Econ 171: Decisions Under Uncertainty

Jordi Martinez-Muñoz

Chapter 1 : Basics of Decision-Making Under Uncertainty

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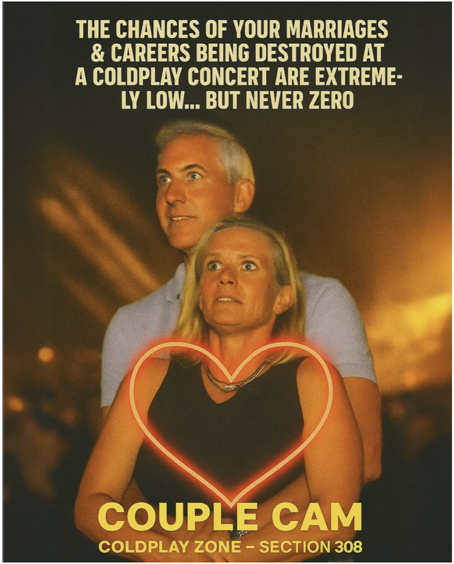

Uncertainty and Anxiety

Anxiety

*An emotion characterized by apprehension and somatic symptoms of tension in which an individual **anticipates** impending danger, catastrophe, or misfortune. The body often mobilizes itself to meet the perceived threat : Muscles become tense, breathing is faster, and the heart beats more rapidly.*

*Anxiety may be distinguished from fear both conceptually and physiologically, although the two terms are often used interchangeably. Anxiety is considered a **future-oriented**, long-acting response broadly focused on a **diffuse threat**, whereas fear is an appropriate, present-oriented, and short-lived response to a clearly identifiable and specific threat.*

Adapted from the APA Dictionary of Psychology

Uncertainty and Discrimination

Alex Chan, 2024, Discrimination Against Doctors : A Field Experiment

- Customers discriminate against Black and Asian doctors when they choose healthcare providers.
- This can be substantially reduced by supplying information on physician quality.

Statistical discrimination arises when customers use easily observable characteristics such as race to infer the expected quality of doctors.

Customers can also hold biased priors about group traits, leading to biased belief discrimination.

Exercise : Classify the Uncertainty

Which type of uncertainty applies to each ?

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Job Choice Example

Decision :

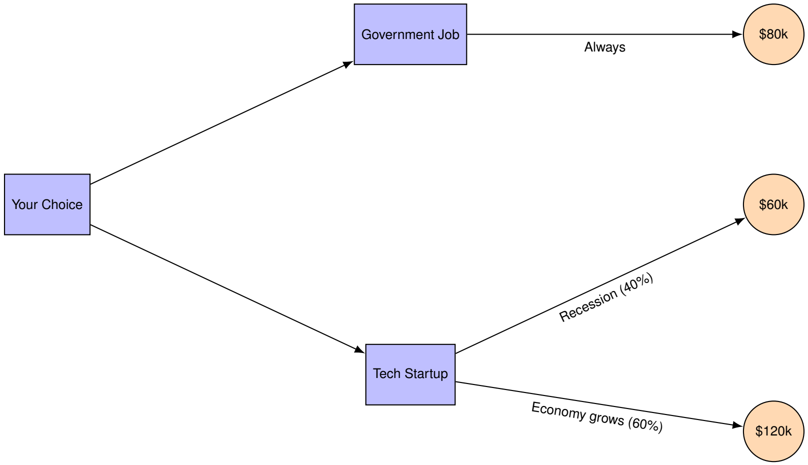
- Choose between a Tech Startup and a Government Job.

Here's how the situation looks :

- **If you accept the tech startup job** and the economy grows strongly, the company does well, and your total compensation will be high — about $120,000 including salary and stock options.
- **But if you take the startup job and the economy falls into a recession**, the company may struggle. Your salary will be cut to only $60,000.
- **On the other hand, if you accept the government job**, you get a steady $80,000 salary regardless of how the economy performs.

You estimate there is a 60% chance the economy will grow and a 40% chance it will go into recession.

Decision Tree Representation



Payoff Matrix Representation

Job Choice	Economy Grows (60%)	Recession (40%)
Tech Startup	$120,000	$60,000
Government Job	$80,000	$80,000

Exercise

A student can choose to study all weekend for next Tuesday's midterm exam, or go to a music festival instead. The midterm could be easy or hard.

- If the student goes to the festival and the exam is hard, the student will get a C- but will have enjoyed the festival.
- If the student goes to the festival and the exam is easy, the student will get an A- and will have enjoyed the festival.
- If the student does not go to the festival and the exam is hard, the student will get a A- but will be had boring weekend.
- If the student does not go to the festival and the exam is easy, the student will get a A but will be had boring weekend.

Exercise : A personal case

Think of a situation in your life in which you have to make an important choice under uncertainty.

Represent this choice in a decision tree or payoff matrix.

Share your story with a classmate. Describe the actions, states, outcomes, and probabilities.

Share also which action you ended up taking and why.

Exercise : A less structured problem

In real life, actions, states, and outcomes won't be given in a text, you rather have to think about them through intuition and knowledge about your environment.

You are the CEO of a company. In a recent survey, workers in the research and design department complained about having too short breaks. They demand at least 30 more minutes of breaks.

What would you do ?

Here is some information for you to consider :

- The company has 120 employees, from which 30 are from the Research and Design department.
- Each research and design team produces on average $1.5 per minute.
- The other workers produce $0.8 per minute.

Relation to Risk Management

Following the *ISO31000*

What is risk management : Coordinated activities to direct and control an organization with regard to risk.

The purpose of risk management is the creation and protection of value. It improves performance, encourages innovation and supports the achievement of objectives.

What is risk ? Effect of uncertainty on objectives

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies).

ISO 31000:2018(E)

Introduction

This document is for use by people who create and protect value in organizations by managing risks, making decisions, setting and achieving objectives and improving performance.

Organizations of all types and sizes face external and internal factors and influences that make it uncertain whether they will achieve their objectives.

Managing risk is iterative and assists organizations in setting strategy, achieving objectives and making informed decisions.

Managing risk is part of governance and leadership, and is fundamental to how the organization is managed at all levels. It contributes to the improvement of management systems.

Managing risk is part of all activities associated with an organization and includes interaction with stakeholders.

Managing risk considers the external and internal context of the organization, including human behaviour and cultural factors.

Managing risk is based on the principles, framework and process outlined in this document, as illustrated in Figure 1. These components might already exist in full or in part within the organization, however, they might need to be adapted or improved so that managing risk is efficient, effective and consistent.

The Process of Risk Management

Relation to Risk Management

Can we translate the Risk Management process into our framework of decisions under uncertainty ?

Exercise

You are in charge of the food for the end-of-year party at one of your clubs.

- It is expected that around 50 people will show up.
- The budget you were promised is around $3,000

Make a simple risk management plan using our decisions under uncertainty framework (actions, states, probabilities, and outcomes).

Chapter 2: Decision Criteria

Uncertainty and Random Variables

- **Random Variables** (or *lotteries*) mathematically describe uncertainty.
- A random variable assigns a number to every possible outcome of a random event.

