MAT 343 LAB 3 - Jordan Ledbetter

Question 1

Enter the matrices E1, E2 and E3 in Example 1 (suppress the output).

```
E1 = eye(4);
E1([4,3],:) = E1([3,4],:)
```

```
E1 = 4×4
     1     0     0     0
     0     1     0     0
     0     0     0     1
     0     0     1     0
```

```
E2 = eye(4);
E2(4,4) = -4
```

```
E2 = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0    -4
```

```
E3 = eye(4);
E3(2,4) = -5
```

```
E3 = 4×4
     1     0     0     0
     0     1     0    -5
     0     0     1     0
     0     0     0     1
```

Generate the matrix A

```
A = floor(10*rand(4,3))
```

```
A = 4×3
     8     6     6
     7     3     1
     1     3     8
     0     0     2
```

*Compute the product E1*A*

```
E1*A
```

```
ans = 4×3
     8     6     6
     7     3     1
     0     0     2
     1     3     8
```

Describe the effect on A of left multiplication by E1:

The multiplication of E1 and A caused matrix A to switch rows 3 and 4.

*Compute the product E2*A*

```
E2*A
```

```
ans = 4×3
     8     6     6
     7     3     1
     1     3     8
     0     0    -8
```

Describe the effect on A of left multiplication by E2:

The multiplication of E2 and A caused row 4 of matrix A to be multiplied by -4.

*Compute the product E3*A*

```
E3*A
```

```
ans = 4×3
     8     6     6
     7     3    -9
     1     3     8
     0     0     2
```

Describe the effect on A of left multiplication by E3

Answer: The multiplication of E3 and A caused the value positioned at (2,2) to change from 0 to 5, then row 2 of matrix A is multiplied by -4.

What is the effect of multiplying a matrix A on the left by an elementary matrix?

Answer: Multiplying a matrix A on the left by an elementary matrix leaves an effect on the rows of matrix A, causing the matrix to undergo elementary row operations

Question 2

Enter the matrix A

```
A = [4,3,4;-8,0,2;4,-1,-1]
```

```
A = 3×3
     4     3     4
    -8     0     2
     4    -1    -1
```

(a)

```
format rat
E1 = eye(3);
E1(2,1) = 2
```

```
E1 =
       1              0              0
       2              1              0
       0              0              1
```

```
E1*A
```

```
ans =
```

```
       4              3              4
       0              6             10
       4             -1             -1
```

```
E2 = eye(3);
E2(3,1) = -1
```

```
E2 =
       1              0              0
       0              1              0
      -1              0              1
```

```
E2*(E1*A)
```

```
ans =
       4              3              4
       0              6             10
       0             -4             -5
```

```
E3 = eye(3);
E3(3,2)=2/3
```

```
E3 =
       1              0              0
       0              1              0
       0              2/3            1
```

```
U = E3 * E2 * E1 * A
```

```
U =
       4              3              4
       0              6             10
       0              *              5/3
```

(b)

```
L = inv(E1) * inv(E2) * inv(E3)
```

```
L =
       1              0              0
      -2              1              0
       1             -2/3            1
```

```
format short
A - L*U
```

```
ans = 3×3
10^-15 ×
         0         0         0
         0         0         0
         0    0.2220         0
```

Question 3

Enter the commands to generate p, E and generate the matrix A.

```
p = [5,4,1,2,3]
```

```
p = 1×5
     5     4     1     2     3
```

```
E = eye(length(p))
```

```
E = 5×5
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
```

```
E = E(p,:)
```

```
E = 5×5
     0     0     0     0     1
     0     0     0     1     0
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
```

```
A = floor(10*rand(5))
```

```
A = 5×5
     8     9     3     4     6
     6     8     2     8     9
     5     2     0     6     7
     3     0     1     8     1
     4     0     5     2     2
```

(a)

Compute the product EA

```
E*A
```

```
ans = 5×5
     4     0     5     2     2
     3     0     1     8     1
     8     9     3     4     6
     6     8     2     8     9
     5     2     0     6     7
```

How are the matrices A and EA related? (be specific)

Answer: Matrices A and EA are both 5x5 matrices with the same values; however, the rows are rearranged in EA. When A is multiplied to the elementary matrix E, EA produces matrix A's matrix values with rows rearranged in the order [5,4,1,2,3].

