Part 2 IO problem set

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1.1

- $(0,0)$
$$\Pi^M_{im} \leq 0 \iff \hat{\Pi}^M_{im} + \epsilon_{im} \leq 0 \iff \epsilon_{im} \leq -\hat{\Pi}^M_{im}, \qquad \forall i \in \{1,2\}.$$
- $(1,1)$
$$\hat{\Pi}^D_{im} + \epsilon_{im} \geq 0 \iff \epsilon_{im} \geq -\hat{\Pi}^D_{im}, \qquad \forall i \in \{1,2\}.$$
- Unique entry by firm j occurs when:
  1. $$\hat{\Pi}^M_{jm} + \epsilon_{jm} \geq 0 \cap \hat{\Pi}^M_{-jm} + \epsilon_{-jm} \leq 0;$$
  2. $$\hat{\Pi}^M_{jm} + \epsilon_{jm} \geq 0 \cap \hat{\Pi}^M_{-jm} + \epsilon_{-jm} \geq 0 \cap \hat{\Pi}^D_{-jm} + \epsilon_{-jm} \leq 0.$$
- The region of multiplicity is given by
$$-\hat{\Pi}^D_{1m} \leq \epsilon_{1m} \leq -\hat{\Pi}^M_{1m} \cap -\hat{\Pi}^D_{2m} \leq \epsilon_{2m} \leq -\hat{\Pi}^M_{2m},$$
corresponding to Region 5 in Figure 1.

Figure 1 illustrates the partition of the (ϵ_1, ϵ_2) space into the nine regions described above.

1.2

The remaining probabilities can be written as follows.

The probability of no entry is

$$\mathbf{P}(D_1 = 0, D_2 = 0) = \int_{-\infty}^{-\pi_1^M} \int_{-\infty}^{-\pi_2^M} \phi_2(u_1, u_2; \rho)\, du_1\, du_2 = \Phi_2(-\pi_1^M, -\pi_2^M; \rho).$$

Assume now that firm 1 moves first, so that equilibrium multiplicity is resolved in favor of outcome $(1,0)$ (Region 4 in Figure 1). Then

$$\mathbf{P}(1,0) = \int_{-\pi_1^M}^{-\pi_1^D} \int_{-\infty}^{-\pi_2^D} \phi_2(u_1, u_2; \rho)\, du_2\, du_1 = \Phi_2(\infty, -\pi_2^D; \rho) - \Phi_2(-\pi_1^M, -\pi_2^D; \rho).$$

Finally, $\mathbf{P}(0,1)$ is implied by the remaining probabilities.

1.3

Table 1 in the Appendix reports estimation results obtained under alternative sign restrictions on the correlation parameter ρ. The log-likelihood is lower when ρ is restricted to be positive, indicating a worse fit relative to the specification with negative correlation. Coefficient estimates also differ substantially across the two cases. In particular, when $\rho < 0$, the estimated competition effect becomes statistically insignificant. This is consistent with the interpretation that negative correlation in unobserved shocks reflects greater heterogeneity between firms, reducing the ability of the model to separately identify competitive effects from idiosyncratic profitability differences.

Table 2 reports estimates of ρ when it is freely estimated. The estimated value is approximately 0.3, suggesting a moderate positive correlation in unobserved shocks. The table also reports results obtained under alternative assumptions about the identity of the firm that moves first. The main differences across specifications arise in the estimated competition effect. When firm 1 (Walmart) is assumed to enter first, the estimated impact of competition on profits is smaller.

This pattern reflects the role of δ and ρ in rationalizing observed entry decisions. When Walmart is assumed to move first, the model attributes a larger share of entry asymmetries to differences in unobserved profitability, implying weaker competitive effects. Conversely, when the smaller firm is assumed to move first, the model requires stronger competitive effects and more similar shocks to rationalize observed outcomes.

Table 3 confirms this interpretation. The estimated value of ρ remains close to that obtained under the assumption that firm 2 moves first, while the estimated effect of Walmart's entry on Kmart's profits increases. This is consistent with Walmart being the larger firm, whose entry plausibly exerts a stronger competitive pressure on Kmart.