Example : Coin flip payoff
If heads, earn \$10 ; if tails, earn \$0.

Define random variable X :

$$X = \begin{cases} 10 & \text{with probability } 0.5 \\ 0 & \text{with probability } 0.5 \end{cases}$$

Discrete vs. Continuous Random Variables

Discrete Random Variables

- Take on countable, listable values.
- Example lottery :

Outcome (x)	$P(x)$
0	0.4
50	0.3
100	0.3

Continuous Random Variables

- Take any value in a range (uncountably many).
- Example : rainfall amount (15.3 mm, 7.008 mm, 17.922 mm), stock returns (2.334%, 7.0001%).
- Probability of any exact value = 0.

Key Properties of Discrete Random Variables

We focus on discrete random variables for this course.

Probability Distribution Function (PDF) :

- Lists all possible outcomes and their probabilities.
- Probabilities must be :
  - Non-negative
  - Sum to 1
- Ensures exhaustivity and mutual exclusivity.

Example lottery PDF :

$$\begin{cases} 0 \text{ with probability } 0.4 \\ 50 \text{ with probability } 0.3 \\ 100 \text{ with probability } 0.3 \end{cases}$$

Cumulative Distribution Function (CDF)

- The **CDF** gives the probability that the outcome is **less than or equal** to a value.
- It shows cumulative probabilities, not single-point probabilities.

Example CDF for our lottery :

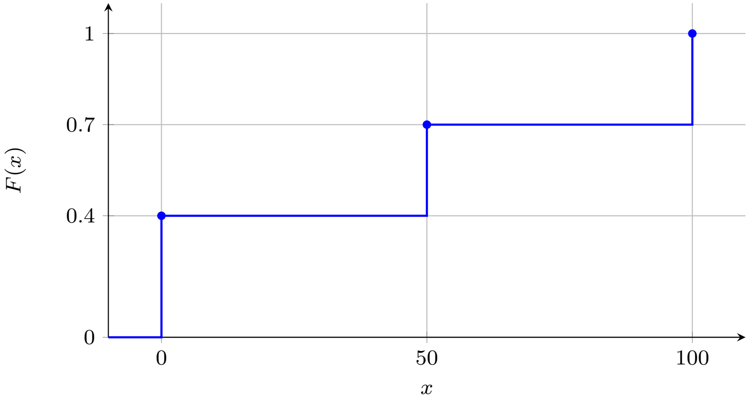
$$F(0) = P(X \leq 0) = 0.4$$

$$F(50) = P(X \leq 50) = 0.4 + 0.3 = 0.7$$

$$F(100) = P(X \leq 100) = 1$$

For values between outcomes (e.g. $F(70)$), sum probabilities for outcomes ≤ 70 : $0.4 + 0.3 = 0.7$

CDF Graph of the Example Lottery



Mean (Expected Value) of a Random Variable

- The **mean** or **expected value** is the average payoff over many repetitions.
- Formula :

$$\mu_X = \mathbb{E}[X] = \sum_i x_i \cdot P(X = x_i)$$

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Example :

$$\mathbb{E}[X] = 0 \times 0.4 + 50 \times 0.3 + 100 \times 0.3 = 45$$

Average expected outcome is \$45.

Variance and Standard Deviation

- **Variance** measures spread : average squared deviation from the mean.
- **Standard deviation** is the square root of variance, in the same units as outcomes.

$$\sigma_X^2 = \text{VAR}(X) = \mathbb{E}[(X - \mathbb{E}[X])^2] = \sum_i P(x_i)(x_i - \mathbb{E}[X])^2,$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

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$$\text{SD}(X) = \sqrt{\text{Var}(X)}$$

Outcome	Probability	Deviation from μ	Squared
0	0.4	-45	2025
50	0.3	5	25
100	0.3	55	3025

$$\text{Var}(X) = 0.4 \times 2025 + 0.3 \times 25 + 0.3 \times 3025 = 1725$$

$$\text{SD}(X) = \sqrt{1725} \approx 41.5$$

Min, Max, and Range

- **Min :** Lowest possible outcome (here, \$0)
- **Max :** Highest possible outcome (here, \$100)
- **Range :** Max – Min = \$100 – \$0 = \$100

Example : Another Lottery

Outcome	Probability	Deviation from μ	Squared
20	0.5	-30	900
80	0.5	30	900

$\mu = 50$

$$\text{Var}(X) = 0.5 \times 900 + 0.5 \times 900 = 900, \quad \text{SD}(X) = 30$$

The lottery here has lower variance (less spread) than the previous one.

max = 80, min=20, range = 60

Skewness and Kurtosis : Shape of the Distribution

Skewness : Measures asymmetry of a distribution.

- Symmetric example : $X = \{1, 2, 3\}$ with equal probabilities.
- Right-skewed example : $Y = \{1, 2, 3\}$ with $P(1) = 0.5, P(2) = 0.25, P(3) = 0.25$.
- Left-skewed example : $Z = \{1, 2, 3\}$ with $P(1) = 0.1, P(2) = 0.2, P(3) = 0.7$.

Kurtosis (not covered here) measures types of peaks.

Measures of Variability

Why Variance is a Better Measure than Range

Variability $\rightarrow$ measure of risk.

Variance is generally preferred measure of variability because it considers *deviations from the mean for each value*, not just the extremes.

Example : Two Lotteries

Lottery A : {-\$100, \$0, \$100}, with probs (0.01, 0.98, 0.01)
Lottery B : {-\$30, \$0, \$30}, with probs (0.3, 0.4, 0.3)

Example : Two Lotteries

Lottery A : {-$100, $0, $100}, with probs (0.01, 0.98, 0.01)
Lottery B : {-$30, $0, $30}, with probs (0.3, 0.4, 0.3)

Both lotteries have mean = $0, but :

- **Range :**
  - Lottery A : $200
  - Lottery B : $60

Example : Two Lotteries

Lottery A : {-\$100, \$0, \$100}, with probs (0.01, 0.98, 0.01)
Lottery B : {-\$30, \$0, \$30}, with probs (0.3, 0.4, 0.3)

Both lotteries have mean = \$0, but :

- **Range :**
  - Lottery A : \$200
  - Lottery B : \$60
- **Variance :**

$$\sigma_A^2 = 0.01 \cdot 100^2 + 0.98 \cdot 0^2 + 0.01 \cdot 100^2 = 200$$
$$\sigma_B^2 = 0.3 \cdot 30^2 + 0.4 \cdot 0^2 + 0.3 \cdot 30^2 = 540$$

Example : Two Lotteries

Lottery A : {-\$100, \$0, \$100}, with probs (0.01, 0.98, 0.01)
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Both lotteries have mean = \$0, but :

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  - Lottery A : \$200
  - Lottery B : \$60
- **Variance :**

$$\sigma_A^2 = 0.01 \cdot 100^2 + 0.98 \cdot 0^2 + 0.01 \cdot 100^2 = 200$$
$$\sigma_B^2 = 0.3 \cdot 30^2 + 0.4 \cdot 0^2 + 0.3 \cdot 30^2 = 540$$

Conclusion : Despite its higher range, Lottery A has *lower variance* than Lottery B.

