APA-L7-python

September 6, 2018

1 APA Laboratori 7 - SVMs

```
In [1]: # Uncomment to upgrade packages
        # !pip install pandas --upgrade
        # !pip install numpy --upgrade
        # !pip install scipy --upgrade
        # !pip install statsmodels --upgrade
        # !pip install scikit-learn --upgrade
        %load_ext autoreload
```

```
In [2]: #%matplotlib notebook
        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import matplotlib.cm as cm
        from IPython.core.interactiveshell import InteractiveShell
        pd.set_option('precision', 3)
        InteractiveShell.ast_node_interactivity = "all"
```

```
In [3]: # Exta imports
        from numpy.random import uniform, normal
        from sklearn.svm import SVC
        from sklearn.model_selection import KFold
        from sklearn.metrics import confusion_matrix, accuracy_score
        from sklearn.svm import SVR
```

```
In [4]: np.random.seed(7)
```

1.1 Modelling artificial 2D sinusoidal data for two-class problems

First we create a simple two-class data set:

```
In [5]: N = 200

        def make_sinusoidals(m,noise=0.2):
          x1 = np.ones(2*m)
          x2 = np.ones(2*m)
```

```
    for i in range(m):
      x1[i] = (i/m) * np.pi
      x2[i] = np.sin(x1[i]) + normal(0,noise,1)


    for j in range(m):
      x1[m+j] = (j/m + 1/2) * np.pi
      x2[m+j] = np.cos(x1[m+j]) + normal(0,noise,1)


    target = [1]*m+[-1]*m

    return pd.DataFrame({'x1':x1,'x2':x2,'t':target})
```

let's generate the data

```
In [6]: dataset = make_sinusoidals(N)
```

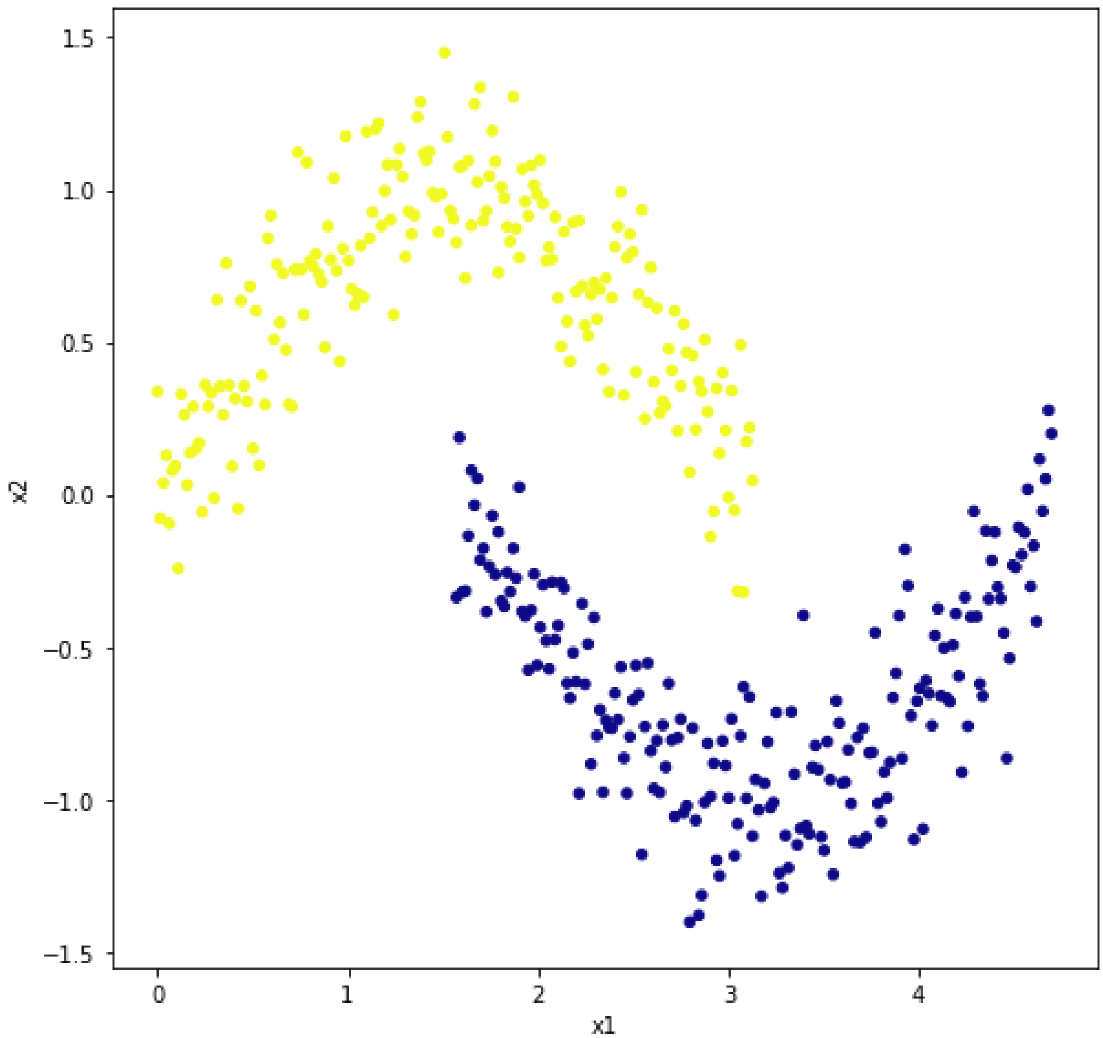
and have a look at it

```
In [7]: dataset.describe()
```

Out[7]:

	x1	x2	t
count	400.000	400.000	400.000
mean	2.348	-0.010	0.000
std	1.201	0.739	1.001
min	0.000	-1.402	-1.000
25%	1.567	-0.675	-1.000
50%	2.348	-0.040	0.000
75%	3.130	0.677	1.000
max	4.697	1.450	1.000

```
In [8]: dataset.plot.scatter(x='x1', y='x2', c='t',
                             colormap='plasma',
                             figsize=(8,8),
                             colorbar=False);
```



Now we wish to fit and visualize different SVM models

1.1.1 model 1: LINEAR kernel, C=1 (cost parameter)

```
In [9]: svc = SVC(C=1, kernel='linear')
        svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

        print('Suports=',svc.n_support_)
```

```
Out[9]: SVC(C=1, cache_size=200, class_weight=None, coef0=0.0,
          decision_function_shape='ovr', degree=3, gamma='auto', kernel='linear',
          max_iter=-1, probability=False, random_state=None, shrinking=True,
          tol=0.001, verbose=False)
```

```
Suports= [26 25]
```

