Exam- Quantitative Methods

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Q1

A stationary recursive competitive equilibrium consists of (i) value and policy functions $V(a,s,e)$, $c(a,s,e)$, $a'(a,s,e)$, (ii) factor prices (r,w), (iii) aggregate quantities (K,N), and (iv) a stationary distribution $\Gamma(a,s,e)$ over individual states, such that:

1. **Households.** Given prices (r,w), the value function $V(a,s,e)$ solves

$$V(a,s,e) = \max_{c,a'\geq 0} \left\{ u(c) + \beta \sum_{s'} \sum_{e'} P_s(s'|s) P_e(e'|e)\, V(a',s',e') \right\}$$

   subject to the budget constraint

$$c + a' = w\,[e + b(1-e)]\, s\,(1-\tau_w) + (1+r)a,$$

   and $c \geq 0$. The associated policy functions are denoted $c(a,s,e)$ and $a'(a,s,e)$.

2. **Firms.** Competitive firms have a standard Cobb–Douglas technology

$$Y = zK^{\alpha}N^{1-\alpha}.$$

   Factor prices equal marginal products:

$$r = \alpha z K^{\alpha-1}N^{1-\alpha} - \delta, \qquad w = (1-\alpha)zK^{\alpha}N^{-\alpha}.$$

3. **Aggregation and market clearing.** The stationary distribution $\Gamma(a,s,e)$ over individual states is consistent with the optimal policy $a'(a,s,e)$ and the exogenous transition matrices P_s and P_e. Given Γ, aggregate capital and effective labor are:

$$K = \int a\, d\Gamma(a,s,e), \qquad N = \int e\, s\, d\Gamma(a,s,e).$$

   The capital market clears when the K used by firms equals the K implied by the distribution of assets:

$$K = \int a\, d\Gamma(a,s,e).$$

   The labor market clears when the N in the production function equals the aggregate effective labor supplied by employed workers:

$$N = \int e\, s\, d\Gamma(a,s,e).$$

   Goods market clearing then holds by Walras' law.

4. **Government budget.** The government finances unemployment benefits by a proportional tax on labor income. In the stationary equilibrium, its budget is balanced:

$$\underbrace{\tau_w\, w \int [\,e + b(1-e)\,]\, s\, d\Gamma(a,s,e)}_{\text{labor income tax revenues}} \;=\; \underbrace{w\, b \int (1-e)\, s\, d\Gamma(a,s,e)}_{\text{UI payments}}.$$