Interpretation.

Decision Criteria
How to choose under uncertainty ?

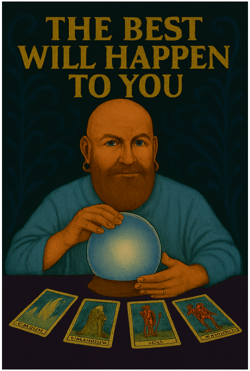
Example : Job Choice Payoff Matrix

	Good Economy	Recession
Startup	\$120k	\$60k
Government	\$80k	\$80k

Maximax Criterion

Optimist

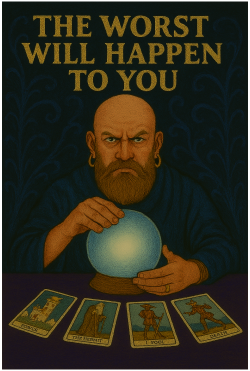
- Assumes the best state will happen regardless of the action taken.
- Pick the action with the highest best outcome



Maximin Criterion

Pessimist/Cautious

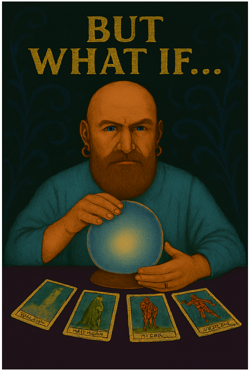
- Assumes the worst state will happen regardless of the action taken.
- Pick the action with the highest worst outcome



Minimax Regret Criterion

Pessimist/Anxious

- Assumes they will regret their choice.
- Pick the lowest potential regret.



Minimax Regret Criterion

Step 1 : Construct Regret Table

	Good Economy	Recession
Startup	\$0k	\$20k
Government	\$40k	\$0k

Step 2 : Maximum Regret for each action

- Startup : max regret = \$20k
- Government : max regret = \$40k

Minimax regret chooses Startup since it minimizes maximum regret.

Probabilistic Criteria : Expected Value

Probabilities :

- Good Economy : 60%
- Recession : 40%

Expected Values (EV) :

$$\text{EV}_{Startup} = 0.6 \times 120 + 0.4 \times 60 = 96k$$

$$\text{EV}_{Government} = 0.6 \times 80 + 0.4 \times 80 = 80k$$

Expected Value Criterion chooses Startup (higher average payoff).

Mean-Variance Criterion

Considers both expected value and risk (variance) :

- Startup : EV = \$96k, **higher variability** (payoffs : 120k or 60k)
- Government : EV = \$80k, **no variability** (constant payoff)

Trade-off : Higher expected payoff vs. higher risk.

Agents may prefer lower variability depending on risk tolerance.

Activity : (Finance) Three Assets

Return rate of an asset (%) : For every \$1 you invest, you receive $\$1 \times (1 + r)$. We call r the (net) return rate.

- **Asset 1 :** Return rates 10% with $p = 0.3$, 0% with $p = 0.7$.
- **Asset 2 :** Return rates 1% with $p = 0.3$, 3% with $p = 0.3$, 5% with $p = 0.4$.
- **Asset 3 :** Return rates 40% with $p = 0.05$, 25% with $p = 0.05$, 0% with $p = 0.9$.

Expected Returns

$$EV(A_1) = 0.1 \times 0.3 = 0.03$$

$$EV(A_2) = 0.01 \times 0.3 + 0.03 \times 0.3 + 0.05 \times 0.4 = 0.032$$

$$EV(A_3) = 0.4 \times 0.05 + 0.25 \times 0.05 = 0.0325$$

Ranking by Expected Return

A_3 highest, A_2 second, A_1 last.

Variance (Risk)

For **Asset 1** :

$$\sigma^2_{A_1} = 0.3 \times (0.10 - 0.03)^2 + 0.7 \times (0 - 0.03)^2$$
$$= 0.3 \times 0.0049 + 0.7 \times 0.0009 = 0.0021$$

For **Asset 2** :

$$\sigma^2_{A_2} = 0.3 \times (0.01 - 0.032)^2 + 0.3 \times (0.03 - 0.032)^2 + 0.4 \times (0.05 - 0.032)^2$$
$$= 0.3 \times 0.000484 + 0.3 \times 0.000004 + 0.4 \times 0.000324 = 0.000276$$

For **Asset 3** :

$$\sigma^2_{A_3} = 0.05 \times (0.40 - 0.0325)^2 + 0.05 \times (0.25 - 0.0325)^2 + 0.9 \times (0 - 0.0325)^2$$
$$= 0.05 \times 0.13405625 + 0.05 \times 0.04730625 + 0.9 \times 0.00105625 = 0.01$$

Ranking by Risk

A_2 least risky, A_1 medium risk, A_3 most risky.

Interpretation

- A_2 **dominates** A_1 : higher return and lower risk.
- A_1 vs A_3 : trade-off between mean and variance.
- A_2 vs A_3 : also a mean–variance trade-off.

Example of First-Order Stochastic Dominance FOSD

Two lotteries :

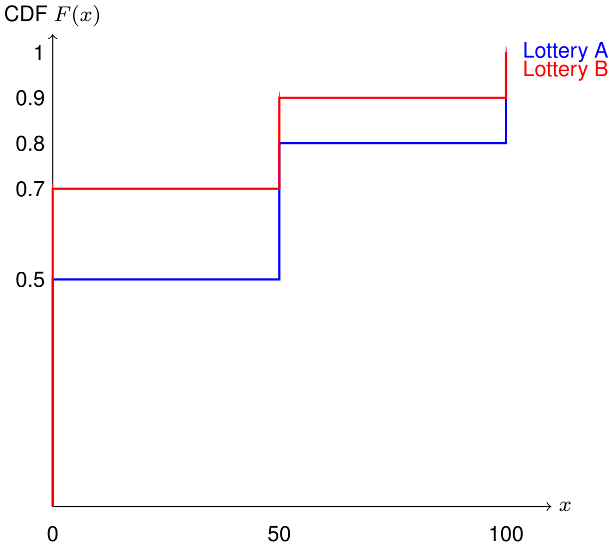
Outcome (x)	0	50	100
Lottery A :	0.5	0.3	0.2
Lottery B :	0.7	0.2	0.1

Cumulative distribution functions (CDF) :

$$\begin{cases} F_A(0) = 0.5, \quad F_B(0) = 0.7 \\ F_A(50) = 0.8, \quad F_B(50) = 0.9 \\ F_A(100) = 1.0, \quad F_B(100) = 1.0 \end{cases}$$

Since $F_A(x) \leq F_B(x)$ for all x, and strictly less for some, **Lottery A FOSD Lottery B**.

Graphical Illustration of FOSD



Interpretation of FOSD

- The blue curve (Lottery A) is always **below or equal** the red curve (Lottery B).
- This means that for any possible number, the probability that Lottery A returns more than that number is greater or equal than the probability that Lottery B returns more than that number.

