Homework 2: set identification

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Question 1

We can easily show that the identified set is a curve by combining the first two moment equalities:

$$P_{01} + P_{10} = -\alpha_1 - \alpha_2 - \alpha_1\alpha_2.$$

We also know that in our data $\hat{P}_{01} = 0.35$ and $\hat{P}_{10} = 0.15$, so that

$$0.5 = -\alpha_1 - \alpha_2 - \alpha_1\alpha_2.$$

Then we can define the identified set Θ_I as

$$\Theta_I = \left\{(\alpha_1, \alpha_2) \in [-1, 0]^2 : 0.5 = -\alpha_1 - \alpha_2 - \alpha_1\alpha_2,\ \exists\, u \in [0, 1]\right\}.$$

To see that this defines a curve in $\mathbb{R}^2$, we can solve explicitly for α_2:

$$\alpha_2 = -\frac{0.5 + \alpha_1}{1 + \alpha_1}, \qquad \alpha_1 \neq -1.$$

Hence the solution set is the graph of a single-valued function, i.e. a one-dimensional curve[1].

Question 2

a)

We can create bounds on the observed probabilities and then define moment inequalities. For $y = (1, 0)$ we have

$$\hat{P}_{01} = -\alpha_1(1 + \alpha_2) + u\,(\alpha_1\alpha_2), \qquad u \in [0, 1].$$

Since $\alpha_1, \alpha_2 \in [-1, 0]$, we have $\alpha_1\alpha_2 \geq 0$, so $\hat{P}_{01}$ is (weakly) increasing in u. Hence

$$-\alpha_1(1 + \alpha_2) \ \leq\ \hat{P}_{01} \ \leq\ -\alpha_1(1 + \alpha_2) + \alpha_1\alpha_2,$$

which yields the two moment inequalities

$$-\alpha_1(1 + \alpha_2) - \hat{P}_{01} \ \leq\ 0, \qquad \hat{P}_{01} + \alpha_1(1 + \alpha_2) - \alpha_1\alpha_2 \ \leq\ 0.$$

Analogously, for $y = (0, 1)$:

$$\hat{P}_{10} = -\alpha_2(1 + \alpha_1) + (1 - u)\,(\alpha_1\alpha_2),$$

which is (weakly) decreasing in u, giving

$$-\alpha_2(1 + \alpha_1) \ \leq\ \hat{P}_{10} \ \leq\ -\alpha_2(1 + \alpha_1) + \alpha_1\alpha_2,$$

and the two inequalities

$$-\alpha_2(1 + \alpha_1) - \hat{P}_{10} \ \leq\ 0, \qquad \hat{P}_{10} + \alpha_2(1 + \alpha_1) - \alpha_1\alpha_2 \ \leq\ 0.$$

Finally, $P_{11} = (1 + \alpha_1)(1 + \alpha_2)$ provides the equality

$$(1 + \alpha_1)(1 + \alpha_2) - \hat{P}_{11} = 0.$$

Thus we have a system of 5 (in)equalities that we can use to estimate the bounds of the identified set.

[1] I think we can use Implicit Function Theorem but this should be enough

b)

Will use CHT and implement it , I think

c)

I can do subsampling. Maybe time consuming but should be straightforward.

d)

Report the results here...

e)

Cox and Shi, to read and then do!