1.4

Table 4 reports estimation results under fixed values of the correlation parameter ρ equal to 0.5 and 1. When shocks are highly correlated, the model attributes a larger share of the observed variation in entry outcomes to strategic interaction, leading to a higher estimated competition effect. Conversely, when shocks are less correlated, heterogeneity in unobserved profitability accounts for a greater fraction of entry decisions, and the estimated competition effect decreases. The specification with $\rho = 1$ converges more quickly and attains a slightly lower log-likelihood, reflecting the tighter structure imposed by perfectly correlated shocks.

Table 5 reports results for the same model when ρ is estimated freely. In this specification, the competition effect and the correlation parameter are not separately identified. The reason is that, in this simplified setting, firms are symmetric in observables, and the only source of cross-firm variation is the unobserved shock. Allowing ρ to approach one enables the model to rationalize the observed distribution of the number of entrants across markets without requiring a distinct competition effect, resulting in a flat likelihood in the (δ, ρ) direction.

This lack of identification also explains why the effects of Z_m cannot be estimated in this model. Since outcomes $(1, 0)$ and $(0, 1)$ are aggregated into the same category $n = 1$, there is no variation in the data that distinguishes between the identities of entering firms. As a result, parameters that shift relative profitability across firms are not identified.

Table 6 presents results for a model with identical firms. This specification is equivalent to the model in Part 1 under the restriction $\rho = 1$, since perfect correlation in a bivariate normal distribution implies identical payoff shocks.

Finally, Table 7 reports results from an ordered logit model for the number of entrants. This specification yields qualitatively similar results. By construction, firms are symmetric, and the ordered logit rationalizes observed entry frequencies through threshold parameters of a latent profitability index. These cutoffs capture market-level profitability and competitive pressures in a reduced-form manner, analogous to the role played by the competition effect in the structural model.

2

2.1

The game can be summarized in 2 equalities and 4 inequalities (or 8 inequalities). Let's start with the equalities:

1. (1,1) $\rightarrow \mathbf{P}(\epsilon_1 \geq -\pi_1^D, \epsilon_2 \geq -\pi_2^D, \rho) = \hat{P_{1,1}} \rightarrow M_1 = \hat{P_{1,1}} - \mathbf{P}(\epsilon_1 \geq -\pi_1^D, \epsilon_2 \geq -\pi_2^D, \rho) = 0$
2. (0,0) $\mathbf{P}(\epsilon_1 \leq -\pi_1^M, \epsilon_2 \leq -\pi_2^M, \rho) = \hat{P_{0,0}} \rightarrow M_2 = \hat{P_{0,0}} - \mathbf{P}(\epsilon_1 \leq -\pi_1^M, \epsilon_2 \leq -\pi_2^M, \rho) = 0$

   Now let's consider the problematic cases (1,0), (0,1). For these, we have already shown that $P_{0,1}$ or $P_{1,0}$ can not be point identified in the data -unless we make assumptions on who moves first etc (as done above) because there is a region of multiplicity where multiple equilibria can rationalize the data.

   However, these probabilities are not devoid of empirical content. Let's take (1,0). We know that the probability we estimate in the data $\hat{P_{0,1}}$ will be bounded between:

   $P^l_{0,1} \leq \hat{P_{0,1}} \leq P^u_{0,1}$, where the lower bound is just the region where 0,1 is equilibrium not in multiplicity. The upper bound is the opposite: the region when we assume the selection function of the multiplicity area always selects this outcome. This gives us two inequalities:

3. $P^l_{0,1} \leq \hat{P_{0,1}} \rightarrow M_3 = \hat{P_{0,1}} - P^l_{0,1} \geq 0$
4. $\hat{P_{0,1}} \leq P^u_{0,1} \rightarrow M_4 = P^u_{0,1} - \hat{P_{0,1}} \geq 0$

   Same exercise can be done with $P_{1,0}$. This leads with 8 moments, 2 equalities (4 inequalities) and 4 inequalities:

5. $P^l_{1,0} \leq \hat{P_{1,0}} \rightarrow M_3 = \hat{P_{1,0}} - P^l_{1,0} \geq 0$
6. $\hat{P_{1,0}} \leq P^u_{1,0} \rightarrow M_4 = P^u_{1,0} - \hat{P_{1,0}} \geq 0$

2.2

First, $\hat{P_{1,1}}, \hat{P_{0,0}}, \hat{P_{1,0}}, \hat{P_{0,1}},$, can be directly estimated with the data. This can be done with non-parametric methods. (do we need to condition on X?).

