Research Statement

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One of the major challenges in the field of complexity theory (both classical and quantum) is the inability to prove unconditional time lower bounds. One way around this is the study of fine-grained complexity, where we use special reductions to prove time lower bounds for many problems in P based on the conjectured hardness of some key problems like the Satisfiability (SAT) problem, 3SUM or APSP. The situation in the quantum regime is no better; almost all known lower bounds are defined in terms of query complexity, which is not very useful for problems whose best-known algorithms take superlinear time. Therefore in order to understand the quantum time complexity of certain problems, employing fine-grained reductions in the quantum setting seems a natural way forward, which most of my PhD work has been about.

1 Contributions in Quantum Fine-Grained Complexity

We studied some existing classical reductions from problems like CNF-SAT, 3SUM and APSP and observed that translating these classical fine-grained reductions directly into the quantum regime is not always trivial. Fortunately, we were able to develop frameworks and proof strategies, using which we are able to circumvent these obstacles and were able to comment on the time complexity of a lot of string comparison, computational geometry, graph theoretic and other related problems.

1.1 A Framework of Quantum Strong Exponential-Time Hypotheses

The strong exponential-time hypothesis (SETH) is a commonly used conjecture in the field of complexity theory. It essentially states that determining whether a CNF formula is satisfiable cannot be done faster than exhaustive search over all possible assignments. This hypothesis and its variants gave rise to a fruitful field of research obtaining (mostly tight) lower bounds for many problems in P whose unconditional lower bounds are very likely beyond current techniques.

Our contribution In this joint work with Harry Buhrman and Florian Speelman, we introduce an extensive framework of Quantum Strong Exponential-Time Hypotheses, as quantum analogues to what SETH is for classical computation. Using the QSETH framework, we are able to translate quantum query lower bounds on black-box problems to conditional quantum time lower bounds for many problems in P. As an example, we provide a conditional quantum time lower bound of $\Omega(n^{1.5})$ for the Longest Common Subsequence and Edit Distance problems. We also show that the n^2 SETH-based lower bound for a recent scheme for Proofs of Useful Work carries over to the quantum setting using our framework.

The conference version of this paper is published in STACS 2021 and also was presented in the non-proceedings track of TQC 2020.

1.2 Fine-Grained Complexity via Quantum Walks

A fundamental conjecture in the classical setting states that the 3SUM problem cannot be solved by (classical) algorithms in time $O(n^{2-\epsilon})$ for an $\epsilon > 0$. Consequently, lower bounds for many problems were concluded based on this conjecture.

Our contribution In this joint work with Harry Buhrman, Bruno Loff, and, Florian Speelman, we further extend the theory of fine-grained complexity in the quantum regime. We formulate an analogous conjecture, the Quantum-3SUM-Conjecture, which states that there exist no sublinear $O(n^{1-\alpha})$ time quantum algorithms for the 3SUM problem. Based on the Quantum-3SUM-Conjecture, we show new lower-bounds on the time complexity of quantum algorithms for several computational problems. These results are proven by adapting to the quantum setting known classical fine-grained reductions from the 3SUM problem. This adaptation is not trivial, since the original classical reductions require sorting the input, and sorting provably cannot be done in sublinear quantum time. We overcome this bottleneck by combining a quantum walk with a classical dynamic datastructure having a certain "history-independence" property. This (general) proof strategy allows us to prove tight lower bounds on several computational geometry problems, on Convolution-3SUM and on the 0-Edge-Weight-Triangle problem, conditional on the Quantum-3SUM-Conjecture.

The conference version of this work appeared in ITCS 2022 and was presented at QIP 2022 and also in the non-proceedings track of TQC 2021.

1.3 Memory Compression with Quantum Random-Access Gates

The quantum fine-grained results from our earlier work lead us to some interesting observations: There are quite a few quantum walk based algorithms that are space *efficient* but are complicated and extremely non-trivial to construct. There are however space *inefficient* analogous algorithms that have a much simpler structure and are far more easy to construct. Unfortunately, making a quantum algorithm space inefficient is not at all appealing especially when maintaining even a few hundred qubits coherently seem like a daunting task. In this project, we present techniques using which we can compress a space inefficient quantum algorithm into a space efficient one with only slight increase in time complexity and slight worsening of error.

Our contribution In this joint work with Harry Buhrman, Bruno Loff and Florian Speelman, we prove the following result. If we have a quantum algorithm that runs in time T and uses M qubits, and which is such that the state of the memory, at any time step, is supported on vectors of Hamming weight at most m, then it may be simulated by another algorithm which uses only $O(m \log M)$ memory. Using some known quantum and classical techniques, we obtain an algorithms running in time $O(T \log T \log M)$. We show how this theorem can be used, in a black-box way, to dramatically simplify several papers, including our 3SUM paper (mentioned above). Broadly speaking, when there exists a need for a space-efficient history-independent quantum data-structure, it is much simpler to construct a space-inefficient, yet sparse, quantum data structure.

Full version of this paper is available on arXiv:2203.05599.