Activity

(1)You are the director of a new product. A potential customer could buy the product or continue using their current one. Your product has a price of \$100, while the current (old) product costs only \$70.

	yours work better	old work better
Buy yours	+\$20 (happiness)	-\$170
Stay with old one	-\$70	-\$70

think of the +\$20 as consumer surplus

(2) You are the commercial director of a manufacturer and want retailers to buy your product (1000 units at a cost of \$8 each, selling price is \$10) :

	sell full stock	sell only half stock
Buy	\$2K	-\$3K
Not buy	-\$0	-\$0

Both maximin and minimax regret criteria do not favor you (only maximax). How could you change this ?

Criteria-robust practices

From the perspective of business/policy makers :

“full refund if you don’t like the product” or “buy-back manufacturer-retailer policies” are examples of practices that reduce “perceived risk” ...

But another way to see it is :

Make your product/policy dominant under as many decision criteria as possible.

Related topics

- Marketing
- Nudging
- Choice architecture
- contract/choice design

Chapter 3: Sequential Problems

Sequential Uncertainty in a Product Launch

Scenario : A (profit-maximizing) startup is deciding whether to produce a gadget. Production cost : $500K.

Revenue depends on the demand size. However, to be able to sell in this market, the startup needs permission from the regulator, which is not certain.

Sequential Uncertainty in a Product Launch

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Production cost : $500K.

Revenue depends on the demand size. However, to be able to sell in this market, the startup needs permission from the regulator, which is not certain.

- **Stage 1 : Regulatory Approval**
  - Pass : probability 0.7
  - Fail : probability 0.3
- **Stage 2 (If regulation passed) : Market Demand**
  - High demand : prob. 0.6 Revenue : $800K
  - Medium demand : prob. 0.3 Revenue : $700K
  - Low demand : prob. 0.1 Revenue : $200K

Sequential Uncertainty in a Product Launch

Payoff Table (Profit in $) :

	Pass–High	Pass–Med	Pass–Low	Fail
Produce	300k	200k	-300k	-500k
Not Produce	0	0	0	0

Note : For **Produce**, states like “Fail–High” are **impossible** ($p = 0$) because demand is only realized if approval passes.

Tabular Representation

State	Profit (Produce) ($)	Profit (Not Produce)	Joint probability
Pass – High	300K	0	$0.7 \times 0.6 = 0.42$
Pass – Medium	200K	0	$0.7 \times 0.3 = 0.21$
Pass – Low	-300K	0	$0.7 \times 0.1 = 0.07$
Fail (no launch)	-500K	0	0.3

Expected Value (Produce)

$$EV(\text{Produce}) = 300{,}000 \cdot 0.42 \; + \; 200{,}000 \cdot 0.21 \; + \; (-300{,}000) \cdot 0.07 \; + \; (-500{,}000) \cdot 0.30$$
$$= \boxed{-\$3K}$$

$$EV(NotProduce) = \boxed{\$0}$$

Decision by EV : Do not produce.

Non-sequential compounded states, relevant ?

Exercise : Two class projects.

- **Option 1 :** Success probability 0.8 ; grade 85% if successful, 50% if unsuccessful.
- **Option 2 :** Success probability 0.6 ; grade 95% if successful, 50% if unsuccessful.

The projects are independent : success of one does not influence the success of the other.

- What are the states in this problem ?
- How does the decision tree look ?
- Compute the choice by EV.
- Compute the choice by minimax regret.

Two Class Projects : Setup

States of the world :

- Both succeed : $0.8 \times 0.6 = 0.48$
- Both fail : $0.2 \times 0.4 = 0.08$
- Only Option 1 succeeds : $0.8 \times 0.4 = 0.32$
- Only Option 2 succeeds : $0.2 \times 0.6 = 0.12$

Because the events are independent, no combination has zero probability.

Two Class Projects : Setup

For **expected value** purposes, we only need each project's success probability :

$$EV(\text{Project 1}) = 85 \cdot 0.8 + 50 \cdot 0.2 = 78$$

$$EV(\text{Project 2}) = 95 \cdot 0.6 + 50 \cdot 0.4 = 77$$

Observation : The EV calculation does not require analyzing all four joint states separately.

Two Class Projects : Setup

Minimax Regret :

	Both Succeed	Only 1 succeeds	Only 2 succeeds	Both Fail
Option 1	10	0	45	0
Option 2	0	35	0	0

Bonus : New Criterion

Expected Regret !

Sometimes called anticipated regret.

Bonus : New Criterion

How would you think about expected regret ?

Bonus : New Criterion

How would you think about expected regret ?

	Both (0.48)	Only 1 (0.32)	Only 2 (0.12)	None (0.08)
Option 1	10	0	45	0
Option 2	0	35	0	0

Bonus : New Criterion

How would you think about expected regret ?

	Both (0.48)	Only 1 (0.32)	Only 2 (0.12)	None (0.08)
Option 1	10	0	45	0
Option 2	0	35	0	0

Is this a reasonable criterion ?

Sequential Actions

A young entrepreneur decides whether to develop an app that connects restaurants with food suppliers or start a physical business supplying food to restaurants.

The app has a small initial cost, but the success rate is only 25%. The physical business has a large initial cost, but the success rate is 80%.

Sequential Actions

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If either project is successful, they can either merge with a restaurant chain or continue working independently.

For the app, the probability of success if they merge is 40%. For the physical store, the probability of success if they merge is 50%.

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A young entrepreneur decides whether to develop an app that connects restaurants with food suppliers or start a physical business supplying food to restaurants.

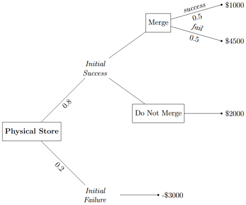
The app has a small initial cost, but the success rate is only 25%. The physical business has a large initial cost, but the success rate is 80%.

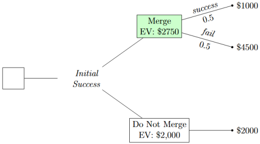
If either project is successful, they can either merge with a restaurant chain or continue working independently.

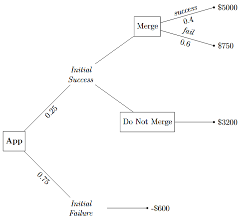
For the app, the probability of success if they merge is 40%. For the physical store, the probability of success if they merge is 50%.

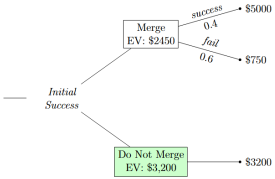
If the app fails, the profit is $-\$600$. If the physical store fails, the profit is $-\$3,000$. If the app succeeds and they don't merge, the profit is \$3,200. If they merge and succeed, the profit is \$5,000. If they merge and do not succeed, the profit is \$750.