Second, we will estimate the rest using model implied probabilities - recycling some results from above, actually -all conditional to X. Let's go by parts:

(a) $p_{00} = \Phi_2(-\pi_1^M, -\pi_2^M, \rho)$
(b) $p_{11} = 1 - \Phi_2(-\pi_1^d, \infty, \rho) - \Phi_2(-\pi_1^d, -\pi_2^d, \rho) - \Phi_2(\infty, -\pi_2^d, \rho)$
(c) For either $(1,0)$ or $(0,1)$ we have:
   - $P^l_{1,0} = \Phi_2(\infty, -\pi_2^d, \rho) - \Phi_2(-\pi_1^m, -\pi_2^d, \rho)$
   - $P^u_{1,0} = 1 - \Phi_2(-\pi_1^m, -\pi_2^d, \rho) - p_{11} - p_{00}$
   - By symmetry $(0,1)$ is just the same reversed.

Where in this case π_i^m, π_i^d are profits under monopoly or duopoly as specified in the beginning for firm $i \in \{1,2\}$.

2.3

We can define the following objective function:

$$Q(\theta) = \int h(M_j(\theta))_+ dFx$$

Where j are the amount of moment inequalities we have and where M designates the moment, which are functions of θ.

The inequalities are written in a positive way. In this sense, we want to penalize deviations for the implied inequalities in the model, so we will construct $Q(\theta)$ so that we only count deviations from our model[1].

2.4

2.4 Estimation of the identified set

To estimate the identified set of structural parameters, we follow the moment-inequality approach of **Ciliberto and Tamer (2009)** and **Chernozhukov, Hong, and Tamer (2007)**. The procedure consists of the following steps.

1. **Discretization of market characteristics.** Let X^c denote the vector of continuous market characteristics. We discretize the support of X^c into two bins for each continuous variable (dbenton, spc, population, urban). Within each bin, we replace the continuous variable by its sample mean. This discretization induces a finite support for (X, Z) and allows for nonparametric estimation of conditional choice probabilities.

2. **Nonparametric estimation of conditional probabilities.** For each combination (x, z) in the discretized support of (X, Z), we estimate the conditional distribution of market outcomes nonparametrically:

$$\hat{P}_n(Y = y \mid X = x, Z = z) = \frac{\sum_{i=1}^{n} \mathbf{1}\{Y_i = y, X_i = x, Z_i = z\}}{\sum_{i=1}^{n} \mathbf{1}\{X_i = x, Z_i = z\}}, \qquad y \in \{(1,1),(1,0),(0,1),(0,0)\}.$$

   Each distinct realization of (x, z) defines a market type in the estimation sample. For each market type, we therefore obtain four empirical conditional probabilities $(\hat{p}_{11}, \hat{p}_{10}, \hat{p}_{01}, \hat{p}_{00})$.

3. **Construction of model-implied moments.** Let θ denote the vector of structural parameters, which includes coefficients on market characteristics, firm-specific intercepts, the common competition effect, and the correlation parameter of the payoff shocks.[2] For a given θ:

   (a) We compute monopoly and duopoly profit indices, $\pi_i^M(x, z; \theta)$ and $\pi_i^D(x, z; \theta)$, using the specified linear profit functions.

   (b) Assuming jointly normal payoff shocks, we compute the model-implied probabilities for each market type:

$$p_{11}(\theta), \quad p_{00}(\theta), \quad p_{10}^L(\theta), \quad p_{10}^U(\theta), \quad p_{01}^L(\theta), \quad p_{01}^U(\theta),$$

   where $p_{10}^L(\theta)$ and $p_{01}^L(\theta)$ denote the lower bounds implied by regions of unique equilibrium, and $p_{10}^U(\theta)$ and $p_{01}^U(\theta)$ denote the corresponding upper bounds implied by equilibrium multiplicity.