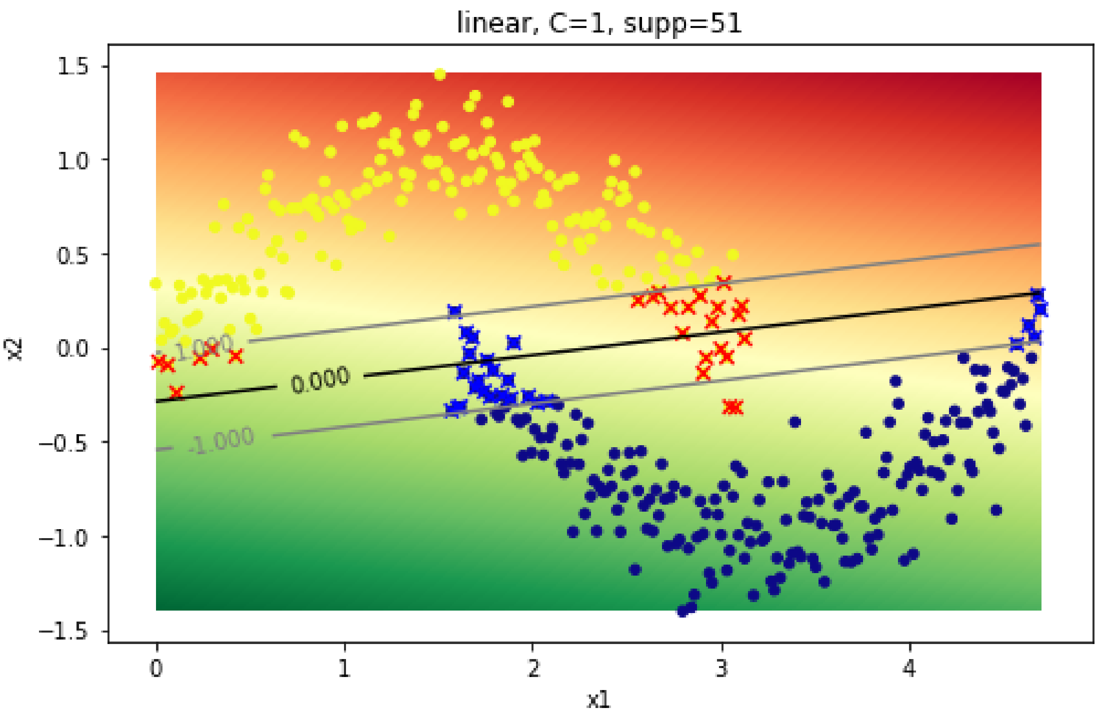
Now we are going to visualize what we have done; since we have artificial data, instead of creating a random test set, we can create a grid of points as test

```
In [10]: def plot_prediction(model, model_name, resol=200, ax=None):
             x_min, x_max = dataset.x1.min(), dataset.x1.max()
             y_min, y_max = dataset.x2.min(), dataset.x2.max()
             xx, yy = np.meshgrid(np.linspace(x_min, x_max, resol),
                                  np.linspace(y_min, y_max, resol))

             Z = model.decision_function(np.c_[xx.ravel(), yy.ravel()])
             Z = Z.reshape(xx.shape)
             if ax is None:
                 fig, ax = plt.subplots(figsize=(8,8))
             ax.imshow(Z, interpolation='bilinear',
                       cmap=cm.RdYlGn,
                       extent=[x_min, x_max, y_min,y_max])
             dataset.plot.scatter(x='x1', y='x2', c='t',
                                  colormap='plasma',
                                  colorbar=False,
                                  ax=ax,
                                  title=model_name+', supp=%d'%np.sum(model.n_support_))
             dataset.iloc[model.support_].plot.scatter(x='x1', y='x2', c='t',
                                                       colormap='bwr',
                                                       colorbar=False,
                                                       ax=ax,marker='x',
                                                       s=40)
             CS=ax.contour(xx, yy, Z, levels=[-1,0,1],colors=['grey','black','grey'])
             plt.clabel(CS, inline=1, fontsize=10)
```

make sure you understand the following results (one by one and their differences)

plot the data, the OSH with margins, the support vectors, ...

```
In [11]: plot_prediction(svc, 'linear, C=1')
```



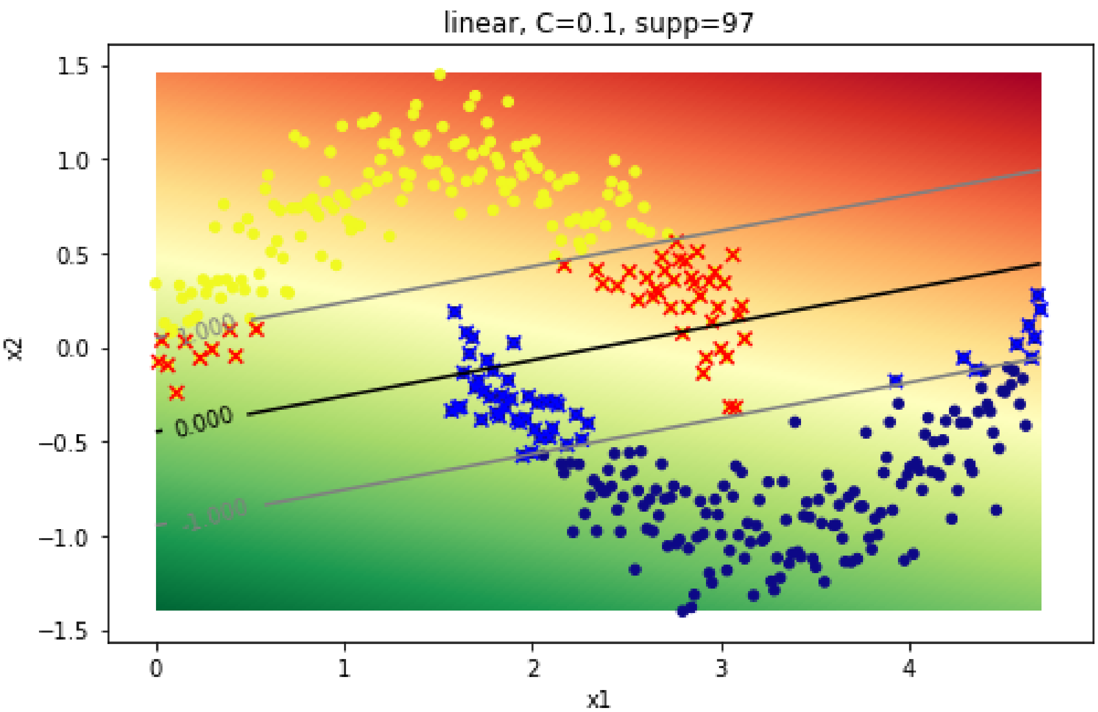
model 2: linear kernel, C=0.1 (cost parameter)

```
In [12]: svc = SVC(C=0.1, kernel='linear')
         svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

         print('Suports=',svc.n_support_)
```

```
Out[12]: SVC(C=0.1, cache_size=200, class_weight=None, coef0=0.0,
           decision_function_shape='ovr', degree=3, gamma='auto', kernel='linear',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)
```

```
Suports= [49 48]
```

```
In [13]: plot_prediction(svc, 'linear, C=0.1')
```



the margin is wider (lower VC dimension), number of support vectors is larger (more violations of the margin)