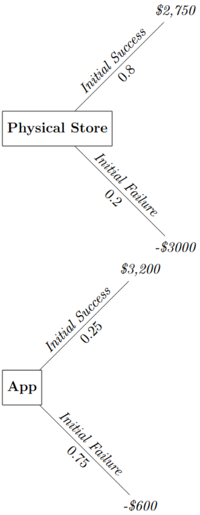
For the physical store, if they succeed and don't merge, the profit is \$500. If they merge and succeed, the profit is \$4,500. If they merge and fail, the profit is \$1,000.
Which project should the young entrepreneur choose under the EV criterion?


success
0.5
$1000
Merge
fail
0.5
$4500
Initial Success
0.8
Do Not Merge
$2000
Physical Store
0.2
Initial Failure
-$3000


success
0.5
$1000
Merge
EV: $2750
fail
0.5
$4500
Initial Success
Do Not Merge
EV: $2,000
$2000


success
0.4
$5000
Merge
fail
0.6
$750
Initial
Success
0.25
Do Not Merge
$3200
App
0.75
Initial
Failure
-$600


success
0.4
$5000
Merge
EV: $2450
fail
0.6
$750
Initial
Success
Do Not Merge
EV: $3,200
$3200

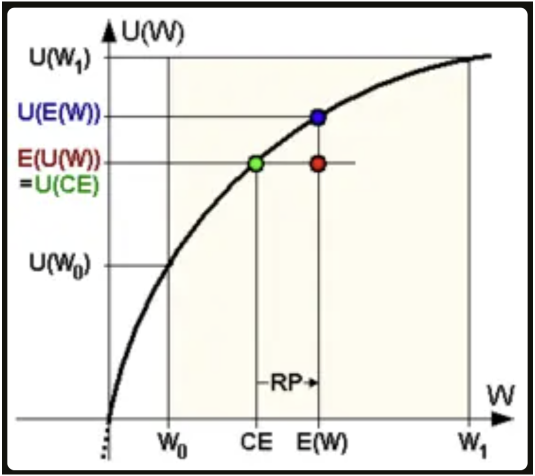

$2,750
Initial Success
0.8
Physical Store
Initial Failure
0.2
-$3000
$3,200
Initial Success
0.25
App
Initial Failure
0.75
-$600

Chapter 5: Risk Preferences

Risk Aversion

For **every** lottery $p \in \Delta(Z)$,

$$EV(p) \succsim p$$



Absolute Risk Aversion

Additive Risk : Gamble does not change with wealth.

Risk preferences may change with wealth :

- Risk preferences do not change with wealth : *CARA*.
- Less risk aversion with more wealth : *DARA*.
- More risk aversion with more wealth : *IARA*.

Absolute Risk Aversion

Additive Risk : Gamble does not change with wealth.

Risk preferences may change with wealth :

- Risk preferences do not change with wealth : *CARA*.
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In terms of the utility function :

$$\text{Arrow-Pratt (A)} = -\frac{u^{''}(z)}{u^{'}(z)}$$

- If A does not depend on z : CARA.
- If A increases with z : IARA.
- If A decreases with z : DARA.

Absolute Risk Aversion

In terms of investment in a risky asset.

Example.

Assume that when the person has \$1000, they invest \$100 in a risky asset (high expected value but more variability than a risk-free asset).

If when the person has \$4000 (more wealth) :

- They invest \$100 in the risky asset (same **amount**) $\Rightarrow$ CARA.
- They invest \$170 in the risky asset (greater **amount**) $\Rightarrow$ DARA.
- They invest \$84 in the risky asset (smaller **amount**) $\Rightarrow$ IARA.

Relative Risk Aversion

Multiplicative Risk : Gamble changes with wealth. For example, *Gaining 10% of your investment with probability 0.50 or losing 10% of your investment with probability 0.5*.

How do risk preferences change with stakes ?

- Risk preferences do not change as stakes increase : *CRRA*.
- Less risk aversion with higher stakes : *DRRA*.
- More risk aversion with higher stakes : *IRRA*.

Relative Risk Aversion

Multiplicative Risk : Gamble changes with wealth. For example, *Gaining 10% of your investment with probability 0.50 or losing 10% of your investment with probability 0.5*.

How do risk preferences change with stakes ?

- Risk preferences do not change as stakes increase : *CRRA*.
- Less risk aversion with higher stakes : *DRRA*.
- More risk aversion with higher stakes : *IRRA*.

In terms of the utility function :

$$\text{Arrow-Pratt (relative) (R)} = -z\frac{u^{''}(z)}{u^{'}(z)}$$

- If R does not depend on z : CRRA.
- If R increases with z : IRRA.
- If R decreases with z : DRRA.

Relative Risk Aversion

In terms of investment in a risky asset.

Example.

Assume that when the person has \$1000, they invest \$100 (10%) in a risky asset (high expected value but more variability than a risk-free asset).

If when the person has \$4000 (more wealth) :

- They invest \$400 (10%) in the risky asset (same **percentage**) $\Rightarrow$ CRRA.
- They invest \$600 (15%) in the risky asset (higher **percentage**) $\Rightarrow$ DRRA.
- They invest \$300 (7.5%) in the risky asset (lower **percentage**) $\Rightarrow$ IRRA.

Summary

Risk Averse if :

- $EV(p) \succsim p$ for every $p \in \Delta(Z)$.
- $u(z)$ is concave ($u''(z) \leq 0$).
- $CE_p \leq EV(p)$ for every p.
- $RP(p) \geq 0$ for every p.
- $A = -\frac{u''}{u'} \geq 0$.
- Do not fully invest in risky assets.

Summary

Between two decision makers,

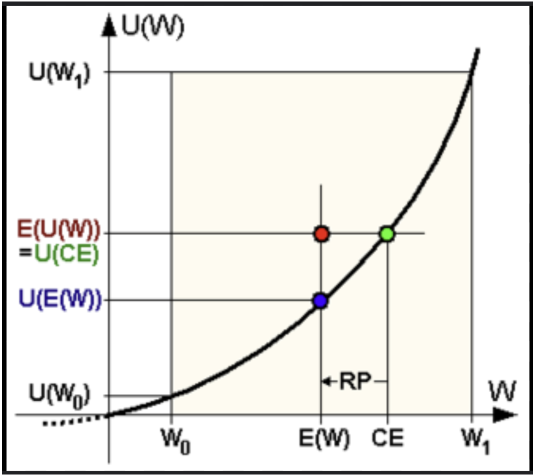
The more risk-averse is the one with :

- More concave u.
- Lower CE.
- Higher RP.
- Higher Arrow-Pratt coefficient (A).
- Lower investment in risky assets (given the same wealth level)

Risk loving/seeking

For **every** lottery $p \in \Delta(Z)$,

$$EV(p) \precsim p$$



Choice Data

- We can't talk about someone being risk-averse (lover) without observing their preferences over all pairs of lotteries ir $u(z)$.
- We can talk about behavior being *consistent with* risk aversion, risk loving, etc.

			Choice		
	Prospects	Ticket Price	Ann	Bob	Charlie
Lottery 1	(1000, 0.2 ; 0, 0.8)	190	Don't buy ticket	Don't buy ticket	buy ticket
Lottery 2	(1000, 0.2 ; 0, 0.8)	120	buy ticket	Don't buy ticket	buy ticket

Exercise.
What can we say about Ann, Bob, and Charlie ?