   (c) We construct a vector of moment conditions $m(\theta \mid x, z)$ that enforces equality restrictions for outcomes $(1, 1)$ and $(0, 0)$ and inequality restrictions for outcomes $(1, 0)$ and $(0, 1)$.

4. **Criterion function.** We define the sample criterion function as

$$Q_n(\theta) = \int H(m(\theta \mid x, z))_+ \, d\hat{F}_{X,Z}(x, z),$$

   where $H(\cdot)_+$ denotes a nonnegative loss function that penalizes violations of the moment inequalities, and $\hat{F}_{X,Z}$ is the empirical distribution of (X, Z). The identified set is approximated by the collection of parameter values θ for which $Q_n(\theta)$ is close to zero.

   Given the size θ we define the grid of parameters using a Uniform distribution for each parameter. We take previous models as reference, and define bounds on the uniform to include them. We run 2000 simulations of θ. In table 2 we can see the distribution of $Q(\theta)$.

   Inference this I would leave if there is time left...It can be done, but it is more prioritary to finish the rest. We will use CHT to make inference...

[1] We could also write the two equalities as 4 inequalities and add them all up.

[2] Specifically, θ includes coefficients on population, spc, urban, and dbenton; regional indicators; a common competition parameter; firm-specific intercepts; and the correlation parameter ρ.

Appendix

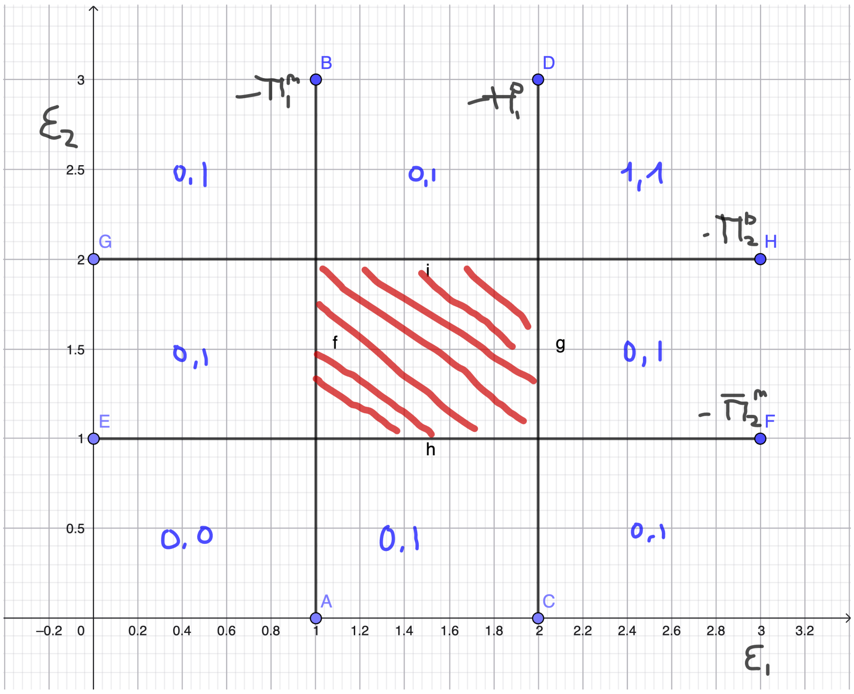

Figure 1: Equilibrium regions.

Table 1: Regression Results

	(1)	(2)
xb		
population	1.780***	1.467***
	(0.076)	(0.076)
spc	1.578***	1.353***
	(0.120)	(0.122)
urban	1.275***	1.034***
	(0.171)	(0.155)
zw		
dbenton	-1.048***	-0.964***
	(0.080)	(0.085)
southern	0.649***	0.705***
	(0.081)	(0.078)
_cons	-12.576***	-10.341***
	(1.029)	(1.003)
zk		
midwest	0.337***	0.385***
	(0.083)	(0.087)
_cons	-19.939***	-17.676***
	(1.031)	(1.085)
comp		
comp	-0.936***	0.082
	(0.084)	(0.063)
Observations	2065.000	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 2: Regression Results

