

# Replication Exercise -Set Identification, Mres

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## Exercise 1

- We can interpret  $\beta_l$  as the increase in production given an increase in labor, keeping prices fixed. Derivation in section 1
- Now we need to assume that both inputs are flexible and static. This is totally unrealistic for capital, as it is quite unlikely that farmers invest in capital non-dynamically (tools/machines last periods, 1920..old technology, even more) and that can quickly adjust their levels of capital in the short run. I derive results in section 1
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## Exercise 2

The moment condition are

$$\mathbf{E} [\nu_{f,t} | \mathcal{I}_{f,t-1}] = 0$$

Given the structure of the problem we can use  $l_{f,t-1}, l_{f,t-2} \dots$  and  $k_{f,t}, k_{f,t-1} \dots$  as instruments. Then we know that  $y_{f,t} = \beta_0 + \beta_l l_{f,t} + \beta_k k_{f,t} + \omega_{f,t}$ , and we have the moments:

$$\mathbf{E} [\hat{\omega}_{f,t} - \rho \omega_{f,t-1} - \beta^a A_{f,t-1} | \mathcal{I}_{f,t-1}] = 0$$

Where we  $\hat{\omega}_{f,t} = y_{f,t} - \beta_0 + \beta_l l_{f,t} + \beta_k k_{f,t}$ . And we can estimate the following GMM:

$$\mathbf{E} [Z_{it-1} \otimes (\hat{\omega}_{f,t} - \rho \omega_{f,t-1} - \beta^a A_{f,t-1})] = 0$$

where, given the assumptions of the exercise,  $Z_{it-1} = \begin{pmatrix} 1 \\ l_{f,t-1} \\ l_{f,t-2} \\ k_{f,t} \\ k_{f,t-1} \end{pmatrix}$

It is important to include  $\beta^a A_{f,t-1}$  in the moment function to support that  $\mathbf{E} [\nu_{f,t} | \mathcal{I}_{f,t-1}] = 0$ . Mergers and acquisitions are not random events: high-productivity firms are more likely to acquire other firms, while low-productivity firms are more likely to be acquired. As a result,  $A_{f,t-1}$  is correlated with past productivity and belongs to the firm information set at t-1, thus the previous condition does not hold.

**B** Estimated in the jupyter notebook. When using the instruments reported above I get the following results:

**C**

The key assumption that we need to discuss is whether all inputs are substitutable or not. But now cotton clearly is not substitutable with labor or capital and this justifies the parametrization of the problem as a Leontief model. (i.e cotton is not productive per se without labor or capital).

**D**

Estimated in the Jupyter Notebook. It is less than the markup computed above, just by definition.

## Exercise 3

**A** In the main setup, assuming acquisitions are exogenous and determined before  $\nu$  is realized, the effect of acquisitions is negative (-65). This is quite unreasonable as we would expect merged firms to operate with higher productivity after a merger. What this could reflect is that the assumption of exogeneity of the acquisition is wrong. Maybe firms merge with some knowledge of this transient shock  $\nu_t$ : for example, negotiations between firms may take long and in our dataset merger is defined when it is officially completed but then come months of negotiation, information etc. if the firm expects a negative shock, maybe it accepts the merger and my coefficient is capturing this downward bias.

First case:

$$\text{cov}(A_{t-1}, \nu) = 0$$

Or that  $\mathbf{E}(\nu | \mathcal{I}_{t-1}) = 0$  where  $A_{t-1} \in \mathcal{I}_{t-1}$

Second case:

$$\text{cov}(A_{t-1}, \nu) \neq 0$$

And then  $\mathbf{E}(\nu | \mathcal{I}_{t-1}) \neq 0$  if  $A_{t-1} \in \mathcal{I}_{t-1}$

A solution to this is add instruments in  $\mathbf{Z}$  that explain variation in  $A_{t-1}$  but that are uncorrelated with  $\nu_t$ . A reasonable instrument can be larger lags of the acquisition dummy:  $A_{t-2}, A_{t-3}, \dots$ .

The issue is that mergers are very uncommon and it is very hard to identify the effects of  $\beta^a$  in our case. The estimation is extremely unstable to the selected instruments and, once we demean the variables, we can no longer identify  $\beta^a$ .

## 1 Appendix

### 1

Under perfect competition and one mobile input only (labor) we have that the firm solves:

$$\begin{aligned} & \min_{L_{f,t}, K_{f,t} \geq 0} L_{f,t}W_{f,t} + K_{f,t}R_{f,t} \\ & \text{s.t. } \bar{Q}_{f,t} = L_{f,t}^{\beta_l} K_{f,t}^{\beta_k} \Omega_{f,t} \end{aligned}$$

From the first order condition on labor:

$$W_{f,t} = \lambda \beta_l \left( \frac{Q_{f,t}}{L_{f,t}} \right)$$

By definition:  $\mu_{f,t} = \frac{p_{f,t}}{\lambda_{f,t}}$

$$W_{f,t} = \frac{p_{f,t}}{\mu_{f,t}} \beta_l \left( \frac{Q_{f,t}}{L_{f,t}} \right)$$

Under perfect competition  $\mu_{f,t}=1$ , then rearranging we get:

$$\beta_l = \frac{W_{f,t}L_{f,t}}{P_{f,t}Q_{f,t}}$$

Which we can estimate with a simple average.

#### Text B

From the same problem above, if we take first order conditions with respect to labor and capital we get:

$$W_{f,t}L_{f,t} = \lambda \beta_l(Q_{f,t})$$

$$R_{f,t}K_{f,t} = \lambda \beta_k(Q_{f,t})$$

If we sum the two expressions:  $W_{f,t}L_{f,t} + R_{f,t}K_{f,t} = \lambda \beta_l(Q_{f,t}) + \lambda \beta_k(Q_{f,t})$

which reduces to:

$$W_{f,t}L_{f,t} + R_{f,t}K_{f,t} = \lambda Q_{f,t}$$

Then if we take 1 or 2 and divide by the same in both sides:

$$\frac{W_{f,t}L_{f,t}}{W_{f,t}L_{f,t} + R_{f,t}K_{f,t}} = \frac{\lambda \beta_l(Q_{f,t})}{\lambda Q_{f,t}}$$

$$\text{This simplifies: } \frac{W_{f,t}L_{f,t}}{W_{f,t}L_{f,t} + R_{f,t}K_{f,t}} = \beta_l$$

Same argument for the other. The markup also comes from the expression from above:

$\lambda = \frac{W_{f,t}L_{f,t}}{Q_{f,t}\beta_l}$  If we substitute  $\beta_l$  into this expression and we rearrange we have:

$$\lambda = \frac{W_{f,t}L_{f,t} + R_{f,t}K_{f,t}}{Q_{f,t}}$$

Thus:  $\mu = \frac{pQ_{f,t}}{W_{f,t}L_{f,t} + R_{f,t}K_{f,t}}$

**2**