

$$L = \sum (\mathbf{r}_{0i} - \vec{p}_0 \vec{q}_i)^2 + \lambda (\|\vec{p}_0\|^2 + \|\vec{q}_i\|^2)$$

$$e_{i0} = \mathbf{r}_{0i} - \vec{p}_0 \vec{q}_i$$

$$L = e_{i0}^2 + \lambda (\|\vec{p}_0\|^2 + \|\vec{q}_i\|^2)$$

$$\frac{dL}{d\vec{p}_0} = \frac{d(e_{i0}^2)}{d\vec{p}_0} + \frac{d(\lambda (\|\vec{p}_0\|^2 + \|\vec{q}_i\|^2))}{d\vec{p}_0} = \frac{d(e_{i0}^2)}{d\vec{p}_0} + \frac{d(\lambda \|\vec{p}_0\|^2)}{d\vec{p}_0} + \frac{d(\lambda \|\vec{q}_i\|^2)}{d\vec{p}_0}$$

$$\frac{dL}{d\vec{p}_0} = 2e_{i0} \frac{de_{i0}}{d\vec{p}_0} + \frac{d(\lambda \|\vec{p}_0\|^2)}{d\vec{p}_0} + 0$$

$$\frac{dL}{d\vec{p}_0} = 2e_{i0} \cdot \frac{d(\mathbf{r}_{0i} - \vec{p}_0 \vec{q}_i)}{d\vec{p}_0} + \frac{d(\lambda \|\vec{p}_0\|^2)}{d\vec{p}_0}$$

$$\frac{dL}{d\vec{p}_0} = 2e_{i0} (-\vec{q}_i) + 2\lambda \vec{p}_0 = -2e_{i0} \vec{q}_i + 2\lambda \vec{p}_0$$

$$\boxed{\frac{dL}{d\vec{p}_0} = -2e_{i0} \vec{q}_i + 2\lambda \vec{p}_0}$$

$$\frac{dL}{d\vec{q}_i} = \frac{d(e_{i0}^2)}{d\vec{q}_i} + \frac{d(\lambda \|\vec{p}_0\|^2)}{d\vec{q}_i} + \frac{d(\lambda \|\vec{q}_i\|^2)}{d\vec{q}_i} =$$

$$\frac{dL}{d\vec{q}_i} = 2e_{i0} \frac{d(e_{i0})}{d\vec{q}_i} + 0 + \frac{d(\lambda \|\vec{q}_i\|^2)}{d\vec{q}_i} =$$

$$\frac{dL}{d\vec{q}_i} = 2e_{i0} (-\vec{p}_0) + 0 + 2\lambda \vec{q}_i = -2e_{i0} \vec{p}_0 + 2\lambda \vec{q}_i$$

$$\boxed{\frac{dL}{d\vec{q}_i} = -2e_{i0} \vec{p}_0 + 2\lambda \vec{q}_i}$$

$$\theta \leftarrow \theta - \eta \frac{\partial L}{\partial \theta} \quad \left\{ \begin{array}{l} \vec{p}_0 = \vec{p}_0 - \eta (-2e_{i0} \vec{q}_i + 2\lambda \vec{p}_0) = \vec{p}_0 + \eta (e_{i0} \vec{q}_i - \lambda \vec{p}_0) \\ \vec{q}_i = \vec{q}_i - \eta (-2e_{i0} \vec{p}_0 + 2\lambda \vec{q}_i) = \vec{q}_i + \eta (e_{i0} \vec{p}_0 - \lambda \vec{q}_i) \end{array} \right.$$

$$L = \sum (r_{vi} - \mu - b_v - b_i - \vec{p}_v \cdot \vec{q}_i)^2 + \lambda (\|\vec{p}_v\|^2 + \|\vec{q}_i\|^2 + b_v^2 + b_i^2)$$

$$e_{vi} = r_{vi} - \mu - b_v - b_i - \vec{p}_v \cdot \vec{q}_i$$

$$\frac{dL}{db_v} = \frac{d(e_{vi}^2)}{db_v} + \frac{d(\lambda \|\vec{p}_v\|^2)}{db_v} + \frac{d(\lambda \|\vec{q}_i\|^2)}{db_v} + \frac{d(\lambda b_v^2)}{db_v} + \frac{d(\lambda b_i^2)}{db_v}$$

$$\frac{dL}{db_v} = 2e_{vi} \cdot \frac{de_{vi}}{db_v} + 0 + 0 + 2\lambda b_v + 0$$

$$\frac{dL}{db_v} = 2e_{vi} \cdot (-1) + 2\lambda b_v$$

$$\boxed{\frac{dL}{db_v} = -2e_{vi} + 2\lambda b_v}$$

~~b_v~~

$$\boxed{b_v = b_v + \eta (e_{vi} - \lambda b_v)}$$