Empirical measures of risk aversion

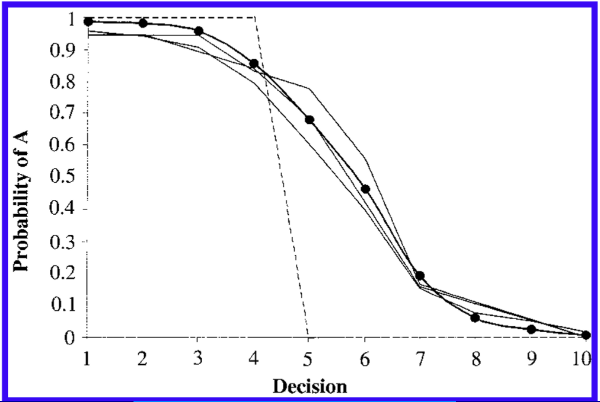
Empirical measures of risk aversion : Holt and Laury, 2002

TABLE 1—THE TEN PAIRED LOTTERY-CHOICE DECISIONS WITH LOW PAYOFFS

Option A	Option B	Expected payoff difference
1/10 of \$2.00, 9/10 of \$1.60	1/10 of \$3.85, 9/10 of \$0.10	\$1.17
2/10 of \$2.00, 8/10 of \$1.60	2/10 of \$3.85, 8/10 of \$0.10	\$0.83
3/10 of \$2.00, 7/10 of \$1.60	3/10 of \$3.85, 7/10 of \$0.10	\$0.50
4/10 of \$2.00, 6/10 of \$1.60	4/10 of \$3.85, 6/10 of \$0.10	\$0.16
5/10 of \$2.00, 5/10 of \$1.60	5/10 of \$3.85, 5/10 of \$0.10	−\$0.18
6/10 of \$2.00, 4/10 of \$1.60	6/10 of \$3.85, 4/10 of \$0.10	−\$0.51
7/10 of \$2.00, 3/10 of \$1.60	7/10 of \$3.85, 3/10 of \$0.10	−\$0.85
8/10 of \$2.00, 2/10 of \$1.60	8/10 of \$3.85, 2/10 of \$0.10	−\$1.18
9/10 of \$2.00, 1/10 of \$1.60	9/10 of \$3.85, 1/10 of \$0.10	−\$1.52
10/10 of \$2.00, 0/10 of \$1.60	10/10 of \$3.85, 0/10 of \$0.10	−\$1.85

Empirical measures of risk aversion : Holt and Laury, 2002

In all of our treatments, the majority of subjects chose the safe option when the probability of the higher payoff was small, and then crossed over to Option B without ever going back to Option A. In all sessions, only 28 of 212 subjects ever switched back from B to A in the first low-payoff decision, and only 14 switched back in the final low-payoff choice. Fewer than one-fourth of these subjects switched back from B to A more than once. The number of such switches was even lower for the high-payoff choices,



Empirical measures of risk aversion : Holt and Laury, 2002

TABLE 3—RISK-AVERSION CLASSIFICATIONS BASED ON LOTTERY CHOICES

Number of safe choices	Range of relative risk aversion for $U(x) = x^{1-r}/(1-r)$	Risk preference classification	Proportion of choices		
			Low real[a]	20x hypothetical	20x real
0–1	$r < -0.95$	highly risk loving	0.01	0.03	0.01
2	$-0.95 < r < -0.49$	very risk loving	0.01	0.04	0.01
3	$-0.49 < r < -0.15$	risk loving	0.06	0.08	0.04
4	$-0.15 < r < 0.15$	risk neutral	0.26	0.29	0.13
5	$0.15 < r < 0.41$	slightly risk averse	0.26	0.16	0.19
6	$0.41 < r < 0.68$	risk averse	0.23	0.25	0.23
7	$0.68 < r < 0.97$	very risk averse	0.13	0.09	0.22
8	$0.97 < r < 1.37$	highly risk averse	0.03	0.03	0.11
9–10	$1.37 < r$	stay in bed	0.01	0.03	0.06

[a] Average over first and second decisions.

Other measures of risk aversion in the literature

BDM, Based on eliciting certainty equivalents. *Becker, G. M., DeGroot, M. H., Marschak, J., 1964. Measuring utility by a single-response sequential method. Behavioral Science 9, 226–232.*

Eckel and Grossman, *Eckel, C. C., & Grossman, P. J. (2008). Forecasting risk attitudes : An experimental study using actual and forecast gamble choices. Journal of Economic Behavior & Organization, 68(1), 1-17.*

Gneezy and Potters, *Gneezy, U., & Potters, J. (1997). An experiment on risk taking and evaluation periods. The quarterly journal of economics, 112(2), 631-645.*

Chapter 6: Insurance

Demand of Insurance : Summary

Initial wealth W, prob. of loss p, loss amount X.

Implicit lottery $L = (W - X, p; W, 1 - p)$

- Full insurance (cover full X), insurance premium (fee) π.

If $\pi = pX$ (expected loss) :

- Risk neutral is indifferent
- Risk averse buys full insurance

If $\pi > pX$:

- Risk averse buy full insurance if $\pi \leq pX + RP_L$

Demand of Insurance : Summary

Partial Insurance :

The individual pays a percentage $\alpha \in (0,1)$ of the insurance premium, and if the bad state occurs, the insurance covers $\alpha \times X$.

Optimization problem :

Find α to maximize

$$EU(\alpha) = p\,u(W - X - \alpha\pi + \alpha X) + (1-p)u(W - \alpha\pi).$$

This has the F.O.C. :

$$\frac{\partial EU(\alpha)}{\partial \alpha} = pu^{'}(W - X - \alpha\pi + \alpha X)(X - \pi) - (1-p)u^{'}(W - \alpha\pi)\pi = 0,$$

so

$$\frac{p(X-\pi)}{(1-p)\pi} = \frac{u^{'}(W - \alpha\pi)}{u^{'}(W - X - \alpha\pi + \alpha X)}.$$

Demand of Insurance : Summary

We can rearrange the F.O.C. into

$$\frac{u^{'}(W - X - \alpha\pi + \alpha X)}{(1-p)\pi} = \frac{u^{'}(W - \alpha\pi)}{p(X-\pi)},$$

Heterogeneity in Preferences

Since CE and RP are related to the demand of full insurance, we can say that

- The more risk-averse person is more willing to pay for insurance.
  - Pay higher insurance premiums for full coverage.
  - Choose greater percentage coverage for a given risk premium.

Supply of Insurance

What is Insurance ?

- Insurance : trading a commitment “pay a small amount now, and the insurer covers large potential losses”.

Is it sustainable ?

- Single client example : 1% chance of a \$100,000 disaster with premium \$1,000 $\Rightarrow$ huge risk for insurer.
- Higher premium ? Is the insurer insane ?

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- Business not sustainable without **pooling**.

Is it sustainable ?

- Single client example : 1% chance of a \$100,000 disaster with premium \$1,000 $\Rightarrow$ huge risk for insurer.
- Higher premium ? Is the insurer insane ?
- Business not sustainable without **pooling**.
- **Law of Large Numbers (LLN)** :
  - Sample average of many independent risks $\rightarrow$ true mean.
  - Many clients $\Rightarrow$ risk is pooled, insurer can pay claims reliably.