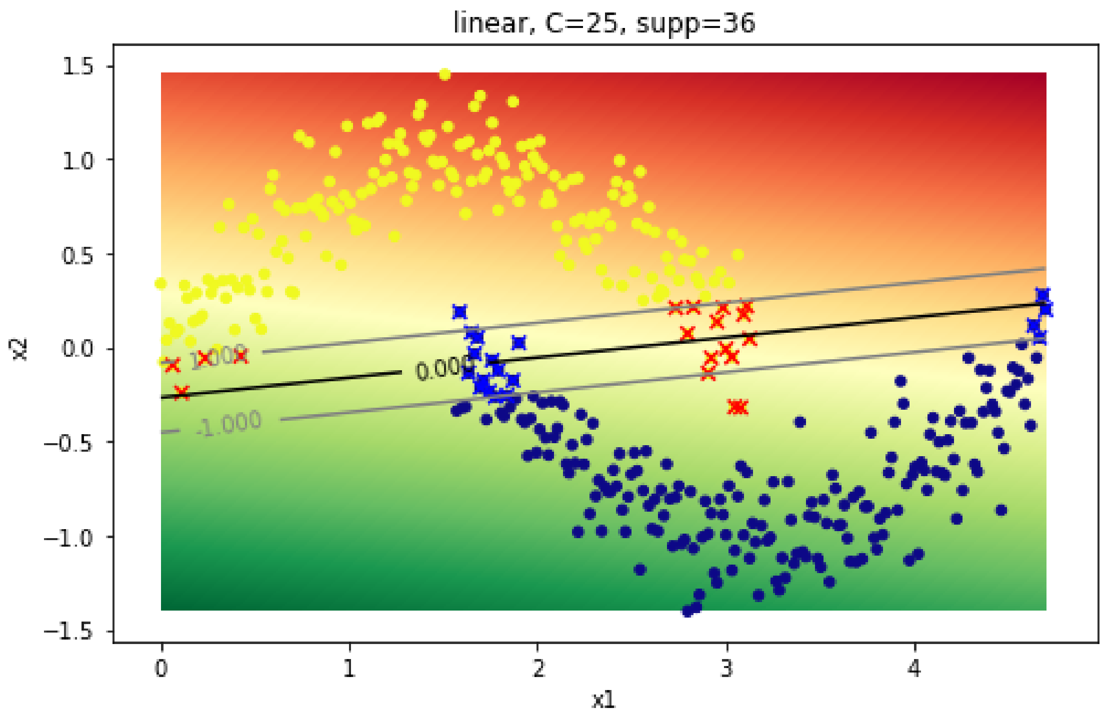
1.1.2 model 3: linear kernel, C=25 (cost parameter)

```
In [14]: svc = SVC(C=25, kernel='linear')
         svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

         print('Suports=',svc.n_support_)
```

```
Out[14]: SVC(C=25, cache_size=200, class_weight=None, coef0=0.0,
           decision_function_shape='ovr', degree=3, gamma='auto', kernel='linear',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)
```

```
Suports= [18 18]
```

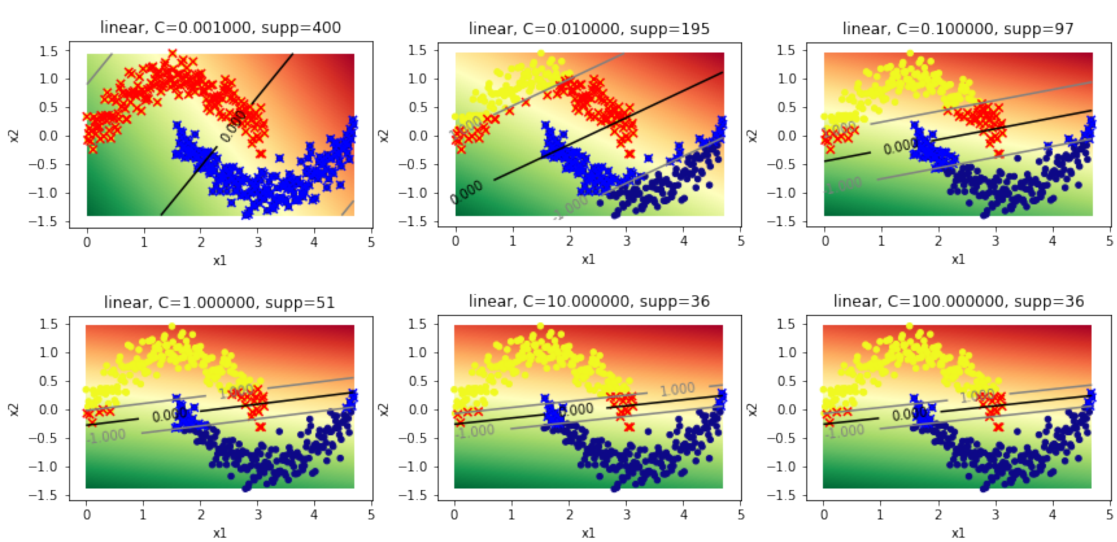
```
In [15]: plot_prediction(svc, 'linear, C=25')
```



the margin is narrower (higher VC dimension), number of support vectors is smaller (less violations of the margin)

Let's put it together, for 6 values of C:

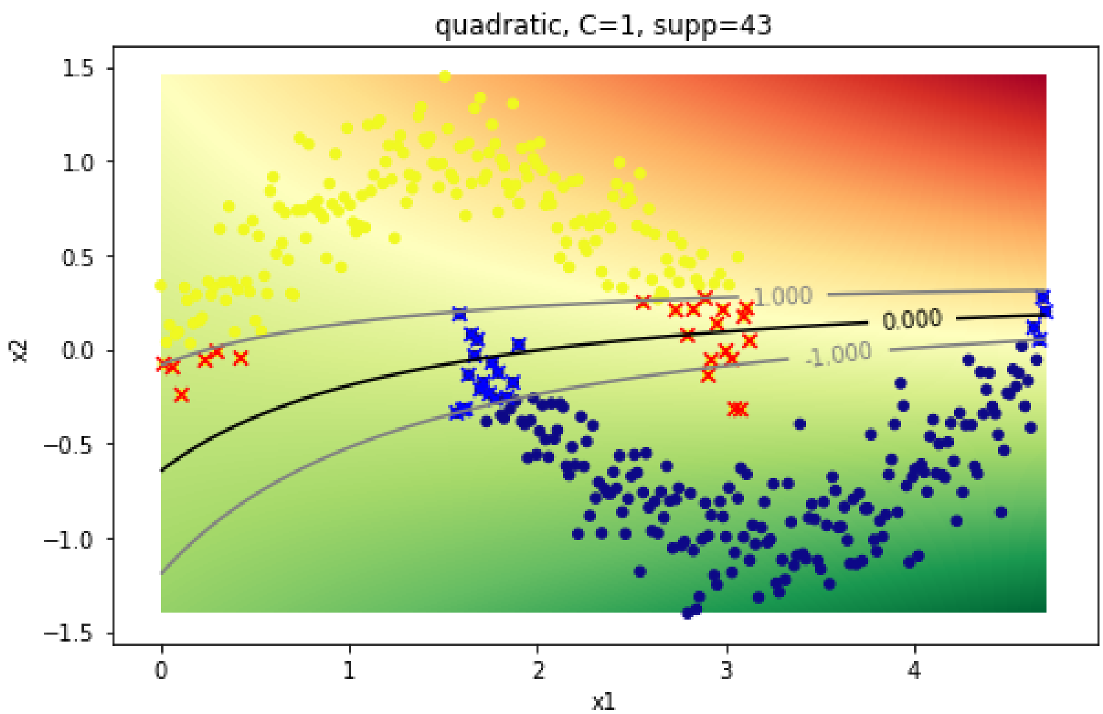
```
In [16]: fig, ax = plt.subplots(2,3,figsize=(12,6))
         for C, ax in zip(10**np.linspace(-3,2,6),ax.ravel()):
             svc = SVC(C=C, kernel='linear').fit(dataset.loc[:,'x1':'x2'], dataset.t)
             plot_prediction(svc, 'linear, C=%f'%C,ax=ax)
         plt.tight_layout()
```



Now we move to a QUADRATIC kernel (polynomial of degree 2); the kernel has the form: $k(x,y) = (< x,y > +coef0)^{degree}$
quadratic kernel, C=1 (cost parameter)

```
In [17]: svc = SVC(C=1, kernel='poly',degree=2,coef0=1)
         svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

         print('Suports=',svc.n_support_)

Out[17]: SVC(C=1, cache_size=200, class_weight=None, coef0=1,
           decision_function_shape='ovr', degree=2, gamma='auto', kernel='poly',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)

Suports= [21 22]
```

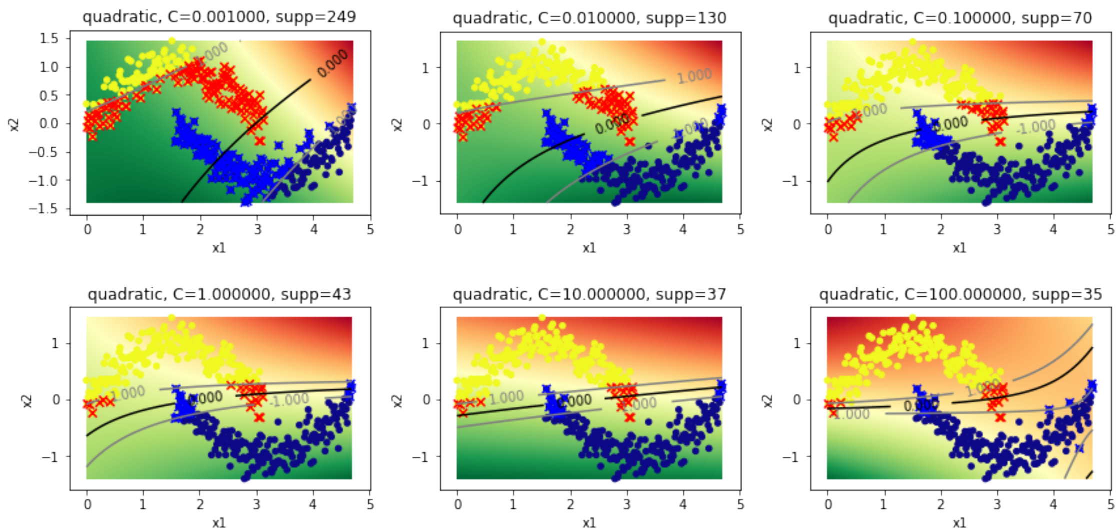
```
In [18]: plot_prediction(svc, 'quadratic, C=1')
```



notice that neither the OSH or the margins are linear (they are quadratic); they are linear in the feature space in the previous linear kernel, both spaces coincide

Let's put it together directly, for 6 values of C:

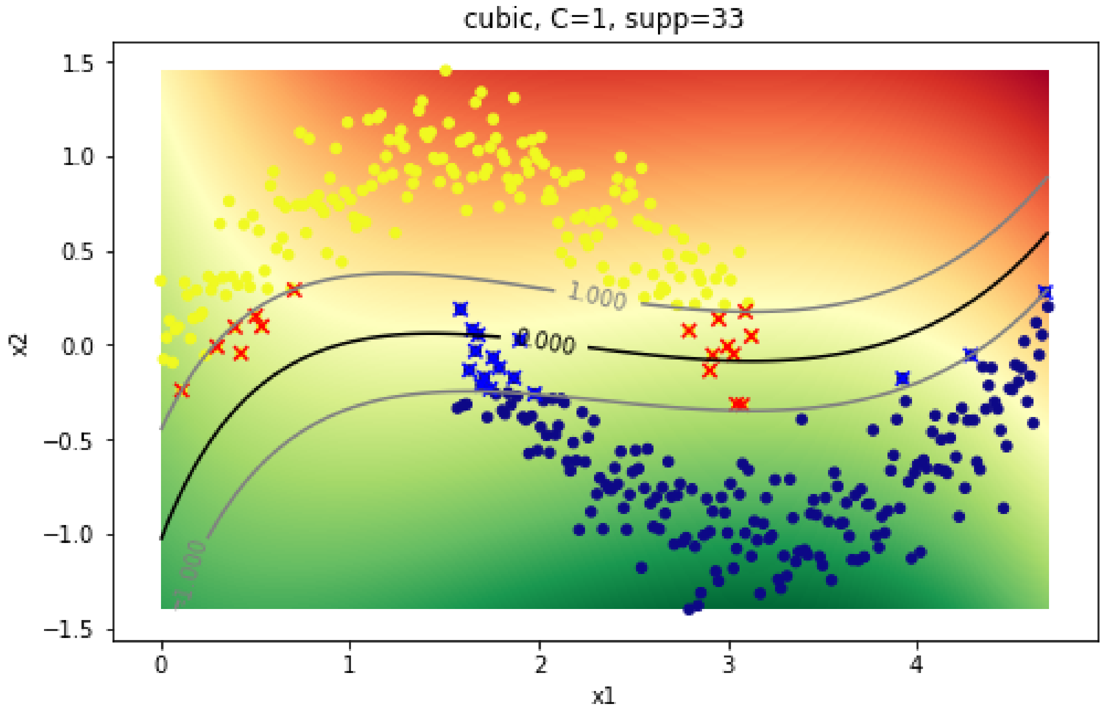
```
In [19]: fig, ax = plt.subplots(2,3,figsize=(12,6))
         for C, ax in zip(10**np.linspace(-3,2,6),ax.ravel()):
             svc = SVC(C=C,
                       kernel='poly',
                       degree=2,
                       coef0=1).fit(dataset.loc[:,'x1':'x2'],
                                    dataset.t)
             plot_prediction(svc, 'quadratic, C=%f'%C,ax=ax)
         plt.tight_layout()
```



