Summary of Research

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One of the major challenges in the field of complexity theory (both classical and quantum) is the inability to prove unconditional time lower bounds. One way around this is the study of fine-grained complexity, where we use special reductions to prove time lower bounds for many problems in P based on the conjectured hardness of some key problems like the Satisfiability (SAT) problem, 3SUM or APSP. The situation in the quantum regime is no better; almost all known lower bounds are defined in terms of query complexity, which is not very useful for problems whose best-known algorithms take superlinear time. Therefore in order to understand the time complexity of certain problems, employing fine-grained reductions in the quantum setting seems a natural way forward, which most of my PhD work has been about.

1 Contributions in Quantum Fine-Grained Complexity

We studied some existing classical reductions from problems like CNF-SAT, 3SUM and APSP and observed that translating these classical fine-grained reductions directly into the quantum regime is not always trivial. With time, we were able to develop frameworks and proof strategies, using which we are able to circumvent these obstacles and were able to comment on the time complexity of a lot of string comparison, computational geometry and other related problems. Some of our results (mentioned below) are published online while the rest are still work in progress.

1.1 A Framework of Quantum Strong Exponential-Time Hypotheses

The strong exponential-time hypothesis (SETH) is a commonly used conjecture in the field of complexity theory. It essentially states that determining whether a CNF formula is satisfiable cannot be done faster than exhaustive search over all possible assignments. This hypothesis and its variants gave rise to a fruitful field of research, fine-grained complexity, obtaining (mostly tight) lower bounds for many problems in P whose unconditional lower bounds are very likely beyond current techniques.

Our contribution. In this joint work with Harry Buhrman and Florian Speelman, we introduce an extensive framework of Quantum Strong Exponential-Time Hypotheses, as quantum analogues to what SETH is for classical computation. Using the QSETH framework, we are able to translate quantum query lower bounds on black-box problems to conditional quantum time lower bounds for many problems in P. As an example, we provide a conditional quantum time lower bound of $\Omega(n^{1.5})$ for the Longest Common Subsequence and Edit Distance problems. We also show that the n^2 SETH-based lower bound for a recent scheme for Proofs of Useful Work carries over to the quantum setting using our framework, maintaining a quadratic gap between verifier and prover. Lastly, we also show that the assumptions in our framework cannot be simplified further with relativizing proof techniques, as they are false in relativized worlds.

The conference version of this work appeared in STACS 2021 and also was presented in the non-proceedings track of TQC 2020.

1.2 Fine-Grained Complexity via Quantum Walks

In this joint work with Harry Buhrman, Bruno Loff, and, Florian Speelman, we further extend the theory of fine-grained complexity in the quantum regime. A fundamental conjecture in the classical setting states that the 3SUM problem cannot be solved by (classical) algorithms in time $O(n^{2-\epsilon})$ for an $\epsilon > 0$.

Our contribution. We formulate an analogous conjecture, the Quantum-3SUM-Conjecture, which states that there exist no sublinear $O(n^{1-\alpha})$ time quantum algorithms for the 3SUM problem. Based on the Quantum-3SUM-Conjecture, we show new lower-bounds on the time complexity of quantum algorithms for several computational problems. Most of our lower-bounds are optimal, in that they match known upper-bounds, and hence they imply tight limits on the quantum speedup that is possible for these problems. These results are proven by adapting to the quantum setting known classical fine-grained reductions from the 3SUM problem. This adaptation is not trivial, however, since the original classical reductions require pre-processing the input in various ways, e.g. by sorting it according to some order, and this pre-processing (provably) cannot be done in sublinear quantum time. We overcome this bottleneck by combining a quantum walk with a classical dynamic datastructure having a certain "history-independence" property. This type of construction has been used in the past to prove upper bounds, and here we use it for the first time as part of a reduction. This general proof strategy allows us to prove tight lower bounds on several computational geometry problems, on Convolution-3SUM and on the 0-Edge-Weight-Triangle problem, conditional on the Quantum-3SUM-Conjecture. We believe this proof strategy will be useful in proving tight (conditional) lower-bounds, and limits on quantum speed-ups, for many other problems.

The conference version of this work is to appear in ITCS 2022 and was presented in the non-proceedings track of TQC 2021.

2 Other Research Contributions

Prior to my PhD in my masters, I had co-authored the following results in the area of Quantum Information Theory.

2.1 Impossibility of cloning of quantum coherence

Our contribution. It is well known that it is impossible to clone an arbitrary quantum state. However, this inability does not lead directly to no cloning of quantum coherence. Here, in this joint work with Dhrumil Patel, Chiranjeevi Vanarasa, Indranil Chakrabarty, and, Arun Kumar Pati, we show that it is impossible to clone the coherence of an arbitrary quantum state. In particular, with an ancillary system as machine state, we show that it is impossible to clone the coherence of states whose coherence is greater than the coherence of the known states on which the transformations are defined. Also, we characterize the class of states for which coherence cloning will be possible for a given choice of machine. Furthermore, we find the maximum range of states whose coherence can be cloned perfectly. The impossibility proof also holds when we do not include machine states. Lastly, we generalize the impossibility of cloning of coherence in terms of dimension of the quantum state and coherence measure taken into consideration.

The journal version of this work appeared in Phys. Rev. A 103 in 2021.

2.2 Non-negativity of conditional von Neumann entropy and global unitary operations

Our contribution. Conditional von Neumann entropy is an intriguing concept in quantum information theory. In this joint work with Indranil Chakrabarty and Nirman Ganguly, we examine the effect of global unitary operations on the conditional entropy of the system. We start with a set containing states with a non-negative conditional entropy and find that some states preserve the non-negativity under unitary operations on the composite system. We call this class of states the absolute conditional von Neumann entropy non-negative (ACVENN) class. We characterize such states for $2 \otimes 2$–dimensional systems. From a different perspective the characterization accentuates the detection of states whose conditional entropy becomes negative after the global unitary action. Interestingly, we show that this ACVENN class of states forms a set which is convex and compact. This feature enables the existence of Hermitian witness operators. With these we can distinguish the unknown states which will have a negative conditional entropy after the global unitary operation. We also show that this has immediate application to superdense coding and state merging, as the negativity of the conditional entropy plays a key role in both these information processing tasks. Some illustrations followed by analysis are also provided to probe the connection of such states with absolutely separable states and absolutely local states.

The journal version of this work appeared in Phys. Rev. A 96 in 2017.

3 On Going Projects

I am currently working on wrapping some more quantum fine-grained results, and, also am part of an ongoing project in the area of quantum query complexity.