	(1)	(2)
xb		
population	1.756***	1.851***
	(0.075)	(0.079)
spc	1.572***	1.649***
	(0.119)	(0.124)
urban	1.255***	1.345***
	(0.169)	(0.180)
zw		
dbenton	-1.068***	-1.013***
	(0.084)	(0.085)
southern	0.684***	0.580***
	(0.082)	(0.089)
_cons	-12.347***	-13.470***
	(1.012)	(1.050)
zk		
midwest	0.350***	0.312***
	(0.085)	(0.081)
_cons	-19.982***	-20.932***
	(1.034)	(1.085)
comp		
comp	-0.737***	-0.953***
	(0.102)	(0.103)
rho		
_cons	0.330***	0.633***
	(0.080)	(0.088)
Observations	2065.000	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Regression Results

	(1)
xb	
population	1.853***
	(0.079)
spc	1.657***
	(0.125)
urban	1.354***
	(0.181)
zw	
dbenton	-1.001***
	(0.085)
southern	0.565***
	(0.089)
_cons	-13.596***
	(1.060)
zk	
midwest	0.314***
	(0.081)
_cons	-21.065***
	(1.100)
comp_k	
comp	-0.891***
	(0.137)
comp_w	
comp	-1.054***
	(0.172)
rho	
_cons	0.649***
	(0.088)
Observations	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Regression Results

	(1)	(2)
xb		
population	1.497***	1.642***
	(0.059)	(0.064)
spc	1.261***	1.379***
	(0.085)	(0.093)
urban	1.160***	1.271***
	(0.144)	(0.158)
_cons	-15.534***	-16.564***
	(0.724)	(0.793)
comp		
comp	-1.201***	-2.187***
	(0.064)	(0.070)
Observations	2065.000	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Regression Results

	(1)
xb	
population	1.641***
	(0.067)
spc	1.379***
	(0.094)
urban	1.271***
	(0.158)
_cons	-16.577***
	(0.948)
comp	
comp	-2.156
	(1.421)
rho	
_cons	0.999***
	(0.067)
Observations	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Regression Results

	(1)
xb	
population	1.642***
	(0.064)
spc	1.379***
	(0.093)
urban	1.271***
	(0.158)
_cons	-16.564***
	(0.793)
cut1	
_cons	-2.187***
	(0.070)
Observations	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Regression Results

	(1)
entry	
population	1.642***
	(0.066)
spc	1.379***
	(0.109)
urban	1.271***
	(0.168)
/	
cut1	16.564***
	(0.913)
cut2	18.751***
	(0.943)
Observations	2065.000

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

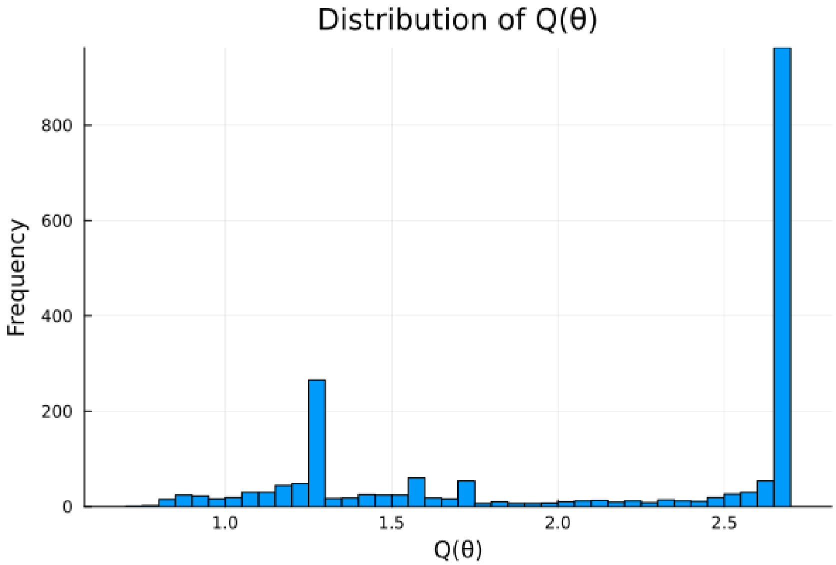

Figure 2: Distribution of the criterion function $Q(\theta)$.