Databse Systems Extra Credit Hw 2

Jordyn Young

February 2026

Practice Problem for Homework 2 Question 4

Question 4 [10 points]. Convert the following set of functional dependencies to minimal basis.
Show all steps:

$F = \{A \to B,\ B \to C,\ A \to C,\ C \to D,\ CD \to E,\ C \to E,\ AB \to G,\ DG \to H\}.$

Solution

compute a minimal basis by doing the following: (1) split RHS attributes, (2) remove extraneous LHS attributes, (3) remove redundant dependencies.

Step 1: Split RHS into single attributes (and remove trivial FDs)
Each FD already has a single attribute on the RHS, and none are trivial.

Step 2: Remove extraneous attributes from LHS Only $AB \to G$ has a multi-attribute LHS.

Check if A is extraneous in $AB \to G$:

$$(B)^+ = \{B\} \xrightarrow{B \to C} \{B, C\} \xrightarrow{C \to D} \{B, C, D\} \xrightarrow{CD \to E} \{B, C, D, E\}.$$

$G \notin (B)^+$, so A is not extraneous.

Check if B is extraneous in $AB \to G$:

$$(A)^+ = \{A\} \xrightarrow{A \to B} \{A, B\} \xrightarrow{B \to C} \{A, B, C\} \xrightarrow{C \to D} \{A, B, C, D\} \xrightarrow{CD \to E} \{A, B, C, D, E\}.$$

$G \notin (A)^+$, so B is not extraneous.

Therefore, no LHS attributes can be removed, and

$$F_2 = F_1.$$

Step 3: Remove redundant dependencies For each FD $X \to Y$, remove it and check whether $Y \in X^+$ under the remaining FDs.

(a) Test redundancy of $A \to C$:
Let $F' = F_2 \setminus \{A \to C\}$. Compute A^+ under F':

$$A^+ = \{A\} \xrightarrow{A \to B} \{A, B\} \xrightarrow{B \to C} \{A, B, C\}.$$

Thus $C \in A^+$, so $A \to C$ is implied by the others and is redundant. Remove it.

(b) Test redundancy of $C \to E$:
Let $F'' = (F_2 \setminus \{A \to C\}) \setminus \{C \to E\}$. Compute C^+ under F'':

$$C^+ = \{C\} \xrightarrow{C \to D} \{C, D\}.$$

$CD \to E$ applies:

$$\{C, D\} \xrightarrow{CD \to E} \{C, D, E\}.$$

Thus $E \in C^+$, so $C \to E$ is implied by the others and is redundant. Remove it.

(c) Check remaining FDs are not redundant (briefly):

- Without $A \to B$, B cannot be derived from A.
- Without $B \to C$, C cannot be derived from B.
- Without $C \to D$, D cannot be derived from C.
- Without $CD \to E$, there is no rule left that produces E.
- Without $AB \to G$, there is no rule left that produces G.
- Without $DG \to H$, there is no rule left that produces H.

thus no further redundancies remain.

Final Minimal Basis

minimal basis is:

$$\boxed{F_{\min} = \{A \to B,\ B \to C,\ C \to D,\ CD \to E,\ AB \to G,\ DG \to H\}.}$$