Expected Profits

- Consider a loss X with probability p.
- Premium π
- N clients (big N)

Revenue for the insurer : $N\pi$

Expected cost : $(Np)X$ *(by the LLN we expect only Np clients to suffer the loss).*

Expected profit :

$$\mathbb{E}[\text{Profit}] = N\pi - NpX = N(\pi - pX)$$

When are profits zero ?

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- When $\pi = pX$ **(Actuarially Fair Premium)**

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When are profits zero ?

- When $\pi = pX$ **(Actuarially Fair Premium)**

If $\pi > pX \Rightarrow \mathbb{E}[\text{Profit}] > 0$

Example

Going back to the previous example, 1% chance of a $100,000.

Consider $\pi = \$1,000$ (actuarially fair)

- 2,000 clients, each paying premium $\$\pi$.
- Expected claims (1% suffer damage) : $20 \times 100,000 = \$2,000,000$.
- Total premiums collected : $2,000 \times 1,000 = \$2,000,000$.

It is sustainable in the sense of : By LLN, Expected profit **NEVER** negative.

With an actuarially fair premium : Expected profits are zero.

Administrative Costs

- Real insurance has additional costs (admin, operations, etc.).
- Real-world premium :

$$\pi = \pi^* + \text{ad}$$

- Premiums slightly above actuarially fair to cover costs and remain sustainable.

Real-World Insurance Contracts

Key Features of Insurance Contracts

- Coverage Limit
- Deductible
- Premium
- Co-insurance

Premium

- Price paid for insurance coverage (monthly, quarterly, annually).
- Determined by expected loss, administrative costs, and contract features.
- π in our framework.

Coverage Limit

- Maximum amount insurer will pay for a covered loss.
- Any excess must be borne by the insured.

Example :
Car insurance coverage limit $20,000. Accident damage $25,000. Insurer pays $20,000, insured pays remaining $5,000.

Deductible

- Amount paid out-of-pocket before coverage begins.
- Reduces premiums and mitigates moral hazard.

Example :
Deductible \$1,000. - Claim \$700 $\rightarrow$ no reimbursement. - Claim \$1,500 $\rightarrow$ insurer reimburses \$1,200 (first \$300 completes deductible).

Co-Insurance

- Percentage of covered costs the insured must pay even after the deductible.
- Shares risk between insurer and insured.
- Notice this is a form of partial coverage.

Example :
Policy includes 20% co-insurance. Loss after deductible $10,000. - Insured pays $2,000 (20%) - Insurer covers $8,000

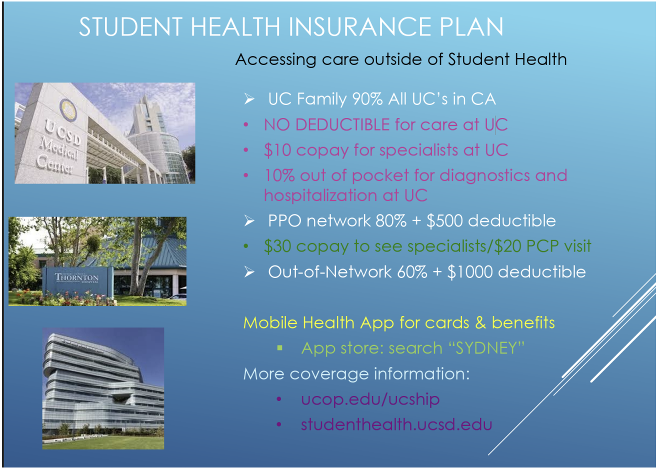
Example UC Ship

<u>S</u>TUDENT <u>H</u>EALTH <u>I</u>NSURANCE <u>P</u>LAN

All students are automatically enrolled in UCSHIP as part of the registration process.

- Cost for AY25/26:
  - $987.00 per quarter (fall, winter, and spring)
  - Spring includes summer
- Most services at SHS are at no cost
- Includes medical, dental, pharmacy & vision
- Worldwide coverage

Example UC Ship



Example UC Ship

Important Questions	Answers	Why This Matters:
What is the overall deductible?	There is no deductible for UC Family providers. For network providers: $500/ person or $1000/family; Out-of-network provider: $1000/person or $2000/family.	Generally, you must pay all the costs from providers up to the deductible amount before this plan begins to pay. If you have other family members on the plan, each family member must meet their own individual deductible until the total amount of deductible expenses paid by all family members meets the overall family deductible.
Are there services covered before you meet your deductible?	Yes, network preventive services, emergency room, urgent care, acupuncture, chiropractic, physician office visits, family planning, medical evacuation, repatriation and prescription drugs.	This plan covers some items and services even if you haven't yet met the deductible amount. But a copayment or coinsurance may apply. For example, this plan covers certain preventive services without cost-sharing and before you meet your deductible. See a list of covered preventive services at https://www.healthcare.gov/coverage/preventive-care-benefits
Are there other deductibles for specific services?	Yes. Pediatric dental: $60/person or $120/family. There are no other specific deductibles.	You must pay all of the costs for these services up to the specific deductible amount before this plan begins to pay for these services.
What is the out-of-pocket limit for this plan?	For UC family providers/network providers: $4500/person or $9000/family. For out-of-network providers: $9000/person or $18000/family.	The out-of-pocket limit is the most you could pay in a year for covered services. If you have other family members in this plan, they have to meet their own out-of-pocket limits until the overall family out-of-pocket limit has been met.
What is not included in the out-of-pocket limit?	Premiums, balance-billed charges and health care this plan doesn't cover.	Even though you pay these expenses, they don't count toward the out-of-pocket limit.

Example UC Ship

Common Medical Event	Services You May Need	UC Family Provider (You will pay the least)	Network Provider (You will pay the least)	Out-of-Network Provider (You will pay the most)	Limitations, Exceptions, & Other Important Information
If you visit a health care provider's office or clinic	Primary care visit to treat an injury or illness	No charge Student Health Services (SHS); $5 copayment/visit (UC Family). Deductible does not apply.	$20 copayment/visit. Deductible does not apply.	40% coinsurance	——————none——————
	Specialist visit	No charge (SHS); $10 copayment/visit (UC Family). Deductible does not apply.	$30 copayment/visit. Deductible does not apply.	40% coinsurance	——————none——————
	Preventive care/screening/ immunization	No charge. Deductible does not apply.	No charge. Deductible does not apply.	Not covered	You may have to pay for services that are not preventive. Ask your provider if the services needed are preventive. Then check what your plan will pay for.
If you have a test	Diagnostic test (x-ray, blood work)	10% coinsurance. Deductible does not apply.	20% coinsurance	40% coinsurance	——————none——————
	Imaging (CT/PET scans, MRIs)	10% coinsurance. Deductible does not apply.	20% coinsurance	40% coinsurance	You should refer to your policy or plan document for details (*see pages 30, 33, 37, 39, & 67).
If you need drugs to treat your illness or	Generic drugs	$5 copayment (SHS), $10 copayment (UC Family)/prescription.	$10 copayment/ prescription at retail pharmacies. Deductible	$10 plus any amount over the allowed amount/ prescription. Deductible	Covers up to a 30-day supply of medications and up to 180-days for oral contraceptives at retail or

Issues with Insurance

Feature-Table

Plan Name	Annual Premium	Annual Deducible	Coinsurance Rate	Maximum Out of Pocket
Purple*	$817	$1,000	10%	$3,500
Blue	$1,321	$750	10%	$3,250
Red	$1,419	$500	10%	$3,000
Black	$1,957	$250	10%	$2,750

* Denotes the dominant option (Not shown to subjects)

Real-World Insurance Markets : Are They Competitive ?