Now we move to a CUBIC kernel (polynomial of degree 3); the kernel has the form:
$k(x,y) = (< x,y > +coef0)^{degree}$
cubic kernel, C=1 (cost parameter)

```
In [20]: svc = SVC(C=1, kernel='poly',degree=3,coef0=1)
         svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

         print('Suports=',svc.n_support_)

Out[20]: SVC(C=1, cache_size=200, class_weight=None, coef0=1,
           decision_function_shape='ovr', degree=3, gamma='auto', kernel='poly',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)

Suports= [16 17]
```

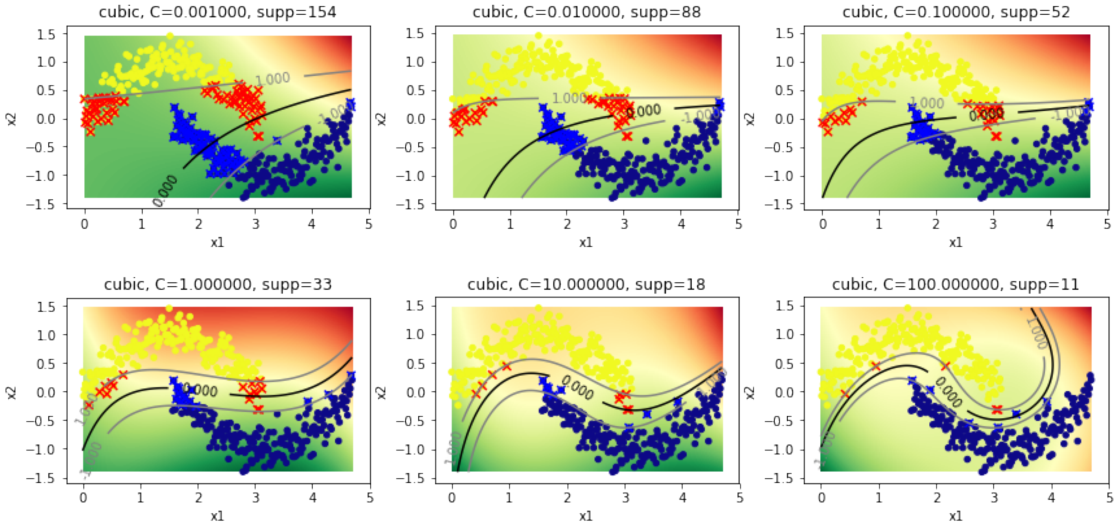
```
In [21]: plot_prediction(svc, 'cubic, C=1')
```



notice that neither the OSH or the margins are linear (they are now cubic); they are linear in the feature space this choice seems much better, given the structure of the classes

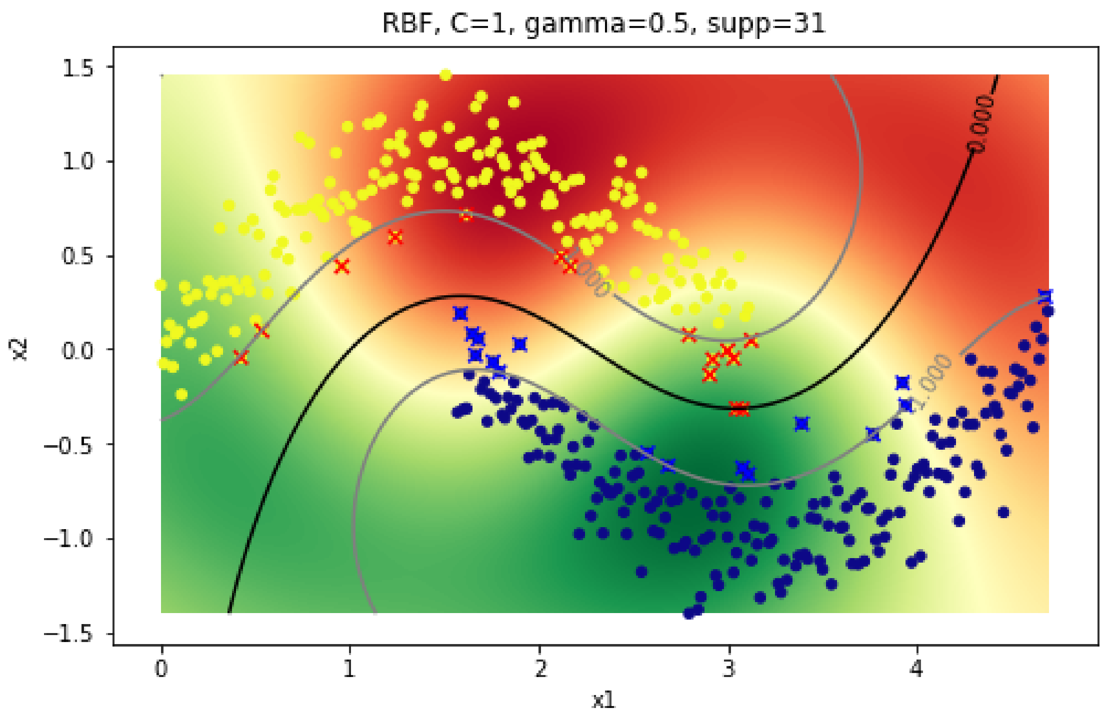
Let's put it together directly, for 6 values of C:

```
In [22]: fig, ax = plt.subplots(2,3,figsize=(12,6))
         for C, ax in zip(10**np.linspace(-3,2,6),ax.ravel()):
             svc = SVC(C=C,
                       kernel='poly',
                       degree=3,
                       coef0=1).fit(dataset.loc[:,'x1':'x2'],
                                    dataset.t)
             plot_prediction(svc, 'cubic, C=%f'%C,ax=ax)
         plt.tight_layout()
```



