      -0.590
         word_freq_addresses        0.8487      0.802      1.058      0.290      -0.724
         word_freq_email            0.1439      0.147      0.978      0.328      -0.144
         word_freq_you              0.1415      0.051      2.782      0.005       0.042
         word_freq_your             0.1381      0.074      1.870      0.061      -0.007
         word_freq_font             0.1757      0.279      0.630      0.529      -0.371
         word_freq_000              1.0192      0.468      2.176      0.030       0.101
         word_freq_lab            -12.4533     10.458     -1.191      0.234     -32.950
         word_freq_labs            -1.6974      1.046     -1.623      0.105      -3.748
         word_freq_telnet          -0.2793      1.874     -0.149      0.882      -3.952
```

```
word_freq_857                   -0.5357      3.463     -0.155      0.877      -7.323
word_freq_data                  -0.4946      0.292     -1.696      0.090      -1.066
word_freq_415                  424.0571   4.52e+07   9.38e-06      1.000   -8.86e+07     8
word_freq_85                    -1.8880      1.518     -1.244      0.214      -4.863
word_freq_technology             0.8304      0.547      1.519      0.129      -0.241
word_freq_1999                   0.2841      0.237      1.197      0.231      -0.181
word_freq_parts                  0.3967      2.682      0.148      0.882      -4.861
word_freq_pm                    -1.1317      0.562     -2.015      0.044      -2.233
word_freq_direct                -0.3583      0.445     -0.806      0.421      -1.230
word_freq_cs                  -354.6653   3.85e+06  -9.21e-05      1.000   -7.55e+06     7
word_freq_meeting               -6.4294      2.328     -2.762      0.006     -10.993
word_freq_original              -1.4478      1.044     -1.387      0.165      -3.493
word_freq_project               -1.7884      0.827     -2.164      0.030      -3.408
word_freq_re                    -0.7498      0.195     -3.842      0.000      -1.132
word_freq_edu                   -4.7957      0.948     -5.059      0.000      -6.654
word_freq_table                 -1.4227      2.442     -0.583      0.560      -6.210
word_freq_conference            -4.2235      1.523     -2.773      0.006      -7.209
char_freq_;                     -1.5962      0.696     -2.295      0.022      -2.960
char_freq_(                     -0.6276      0.413     -1.519      0.129      -1.437
char_freq_[                     -0.0500      0.979     -0.051      0.959      -1.969
char_freq_!                      0.2124      0.061      3.471      0.001       0.092
char_freq_$                      5.7022      1.098      5.193      0.000       3.550
char_freq_#                      1.2748      1.654      0.771      0.441      -1.966
capital_run_length_average       0.7272      0.786      0.925      0.355      -0.813
capital_run_length_longest       0.5299      0.448      1.182      0.237      -0.349
capital_run_length_total         1.8490      0.311      5.948      0.000       1.240
about_money                      0.6960      0.117      5.926      0.000       0.466
==========================================================================================
"""
```

We can see that there are some variables that have small weights and are probably not very relevant. The R notebook uses stepwise variable selection to simplify the model.

Statsmodels does not have stepwise variable selection, but we can use crossvalidated Recursive Forward Elimination (RFE) with the implementation of logistic regression from scikit learn. RFE does the same thing as stepwise variable selection but uses accuracy to select the best model using cross validation. The implentation of logistic regression in scikit-learn is more sofisticated and uses regularization so the results will be different than in the R notebook.

In [48]:
```
# we use L1 regularization to make 0 a large number
# of the weigths, the lower the C the more attributes will be discarded
logreg = LogisticRegression(solver='liblinear',penalty='l1',C=1)
#njobs = -1 means that all the cores from the CPU are used
rfe = RFECV(estimator=logreg,cv=10,n_jobs=-1)
rfe.fit(train.loc[:,:'about_money'],train.Class);
```

In [49]:
```
print('Features Selected:',rfe.n_features_)
print('\n Ranking of features')
sel = pd.DataFrame({'features': train.columns[:-1],
```

```
                         'ranking': rfe.ranking_,
                         'selected':rfe.support_})
    sel.sort_values(by='ranking')
```

