PS1- Estimate Dynamic Models with CCP

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Exercise 1

After correcting for timing by using lagged mileage, I estimate a probit model of replacement as a function of mileage, which can be seen in Table 1. In the baseline linear specification, the coefficient on mileage is positive and statistically significant, indicating that the probability of replacement increases with accumulated mileage. The replacement probability increases strongly and significantly with mileage in all other specifications. Polynomial specifications introduce curvature, but the cubic term is insignificant and overfits at high mileage. In Table 2, I show how the different models compare in terms of information criteria. We can see that the log specification performs well under both criteria. Finally, in Figure 1, I show how the four specifications compare in terms of prediction. We can see again that the log specification appears to be the best, as it provides the smoothest behavior. The cubic specification seems to generate explosive predictions at high mileage, while the quadratic specification becomes non-increasing at some points.
I therefore select the log specification as the preferred policy function estimate.

Exercise 2

In table 3 I show the results for ρ and σ^ρ. We can see that the mean increase in miles of buses that don't invest is 0.19 which is 1.900 miles per bus. The σ^ρ is 0.11, which is also reasonable.

Exercise 3

In table 4 I report the results of x_{iY}^j for $j \in \{1, 2, 3\}$ and $Y \in \{0, 1\}$. We can see that the values are also intuitive. The replacement stream costs are higher for buses that replace today, as they assume the fixed cost of replacement. However, we can see that then their mileage stream has a lesser cost if they invest today, reflecting better health of the engine after replacement. Finally, The shock component differs slightly because the probability of future replacement differs across the two initial conditions, which affects the log probability term entering the expected value

Exercise 4

Finally, in table 5 I show the results of the structural model. We can see that the replacement fixed cost is 9.55 while the mileage cost 0.007. Both are precisely estimated. The magnitudes ...

Exercise 5

Intuition?

1 Appendix

	change			
	(1)	(2)	(3)	(4)
(Intercept)	-3.328***	-4.297***	-6.652***	-5.387***
	(0.136)	(0.419)	(1.501)	(0.429)
lag_mileage	0.063***	0.185***	0.608*	
	(0.007)	(0.046)	(0.247)	
mileage2		-0.003**	-0.027*	
		(0.001)	(0.013)	
mileage3			0.000	
			(0.000)	
log_mileage				1.112***
				(0.145)
N	7,250	7,250	7,250	7,250
Pseudo R^2	0.132	0.145	0.151	0.145

Table 1: Model comparison probit specifications.

	Model	AIC	BIC
1	Quadratic	632.31	652.98
2	Cubic	629.98	657.54
3	Log	630.83	644.61
4	Linear	640.29	654.07

Table 2: Model comparison using AIC and BIC.

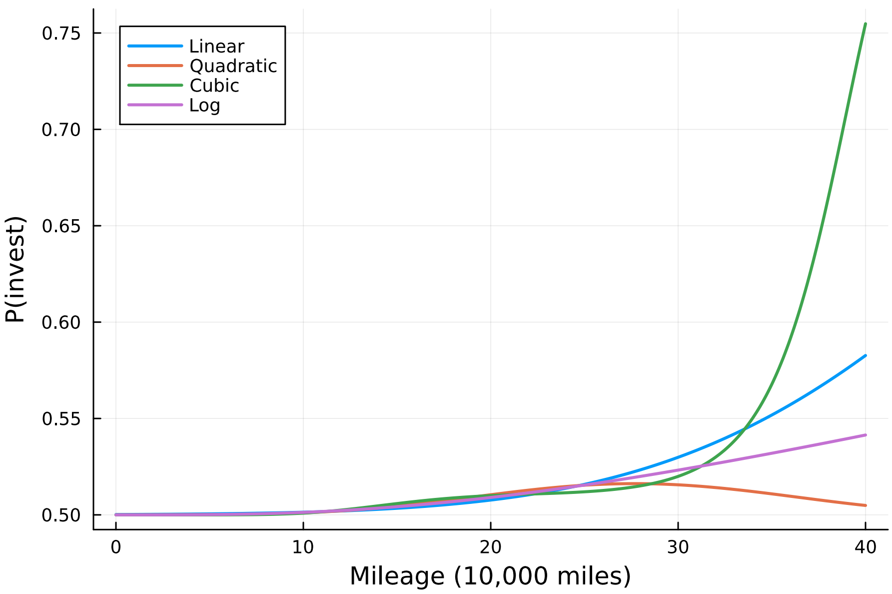

Figure 1: Estimated probability of engine replacement as a function of mileage under alternative probit specifications. Mileage is measured in units of 10,000 miles.

Parameter	Value
ρ (mean increase)	0.190387
σ (std. dev.)	0.113453

Table 3: Mean and standard deviation of mileage increases for buses that do not invest.

Component	Invest Today	No Invest Today
x_1 (replacement stream)	-1.1757	-0.5727
x_2 (mileage stream)	-471.423	-688.320
x_3 (shock term)	37.2352	39.0315

Table 4: Average simulated value function components over observations.

Parameter	Estimate	Std. Error
RC (replacement cost)	9.5537	0.8303
μ (mileage cost)	0.007413	0.001088

Table 5: Structural parameter estimates from the dynamic logit model. The coefficient on v_3 is fixed at 1 as implied by the Type I Extreme Value assumption.