Compute the product AE

```
A*E
```

```
ans = 5×5
     3     4     6     9     8
     2     8     9     8     6
     0     6     7     2     5
     1     8     1     0     3
     5     2     2     0     4
```

How are the matrices A and AE related? (be specific)

Answer: Matrices A and AE are both 5x5 matrices however, the columns are rearranged in AE. When the elementary matrix E is multiplied to matrix A, AE produces matrix A's matrix values with columns rearranged in the order [3,4,5,2,1].

(b)

Compute E^{-1} and E^T

```
E^-1
```

```
ans = 5×5
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
     0     1     0     0     0
     1     0     0     0     0
```

```
E'
```

```
ans = 5×5
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
     0     1     0     0     0
     1     0     0     0     0
```

How are E^{-1} and E^T related?

Answer: Matrices E^-1 and E^T both have the same dimensions and values.

Question 4

Enter the matrix A and the vector b

```
format short
A = [4,1,6,-5;-6,-7,-4,1;-6,-8,-1,-2;-7,-2,-2,-1]
```

```
A = 4×4
     4     1     6    -5
    -6    -7    -4     1
    -6    -8    -1    -2
    -7    -2    -2    -1
```

```
b = [-1;-9;-2;32]
```

```
b = 4×1
    -1
    -9
    -2
    32
```

(a)

Compute the LU factorization and verity that PA=LU.

```
[L,U,P] = lu(A)
```

```
L = 4×4
    1.0000         0         0         0
    0.8571    1.0000         0         0
   -0.5714    0.0227    1.0000         0
    0.8571    0.8409   -0.5962    1.0000
U = 4×4
   -7.0000   -2.0000   -2.0000   -1.0000
         0   -6.2857    0.7143   -1.1429
         0         0    4.8409   -5.5455
         0         0         0   -0.4883
P = 4×4
     0     0     0     1
     0     0     1     0
     1     0     0     0
     0     1     0     0
```

```
P*A - L*U
```

```
ans = 4×4
10^-15 ×
         0         0         0         0
         0         0         0         0
         0         0   -0.8882         0
         0         0         0   -0.2220
```

(b)

Solve the system Ax = b and store the solution in `x_lu`*.*

```
m = P*b
```

```
m = 4×1
    32
    -2
    -1
    -9
```

```
y = L\m
```

```
y = 4×1
   32.0000
  -29.4286
   17.9545
   -0.9765
```

```
x_lu = U\y
```

```
x_lu = 4×1
   -8.0000
    5.0000
    6.0000
    2.0000
```

(c)

Enter the vector x (exact solution) and compute `norm(x_lu-x)`

```
x = L\m
```

```
x = 4×1
   32.0000
  -29.4286
   17.9545
   -0.9765
```

```
norm(x_lu-x)
```

```
ans = 54.1950
```

Question 5

Generate the matrix A and the vectors A, x and b (use semicolon to suppress the output).

```
A = rand(800);
x = ones(800,1);
b = A*x;
```

(a)

Solve using the rref and calculate the elapsed time (the only output should be the time)

```
tic; R = rref([A, b]); x_rref = R(:,end); toc
```

```
Elapsed time is 2.144374 seconds.
```

(b)

Solve using the LU factorization and calculate the elapsed time (the only output should be the time)

```
tic; [L,U,P] = lu(A); y = L\(P*b); x_lu = U\y; toc;
```

```
Elapsed time is 0.014415 seconds.
```

Which method is faster?

The method using LU factorization produces a much faster elapsed time than the method using RREF.

(c)

Compare the accuracy of the solutions from part (a) and part (b)

```
norm(x_rref - x)
```

```
ans = 1.7565e-10
```

```
norm(x_lu - x)
```

```
ans = 3.9600e-10
```

How does the accuracy of the two method compare?

Answer: I do not believe that LU factorization is particularly more or less accurate than RREF. After running this code 5 times, in 3 of the instances, RREF was more accurate and in 2 of the instances, LU factorization was more accurate.