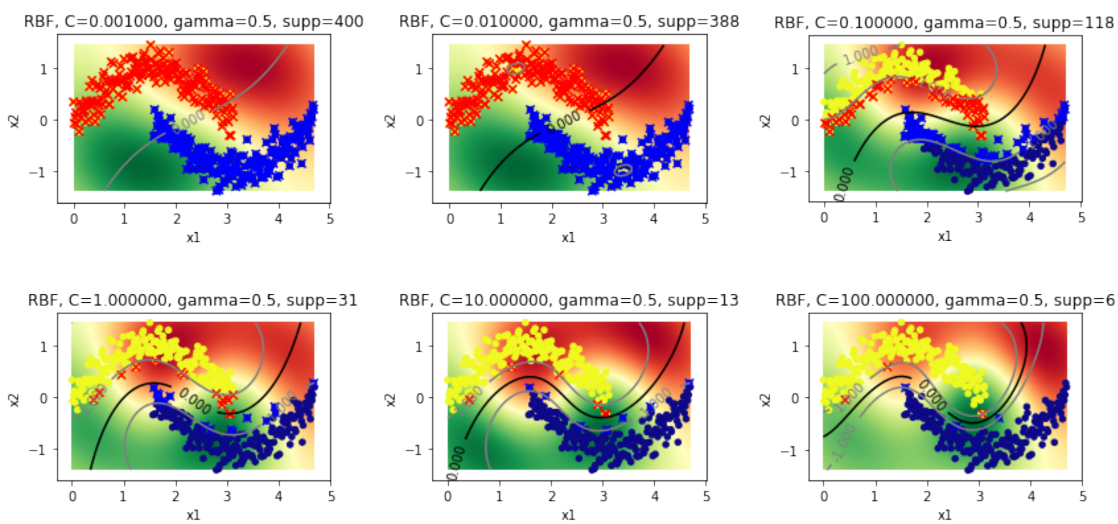
Finally we use the Gaussian RBF kernel (polynomial of infinite degree; the kernel has the form: $k(x,y) = exp(-gamma||x-y||^2)$
RBF kernel, C=1 (cost parameter)

```
In [23]: svc = SVC(C=1, kernel='rbf',gamma=0.5)
         svc.fit(dataset.loc[:,'x1':'x2'], dataset.t)

         print('Suports=',svc.n_support_)

Out[23]: SVC(C=1, cache_size=200, class_weight=None, coef0=0.0,
           decision_function_shape='ovr', degree=3, gamma=0.5, kernel='rbf',
           max_iter=-1, probability=False, random_state=None, shrinking=True,
           tol=0.001, verbose=False)

Suports= [16 15]


In [24]: plot_prediction(svc, 'RBF, C=1, gamma=0.5')
```



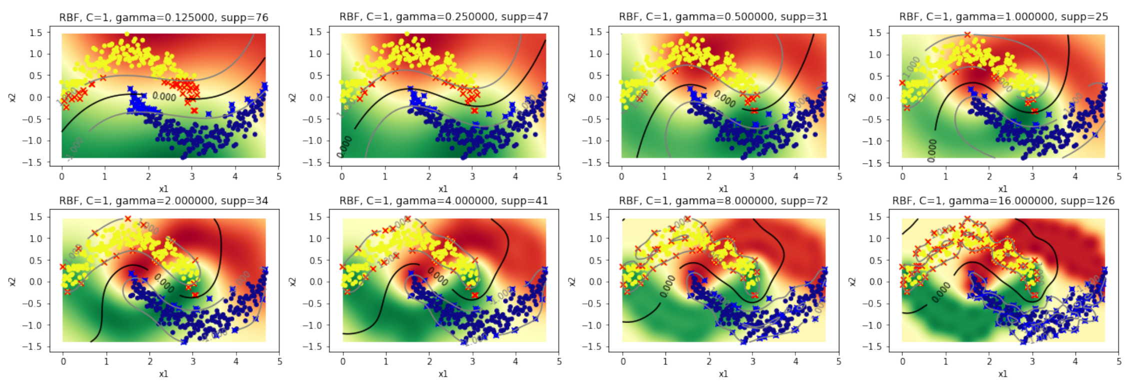
Let's put it together directly, for 6 values of C, holding gamma constant = 0.5:

```
In [25]: fig, ax = plt.subplots(2,3,figsize=(12,6))
         for C, ax in zip(10**np.linspace(-3,2,6),ax.ravel()):
             svc = SVC(C=C,
                       kernel='rbf',
                       gamma=0.5).fit(dataset.loc[:,'x1':'x2'],
                                      dataset.t)
             plot_prediction(svc, 'RBF, C=%f, gamma=0.5'%C,ax=ax)
         plt.tight_layout()
```



Now for 8 values of gamma, holding C constant = 1:

```
In [26]: fig, ax = plt.subplots(2,4,figsize=(18,6))
         for gamma, ax in zip(2**np.linspace(-3,4,8),ax.ravel()):
             svc = SVC(C=1,
                       kernel='rbf',
                       gamma=gamma).fit(dataset.loc[:,'x1':'x2'],
                                        dataset.t)
             plot_prediction(svc, 'RBF, C=1, gamma=%f'%gamma,ax=ax)
         plt.tight_layout()
```



In practice we should optimize both (C,gamma) at the same time
How? Using cross-validation or trying to get "good" estimates analyzing the data
Now we define a utility function for performing k-fold CV: a typical choice is k=10

```
In [27]: k = 10
```

```
In [28]: def train_svm_kCV(C,  k, kCV=10, params={'kernel':'linear'}):
             merr = 0.0
             for i in range(kCV):
                 cv = KFold(n_splits=k)
                 for tr_ind,ts_ind in cv.split(dataset):
                     svc = SVC(C=C,
                               **params).fit(dataset.iloc[tr_ind,[0,1]],
                                             dataset.t.iloc[tr_ind])
                     pred = svc.predict(dataset.iloc[ts_ind,[0,1]])
                     merr += (1-accuracy_score(dataset.t.iloc[ts_ind],pred))
             return merr/(kCV*k)*100
```

```
In [29]: C = 10

         VA_error_linear = train_svm_kCV (C=C, k=10, params={'kernel':'linear'})
         VA_error_linear
```

Out[29]: 17.749999999999996

procedure is to choose the model with the lowest CV error and then refit it with the whole learning data, then use it to predict the test set; we will do this at the end

Fit an SVM with quadratic kernel

```
In [30]: VA_error_poly_2 = train_svm_kCV (C=C, k=10,
                                          params={'kernel':'poly', 'degree':2,'coef0':1})
         VA_error_poly_2
```

Out[30]: 25.250000000000007

Fit an SVM with cubic kernel

```
In [31]: VA_error_poly_3 = train_svm_kCV (C=C, k=10,
                                          params={'kernel':'poly', 'degree':3,'coef0':1})
         VA_error_poly_3
```

Out[31]: 6.7500000000000036

```
In [32]: VA_error_rbf = train_svm_kCV (C=C, k=10, params={'kernel':'rbf'})
         VA_error_rbf
```

Out[32]: 1.25

Now in a real scenario we should choose the model with the lowest CV error which in this case is the RBF (we get a very low CV error because this problem is easy for a SVM)

so we choose RBF and C=1 and refit the model in the whole training set (no CV)

```
In [33]: svc = SVC(C=1,  kernel='rbf').fit(dataset.loc[:,'x1':'x2'], dataset.t)
```

and make it predict a test set:

let's generate the test data

```
In [34]: dataset_test = make_sinusoidals (1000)
```