```
Features Selected: 38

 Ranking of features
```

Out[49]:

	features	ranking	selected
24	word_freq_data	1	True
27	word_freq_technology	1	True
28	word_freq_1999	1	True
30	word_freq_pm	1	True
31	word_freq_direct	1	True
32	word_freq_cs	1	True
33	word_freq_meeting	1	True
34	word_freq_original	1	True
35	word_freq_project	1	True
26	word_freq_85	1	True
36	word_freq_re	1	True
39	word_freq_conference	1	True
40	char_freq_;	1	True
41	char_freq_(	1	True
43	char_freq_!	1	True
44	char_freq_$	1	True
45	char_freq_#	1	True
46	capital_run_length_average	1	True
47	capital_run_length_longest	1	True
37	word_freq_edu	1	True
48	capital_run_length_total	1	True
49	about_money	1	True
11	word_freq_will	1	True
10	word_freq_receive	1	True
21	word_freq_labs	1	True
5	word_freq_over	1	True
4	word_freq_our	1	True
14	word_freq_addresses	1	True
15	word_freq_email	1	True
6	word_freq_remove	1	True
16	word_freq_you	1	True
17	word_freq_your	1	True
18	word_freq_font	1	True
19	word_freq_000	1	True
20	word_freq_lab	1	True
1	word_freq_address	1	True
3	word_freq_3d	1	True
7	word_freq_internet	1	True

```
9                 word_freq_mail        2     False
2                  word_freq_all        3     False
0                 word_freq_make        4     False
42                   char_freq_[        5     False
29               word_freq_parts        6     False
8                word_freq_order        7     False
38               word_freq_table        8     False
13              word_freq_report        9     False
23                 word_freq_857       10     False
22              word_freq_telnet       11     False
25                 word_freq_415       12     False
12              word_freq_people       13     False
```

We get the extimator from the RFE and the list of selected variable to slice the data matrix

```
In [50]: resultM1 = rfe.estimator_
         sel_features = list(sel.features[sel.selected])
         sel_features
```

```
Out[50]: ['word_freq_address',
          'word_freq_3d',
          'word_freq_our',
          'word_freq_over',
          'word_freq_remove',
          'word_freq_internet',
          'word_freq_receive',
          'word_freq_will',
          'word_freq_addresses',
          'word_freq_email',
          'word_freq_you',
          'word_freq_your',
          'word_freq_font',
          'word_freq_000',
          'word_freq_lab',
          'word_freq_labs',
          'word_freq_data',
          'word_freq_85',
          'word_freq_technology',
          'word_freq_1999',
          'word_freq_pm',
          'word_freq_direct',
          'word_freq_cs',
          'word_freq_meeting',
          'word_freq_original',
          'word_freq_project',
          'word_freq_re',
          'word_freq_edu',
          'word_freq_conference',
```

```
         'char_freq_;',
         'char_freq_(',
         'char_freq_!',
         'char_freq_$',
         'char_freq_#',
         'capital_run_length_average',
         'capital_run_length_longest',
         'capital_run_length_total',
         'about_money']
```

We define now a convenience function:

'P' is a parameter; whenever our filter assigns spam with probability at least P then we predict spam

```
In [51]: def spam_acc(P=0.5):
             # We use predict_proba instead of prediction to obtain
             # the probabilities of the classes and
             # we select only the probability for class 1 as
             # the other is just the complementary

             # Accuracy in training
             pred = resultM1.predict_proba(train.loc[:,sel_features])[:,1]
             lab_tr = [1 if i>=P else 0 for i in pred]
             df_tr=confusion(train.Class,lab_tr, classes=['nospam','spam'])

             # Accuracy in test
             pred = resultM1.predict_proba(test.loc[:,sel_features])[:,1]
             lab_ts = [1 if i>=P else 0 for i in pred]
             df_ts=confusion(test.Class,lab_ts, classes=['nospam','spam'])

             return df_tr, (1-accuracy_score(train.Class,lab_tr))*100,\
                    df_ts, (1-accuracy_score(test.Class,lab_ts))*100
```

```
In [52]: c_tr,e_tr,c_ts,e_ts= spam_acc()
         c_tr
         print(f'Training error: {e_tr}%')
         c_ts
         print(f'Test error: {e_ts}%')
```

```
Out[52]: Predicted  nospam  spam
         Actual
         nospam        759    90
         spam           67  1093
```

```
Training error: 7.81483325037332%
```

```
Out[52]: Predicted  nospam  spam
         Actual
```

```
        nospam        382    41
        spam           32   535

Test error: 7.373737373737377%
```

Although the errors are quite low still one could argue that we should try to lower the probability of predicting spam when it is not We can do this (at the expense of increasing the converse probability) by:

```
In [53]: c_tr,e_tr,c_ts,e_ts= spam_acc(0.7)
         c_tr
         print(f'Training error: {e_tr}%')
         c_ts
         print(f'Test error: {e_ts}%')

Out[53]: Predicted  nospam  spam
         Actual
         nospam        801    48
         spam          156  1004

Training error: 10.154305624688897%


Out[53]: Predicted  nospam  spam
         Actual
         nospam        403    20
         spam           78   489

Test error: 9.8989898989896%
```

So we get a much better spam filter; notice that the filter has a very low probability of predicting spam when it is not (which is the delicate case), of about

```
In [54]: c_ts.loc['nospam','spam'] /c_ts.loc['nospam'].sum()*100

Out[54]: 4.7281323877068555
```