1.4 Quantum Fine-grained Reductions from APSP problem

The All Pairs Shortest Path (APSP) problem is one of the well studied problems in graph algorithms. It is defined as follows: Given a graph G of nodes and weighted directed edges, output the length of the shortest paths between every pair of nodes in G. The best known classical algorithm to solve APSP on a graph of n nodes runs in $O(n^3)$ time. No significant improvement to this run-time is known. The APSP problem has been useful in obtaining $\Omega(n^3)$ classical lower bounds for a variety of computational problems, for example, Matrix Multiplication over Semiring[1], Detecting Negative Triangle, Zero Edge Weight Triangle, Matching Triangles, Triangle Collection, etc. Subcubic algorithms for any of these problems imply a subcubic algorithm for APSP. Moreover, most of these problems are cubic-equivalent, i.e., these problems either all have truly subcubic algorithms, or none of them do.

Our contribution In this joint work with Harry Buhrman, Florian Speelman and Keon Leinsje we are attempting to understand the quantum complexities of the APSP problem and the other related problems. There exists a $O(n^{2.5})$ quantum time algorithm for solving APSP, which we conjecture to be optimal. Even though most of the classical reductions from APSP are easily quantizable, the cubic equivalences that hold in the classical setting no longer hold in the quantum setting. We are able to present (non-trivial) quantum algorithms for many of these problems which matches the lower bounds from these reductions.

This is currently a work in progress.

2 Other Research Projects

2.1 Improved Quantum Query Upper Bounds Based on Classical Decision Trees

One of the other approaches towards understanding hardness of a problem is to set the goals a little bit lower and try to understand this problem on a simpler model of computation. Perhaps the simplest model of computation is the Decision Tree. Decision tree complexities in the classical, randomized, and quantum settings are well defined and very well studied.

Our contribution In this joint work with Arjan Cornelissen and Nikhil S. Mande, we first define the randomized decision tree size complexity of a relation $f \subseteq \{0,1\}^n \times R$, let's denote by $RDTsize(f)$, and explore connections with its bounded-error quantum query complexity $Q(f)$. We prove that $Q(f) \leq O(\sqrt{RDTsize(f)})$ for any relation f. We do so by giving an explicit span program and dual adversary solution witnessing the same.

Full version of this paper is available on arXiv:2203.02968.

[1] This should not be mistaken for the usual Matrix Multiplication problem which has a $O(n^\omega)$ time algorithm with ω being the matrix multiplication constant currently at 2.3728596.

3 Research Contributions during Masters

Prior to my PhD in my masters, I had co-authored the following results in the area of Quantum Information Theory.

3.1 Impossibility of cloning of quantum coherence

Our contribution. It is well known that it is impossible to clone an arbitrary quantum state. However, this inability does not lead directly to no cloning of quantum coherence. Here, in this joint work with Dhrumil Patel, Chiranjeevi Vanarasa, Indranil Chakrabarty, and, Arun Kumar Pati, we show that it is impossible to clone the coherence of an arbitrary quantum state. In particular, with an ancillary system as machine state, we show that it is impossible to clone the coherence of states whose coherence is greater than the coherence of the known states on which the transformations are defined. Also, we characterize the class of states for which coherence cloning will be possible for a given choice of machine. Furthermore, we find the maximum range of states whose coherence can be cloned perfectly. The impossibility proof also holds when we do not include machine states. Lastly, we generalize the impossibility of cloning of coherence in terms of dimension of the quantum state and coherence measure taken into consideration.

The journal version of this work appeared in Phys. Rev. A 103 in 2021.

3.2 Non-negativity of conditional von Neumann entropy and global unitary operations

Our contribution. Conditional von Neumann entropy is an intriguing concept in quantum information theory. In this joint work with Indranil Chakrabarty and Nirman Ganguly, we examine the effect of global unitary operations on the conditional entropy of the system. We start with a set containing states with a non-negative conditional entropy and find that some states preserve the non-negativity under unitary operations on the composite system. We call this class of states the absolute conditional von Neumann entropy non-negative (ACVENN) class. We characterize such states for $2\otimes2$–dimensional systems. From a different perspective the characterization accentuates the detection of states whose conditional entropy becomes negative after the global unitary action. Interestingly, we show that this ACVENN class of states forms a set which is convex and compact. This feature enables the existence of Hermitian witness operators. With these we can distinguish the unknown states which will have a negative conditional entropy after the global unitary operation. We also show that this has immediate application to superdense coding and state merging, as the negativity of the conditional entropy plays a key role in both these information processing tasks. Some illustrations followed by analysis are also provided to probe the connection of such states with absolutely separable states and absolutely local states.

The journal version of this work appeared in Phys. Rev. A 96 in 2017.

4 Future Directions

- I would like to further extend my quest for the fine-grained reductions in the quantum setting, especially for problems for which classical reductions are not known and prove tight bounds for the same.
- I am also interested in understanding time complexity of the quantum query algorithms with a hope to give more structure to these algorithms as it exists in the query setting. For example, is it possible to have a time analogue for Reichardt's composition theorem for quantum query bounds (arXiv:1005.1601)?
- Additionally, I am also interested in decision tree complexity (both classical and quantum) of relations when we have access to more powerful input oracles (for example, say if we can query the OR function on a subset of indices as opposed to on a single index) and understand the connections between them.