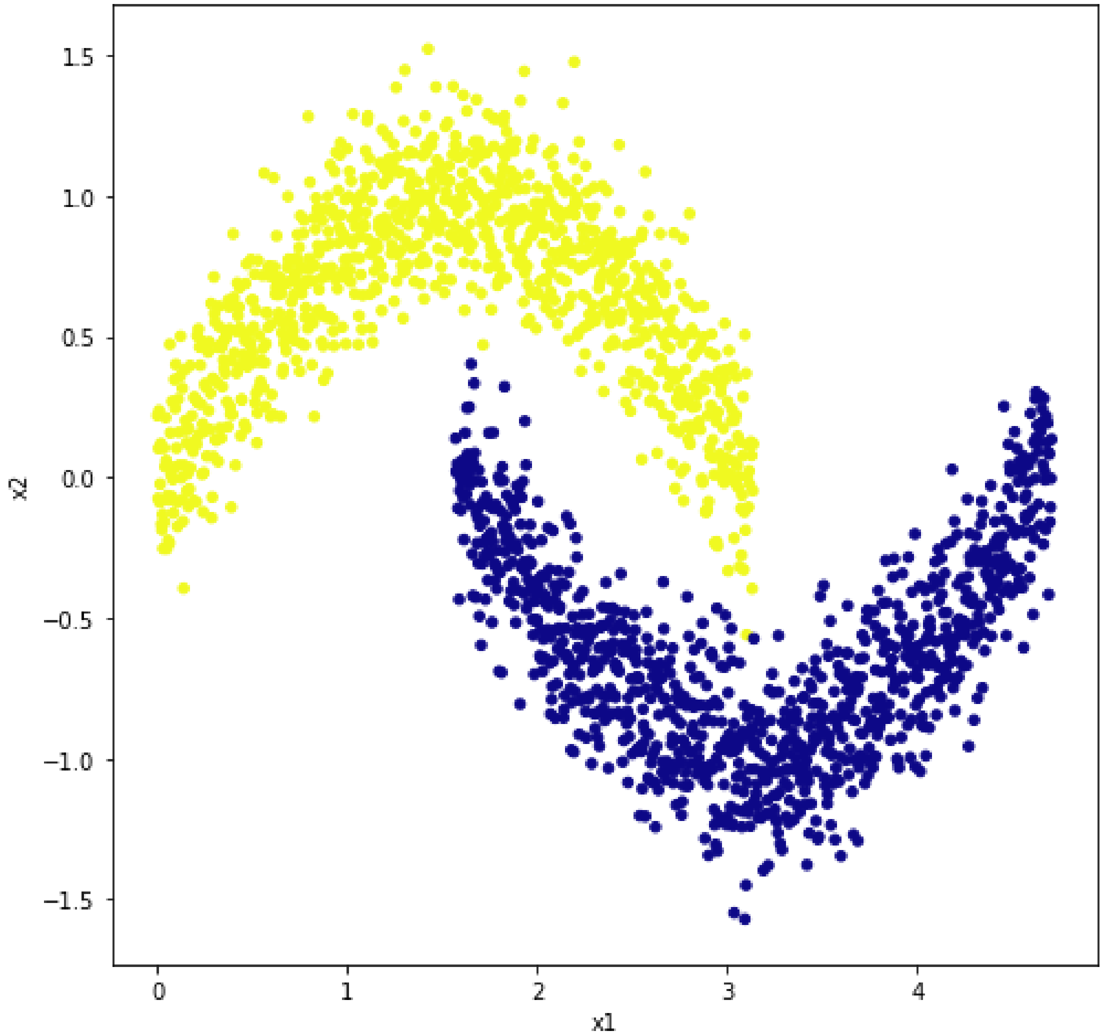
and have a look at it

```
In [35]: dataset_test.describe()
```

Out[35]:

	x1	x2	t
count	2000.000	2000.000	2000.0
mean	2.355	0.005	0.0
std	1.200	0.730	1.0
min	0.000	-1.570	-1.0
25%	1.570	-0.653	-1.0
50%	2.355	0.003	0.0
75%	3.139	0.688	1.0
max	4.709	1.523	1.0

In [36]:
```
dataset_test.plot.scatter(x='x1', y='x2', c='t',
                          colormap='plasma',
                          figsize=(8,8),
                          colorbar=False);
```



In [37]:
```
pred = svc.predict(dataset_test.loc[:,'x1':'x2'])
df_tr=pd.DataFrame(confusion_matrix(dataset_test.t, pred),
                   index=['-1','1'], columns=['-1','1'])
df_tr
(1-accuracy_score(dataset_test.t, pred))*100
```

Out[37]:

	-1	1
-1	996	4
1	5	995

Out[37]: 0.44999999999999485

In a real setting we should also optimize the value of C, again with CV; all this can be done very conveniently using GridSearchCV to do automatic grid-search (very much as we did in the last laboratory for nnet)

other packages provide with heuristic methods to estimate the gamma in the RBF kernel (see below)

Here you can find an interactive tool that allows to play with SVM

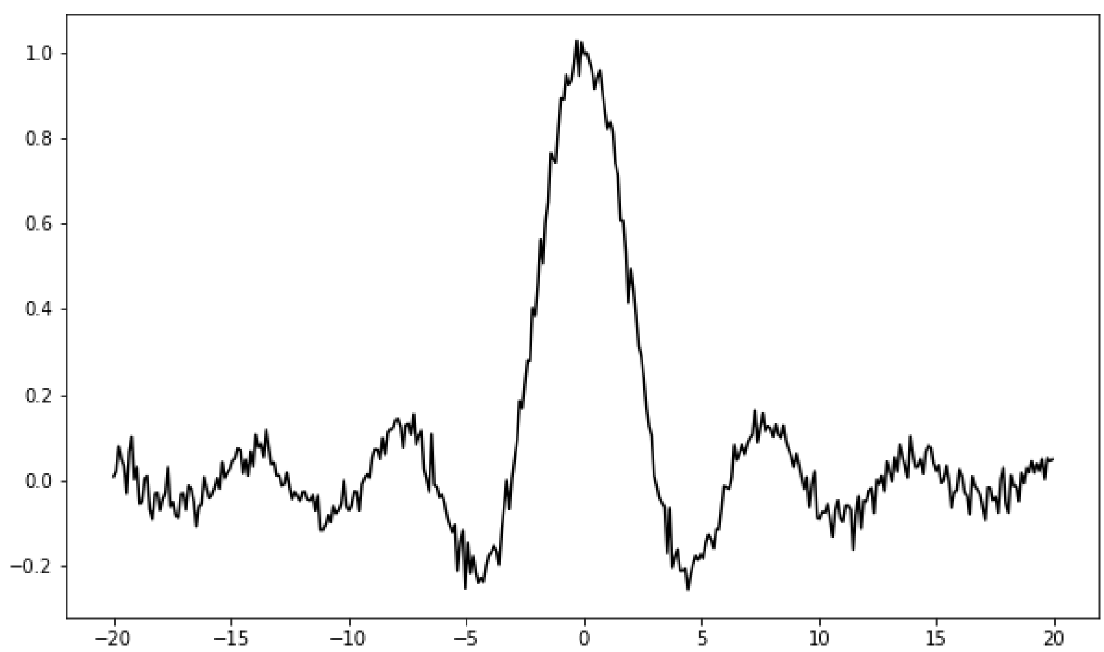
1.2 Playing with the SVM for regression and 1D data

