Replication Exercise -Set Identification, Mres

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Exercise 1

- We can interpret β_l as the increase in production given an increase in labor, keeping prices fixed. Derivation in section 1
- Now we need to assume that both inputs are flexible and static. This is totally unrealistic for capital, as it is quite unlikely that farmers invest in capital non-dynamically (tools/machines last periods, 1920..old technology, even more) and that can quickly adjust their levels of capital in the short run. I derive results in section 1
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Exercise 2

The moment condition are

$$\mathbf{E}\left[\nu_{f,t}|\mathcal{I}_{f,t-1}\right] = 0$$

Given the structure of the problem we can use $l_{f,t-1}, l_{f,t-2}\dots$ and $k_{f,t}, k_{f,t-1}\dots$ as instruments.
Then we know that $y_{f,t} = \beta_0 + \beta_l l_{f,t} + \beta_k k_{f,t} + \omega_{f,t}$, and we have the moments:

$$\mathbf{E}\left[\hat{\omega_{f,t}} - \rho\hat{\omega_{f,t-1}} - \beta^a A_{f,t-1}|\mathcal{I}_{f,t-1}\right] = 0$$

Where we $\hat{\omega_{f,t}} = y_{f,t} - \beta_0 + \beta_l l_{f,t} + \beta_k k_{f,t}$ And we can estimate the following GMM:

$$\mathbf{E}\left[Z_{it-1} \otimes (\hat{\omega_{f,t}} - \rho\hat{\omega_{f,t-1}} - \beta^a A_{f,t-1})\right] = 0$$

where, given the assumptions of the exercise, $Z_{it-1} = \begin{pmatrix} 1 \\ l_{f,t-1} \\ l_{f,t-2} \\ k_{f,t} \\ k_{f,t-1} \end{pmatrix}$

It is important to include $\beta^a A_{f,t-1}$ in the moment function to support that $\mathbf{E}\left[\nu_{f,t}|\mathcal{I}_{f,t-1}\right] = 0$. Mergers and acquisitions are not random events: high-productivity firms are more likely to acquire other firms, while low-productivity firms are more likely to be acquired. As a result, $A_{f,t-1}$ is correlated with past productivity and belongs to the firm information set at t-1, thus the previous condition does not hold.

1 Appendix

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Under perfect competition and one mobile input only (labor) we have that the firm solves:

$$\min_{L_{f,t},\,K_{f,t}\geq 0} L_{f,t}W_{f,t} + K_{f,t}R_{f,t}$$
$$\text{s.t. } \bar{Q}_{f,t} = L_{f,t}^{\beta_l}K_{f,t}^{\beta_k}\Omega_{f,t}$$

From the first order condition on labor:

$$W_{f,t} = \lambda\beta_l\left(\frac{Q_{f,t}}{L_{f,t}}\right)$$

By definition: $\mu_{f,t} = \frac{p_{f,t}}{\lambda_{f,t}}$

$$W_{f,t} = \frac{p_{f,t}}{\mu_{f,t}} \beta_l \left(\frac{Q_{f,t}}{L_{f,t}} \right)$$

Under perfect competition $\mu_{f,t}$=1, then rearranging we get:

$$\beta_l = \frac{W_{f,t}L_{f,t}}{P_{f,t}Q_{f,t}}$$

Which we can estimate with a simple average.

Text B

From the same problem above, if we take first order conditions with respect to labor and capital we get:
$W_{f,t}L_{f,t} = \lambda\beta_l(Q_{f,t})$
$R_{f,t}K_{f,t} = \lambda\beta_k(Q_{f,t})$
If we sum the two expressions: $W_{f,t}L_{f,t} + R_{f,t}K_{f,t} = \lambda\beta_l(Q_{f,t}) + \lambda\beta_k(Q_{f,t})$
which reduces to:
$W_{f,t}L_{f,t} + R_{f,t}K_{f,t} = \lambda Q_{f,t}$
Then if we take 1 or 2 and divide by the same in both sides:
$\frac{W_{f,t}L_{f,t}}{W_{f,t}L_{f,t}+R_{f,t}K_{f,t}} = \frac{\lambda\beta_l(Q_{f,t})}{\lambda Q_{f,t}}$
This simplifies: $\frac{W_{f,t}L_{f,t}}{W_{f,t}L_{f,t}+R_{f,t}K_{f,t}} = \beta_l$
Same argument for the other. The markup also comes from the expression from above:
$\lambda = \frac{W_{f,t}L_{f,t}}{Q_{f,t}\beta_l}$ If we substitute β_l into this expression and we rearrange we have:
$\lambda = \frac{W_{f,t}L_{f,t}+R_{f,t}K_{f,t}}{Q_{f,t}}$
Thus: $\mu = \frac{pQ_{f,t}}{W_{f,t}L_{f,t}+R_{f,t}K_{f,t}}$

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