- Health insurance markets in the U.S. are highly concentrated : a few firms capture most enrollments.
- Insurers can then raise premiums above competitive levels or reduce benefits.

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- Insurers can then raise premiums above competitive levels or reduce benefits.
- **AMA report (2013 and 2024) :**
  - In 89% of MSAs, at least one insurer had $\geq 30\%$ market share.
  - In 47% of MSAs, one insurer had $\geq 50\%$ share.
- **NAIC mid-2024 report :** Net income \$16B (14% drop from 2023), 2.7% profit margin.

Adverse Selection : Buyers Know More Than Sellers

- High-risk individuals are more likely to buy insurance than low-risk individuals.
- This increases premiums, driving out lower-risk participants (*death spiral*).

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  - Healthy individuals opt out $\Rightarrow$ pool becomes riskier $\Rightarrow$ premiums raise $\Rightarrow$ relatively healthier customers opt out...

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- **Example :** Health insurance :
  - individuals expecting high medical bills opt in.
  - Healthy individuals opt out $\Rightarrow$ pool becomes riskier $\Rightarrow$ premiums raise $\Rightarrow$ relatively healthier customers opt out...
- **Responses :** risk-based pricing, medical exams, pre-existing condition clauses.
- Note : These are not allowed in U.S. health insurance after the ACA (2010).

Moral Hazard : Taking More Risks Because of Insurance

- Insured individuals may engage in riskier behavior, knowing they are protected.
- **Example :** Someone with full auto insurance may drive less carefully.
- Problem : probability of a bad state increases $\Rightarrow$ actuarially fair premium is insufficient.
- **Solutions :**
  - Deductibles
  - Co-insurance
  - Monitoring mechanisms (e.g., maintenance records)

Alternative : Social Insurance

- Alternative to private insurance when market frictions are high.
- Built on solidarity : everyone contributes, everyone is covered.
- **Example scheme :**
  - Workers pay a fixed % of salary.
  - Others : Employers and government also contribute.
  - Coverage for workers and dependents, regardless of income, disease, or pre-existing conditions.
- Features :
  - No opt-out (young/healthy subsidize old/sick).
  - Problems : overcrowding, overuse, reduced individual freedom.

Chapter 7: Risk

Risk

We never really defined what risk is (or more importantly, when a lottery is riskier).

Variance is used in finance as a measure of risk (variability)

Risk

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Variance is used in finance as a measure of risk (variability)

Now : *Lottery* q *is riskier than lottery* p*, if every risk-averse person prefers* p *over* q*.*

Revisiting Mean-Var criterion

Assume $u(z) = az - bz^2$ (with $b \geq 0$

- Risk aversion : $u^{''} = -2b < 0$.
- If X is a random variable.
- $EU(X) = aE[X] + bE[X^2]$
  (using $VAR(X) = E[X^2] - (E[X])^2$)

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- $EU(X) = aE[X] + bE[X^2]$
  (using $VAR(X) = E[X^2] - (E[X])^2$)
- $EU(X) = aE[X] - b(E[X])^2 - bVAR(X)$

More variance $\rightarrow$ less expected utility for risk-averse people *(with quadratic utility)*.

Two Criteria to evluate which lottery is riskier

- We will only compare lotteries with same expected value (partial ordering).
- We will look at criteria that looks at the entire distribution (like the FOSD).

Mean Preserving Spread

Given some lottery p,

q is a mean preserving spread if :

$$q = p + \epsilon$$

where $E[\epsilon] = 0$ (noise)

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Example

- if $p = (10, 1)$
- $e = (-5, 0.5; +5, 0.5)$
- Then $q = p + \epsilon = (10 - 5, 0.5; 10 + 5, 0.5)$ is a mean-preserving spread of p.

Notice that $E[q] = E[p] = 10$.

Same mean, but some probability weight is moved to the extremes of the distribution

Mean Preserving Spread

Another example : If $p = (10, 0.4; 20, 0.6)$

$$q = (0, 0.2; 10, 0.3; 20, 0.2; 30, 0.3)$$

is a mean preserving spread of p :

$q = p + \epsilon$ with $\epsilon = (-10, 0.5; +10, 0.5)$

Mean Preserving Spread

If $q, p \in \Delta(Z)$ and q is a mean-preserving spread of q,

then, for **every** risk averse decision maker $p \succsim q$.

According to this classification, q is riskier than p!

Second Order Stochastic Dominance

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$$\sum_{i=1}^{k} F_p(x_i) \leq \sum_{i=1}^{k} F_q(x_i), \quad \text{for all } k = 1, \ldots, n,$$

Second Order Stochastic Dominance

Example. Consider two lotteries p and q :

$$p = (0, 0.25; 1, 0.25; 2, 0.25; 3, 0.25) \qquad q = (0, 0.4; 1, 0.1; 2, 0.1; 3, 0.4)$$

The CDFs are :

$$F_p = \begin{cases} 0.25 & 0 \leq z < 1, \\ 0.5 & 1 \leq z < 2, \\ 0.75 & 2 \leq z < 3, \\ 1 & 3 \leq z, \end{cases} \qquad F_q = \begin{cases} 0.4 & 0 \leq z < 1, \\ 0.5 & 1 \leq z < 2, \\ 0.6 & 2 \leq z < 3, \\ 1 & 3 \leq z, \end{cases}$$

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Example. Consider two lotteries p and q :

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Now compute the cumulative sums :

$F_p(0) = 0.25 < F_q(0) = 0.4$

$F_p(0) + F_p(1) = 0.75 < 0.9 F_q(0) + F_q(1)$

$F_p(0) + F_p(1) + F_p(2) = 1.5 \leq 1.5 = F_q(0) + F_q(1) + F_q(2)$

$F_p(0) + F_p(1) + F_p(2) + F_p(3) = 2.5 \leq 2.5 = F_q(0) + F_q(1) + F_q(2) + F_q(3)$

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In the previous example, no lottery FOSD the other.

In general :

$$FOSD \Rightarrow SOSD$$

But the inverse is not true !

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Neccesary conditions for SOSD :

If $p \succsim_{SOSD} q$, the following **must** be true

- $E[p] \geq E[q]$
- $min(p) \geq min(q)$

Second Order Stochastic Dominance

If $q, p \in \Delta(Z)$ and $p \succsim_{SOSD} q$,

then, for **every** risk averse decision maker $p \succsim q$.

According to this classification, q is riskier than p !