Now we do regression; we have an extra parameter: the 'epsilon', which controls the width of the epsilon-insensitive tube (in feature space)

```
In [38]: A=20
```

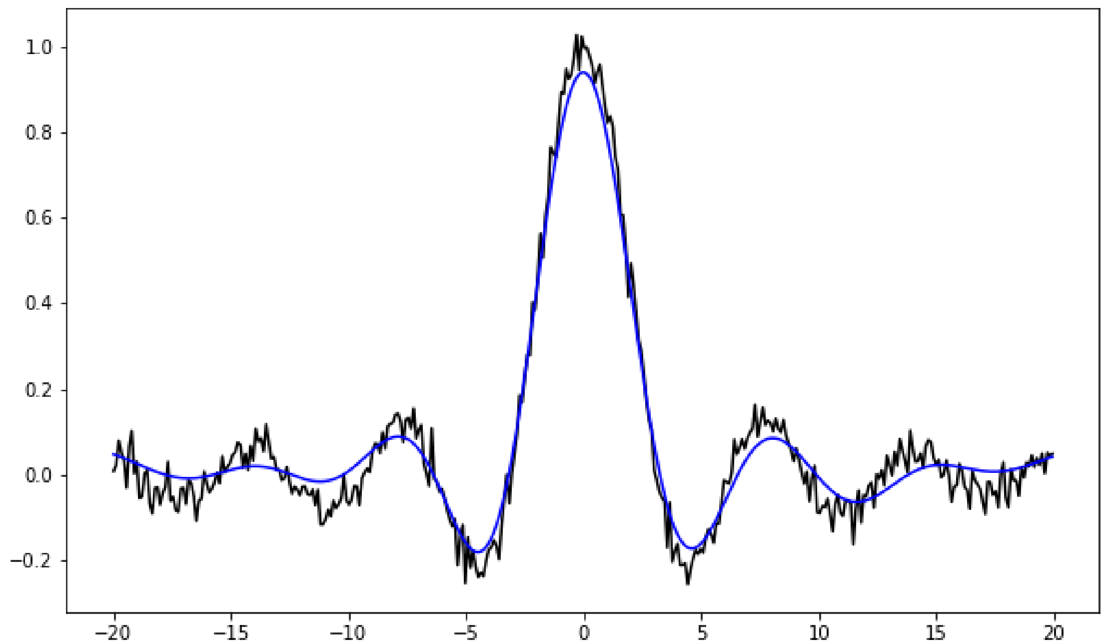
a really nice-looking function

```
In [39]: x = np.arange(-A,A,0.11)
         y = (np.sin(x)/x) + normal(scale=0.03,size=len(x))
         x = np.array(x).reshape(-1, 1)
         fig, ax = plt.subplots(figsize=(10,6))
         plt.plot(x,y,'k-');
```



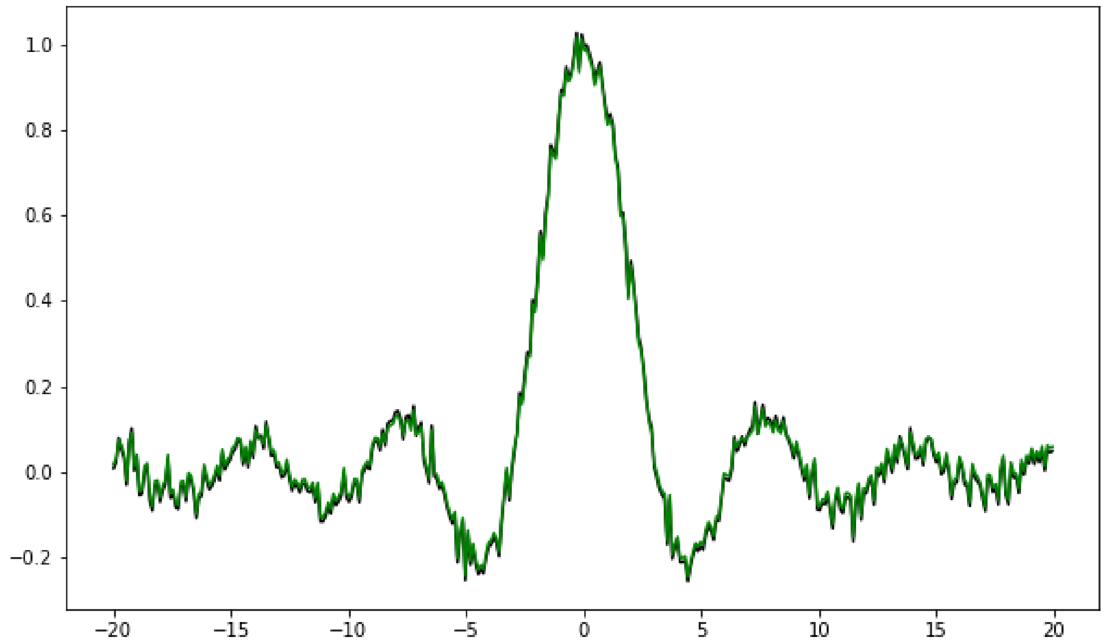
With this choice of the 'epsilon', 'gamma' and C parameters, the SVM underfits the data (blue line)

```
In [40]: svr = SVR(epsilon=0.1,gamma=0.1).fit(x,y)
         fig, ax = plt.subplots(figsize=(10,6))
         plt.plot(x,y,'k-')
         plt.plot(x,svr.predict(x),'b-');
```



With this choice of the 'epsilon', 'gamma' and C parameters, the SVM overfits the data (green line)

```
In [41]: svr = SVR(epsilon=0.01,gamma=200, C=100).fit(x,y)
         fig, ax = plt.subplots(figsize=(10,6))
         plt.plot(x,y,'k-')
         plt.plot(x,svr.predict(x),'g-');
```



With this choice of the 'epsilon', 'gamma' and C parameters, the SVM has a very decent fit (red line)

In [42]:
```
svr = SVR(epsilon=0.01,gamma=1).fit(x,y)
fig, ax = plt.subplots(figsize=(10,6))
plt.plot(x,y,'k-')
plt.plot(x,svr.predict(x),'r-');
```