U-Spin Sum Rules

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1 Background

There is a relation to the decay amplitudes and the reduced matrix elements of processes related by U-spin. These relations are the sum rules.

Since U-spin is an approximate symmetry, we are interested in at which order of U-spin breaking do these sum rules disappear. We break U-spin by introducing a spurion via the mass of the strange quark.

The end goal of this project is to essentially find an algorithm that when given a process, will find at which order the sum rules disappear.

The U-spin sum rules done by Grossman and Schact at hep-ph/1811.11188, when looking at the full U-spin limit will yield a 16×16 matrix full of reduced matrix elements and sum rules. We will instead start with a simpler process of $D_0 \to P^+P^-$, where P^+ is the U-spin doublet of $\begin{pmatrix}\pi^+ & K^+\end{pmatrix}$. This leads to a 4×4 matrix, and the relation between the decay amplitudes and the reduced matrix elements are purely from the Clebsch-Gordan coefficients and nothing to do with the weak Hamiltonian or the CKM. So where are these sum rules coming from? They are actually coming from the degeneracy in the reduced matrix elements, when you take the full U-spin limit, or take an effective coupling as small.

After this we will study the process $B \to P^+P^-$, where B is the U-spin doublet $\begin{pmatrix}B & B_s\end{pmatrix}$. This time we end up with an 8×8 matrix. After this is when we look at the 16×16 matrix, discussed above.

2 $D^0 \to P^+P^-$

We use the following conventions based on 1503.06759 and hep-ph/0609089:

$$\left|\pi^+\right\rangle \equiv \left|u\bar{d}\right\rangle = -\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \tag{2.1}$$

$$-\left|\pi^-\right\rangle \equiv \left|\bar{u}d\right\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle \tag{2.2}$$

$$\left|K^+\right\rangle \equiv \left|u\bar{s}\right\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle \tag{2.3}$$

$$-\left|K^-\right\rangle \equiv \left|\bar{u}s\right\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \tag{2.4}$$

$$-\left|D^0\right\rangle \equiv \left|c\bar{u}\right\rangle = \left|0, 0\right\rangle \tag{2.5}$$

We use $P^- \otimes P^+$ for the group theoretical decomposition for final states in a similar fashion as 1811.11188, hence

$$\left|\pi^+\pi^-\right\rangle = \frac{1}{\sqrt{2}}\left|0,0\right\rangle + \frac{1}{\sqrt{2}}\left|1,0\right\rangle \tag{2.6}$$

$$\left|K^+K^-\right\rangle = \frac{1}{\sqrt{2}}\left|0,0\right\rangle - \frac{1}{\sqrt{2}}\left|1,0\right\rangle \tag{2.7}$$

$$\left|\pi^+K^-\right\rangle = \left|1,-1\right\rangle \tag{2.8}$$

$$\left|K^+\pi^-\right\rangle = -\left|1,1\right\rangle \tag{2.9}$$

2.1 Zeroth Order

The Hamiltonian for the full U-spin limit is

$$\mathcal{H}_{eff} = \Sigma_{SCS}(1,0) + \Delta_{SCS}(0,0) + \Sigma_{CF}(1,-1) + \Sigma_{DCS}(1,1) \tag{2.10}$$

where

$$\Sigma_{SCS} = \frac{V_{cs}^* V_{us} - V_{cd}^* V_{ud}}{2} \tag{2.11}$$

$$\Delta_{SCS} = -\frac{V_{cb}^* V_{ub}}{2} \tag{2.12}$$

$$\Sigma_{CF} = V_{cs}^* V_{ud} \tag{2.13}$$

$$\Sigma_{DCS} = V_{cd}^* V_{us} \tag{2.14}$$

Acting $\mathcal{H}_{eff}$ on the initial state, with $\langle F|\,\mathcal{H}_{eff}\,|I\rangle$, we get the following table

Decay ampl. $\mathcal{A}_0$	$\Sigma\,\langle 1|\,1\,|0\rangle$	$\Delta\,\langle 0|\,0\,|0\rangle$
$\mathcal{A}_0(D_0 \to \pi^+\pi^-)$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\mathcal{A}_0(D_0 \to K^+K^-)$	$-1/\sqrt{2}$	$1/\sqrt{2}$
$\mathcal{A}_0(D_0 \to \pi^+K^-)$	1	0
$\mathcal{A}_0(D_0 \to K^+\pi^-)$	-1	0

We have 2 reduced matrix elements and 4 decay amplitudes, so we have 2 sum rules.

$$\mathcal{A}_0(D^0 \to \pi^+K^-) = -\mathcal{A}_0(D^0 \to K^+\pi^-) \tag{2.15}$$

$$\mathcal{A}_0(D^0 \to \pi^+K^-) = \frac{1}{\sqrt{2}}[\mathcal{A}_0(D^0 \to \pi^+\pi^-) - \mathcal{A}_0(D^0 \to K^+K^-)] \tag{2.16}$$

Now, if we impose that Δ_{SCS} is small (in the limit as it goes to 0), we can basically ignore the last column, leaving us with only 1 reduced matrix element, hence an additional sum rule of the form

$$\mathcal{A}_0(D^0 \to \pi^+\pi^-) = -\mathcal{A}_0(D^0 \to K^+K^-) \tag{2.17}$$

2.2 First Order U-Spin Breaking

From this point on we are assuming that there are only two generations of quarks. The U-spin breaking gives rise to a triplet spurion operator (1,0),

$$\mathcal{H}_X = \mathcal{O}(\epsilon)\Bigg(\Sigma_{SCS}\left[\sqrt{\frac{2}{3}}(2,0) - \sqrt{\frac{1}{3}}(0,0)\right] + \Delta_{SCS}(1,0) \tag{2.18}$$

$$+ \Sigma_{CF}\left[\sqrt{\frac{1}{2}}(2,-1) + \sqrt{\frac{1}{2}}(1,-1)\right] \tag{2.19}$$

$$+ \Sigma_{DCS}\left[\sqrt{\frac{1}{2}}(2,1) - \sqrt{\frac{1}{2}}(1,1)\right]\Bigg) \tag{2.20}$$

Acting on the initial state as before, we get the following table, note that these are of $\mathcal{O}(\epsilon)$,

Decay ampl. $\mathcal{A}_X$	$\Sigma\langle 1\| 1\|0\rangle$	$\Sigma\langle 0\| 0\|0\rangle$	$\Delta\langle 1\| 1\|0\rangle$
$\mathcal{A}_X(D_0 \to \pi^+\pi^-)$	0	$-1/\sqrt{6}$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}_X(D_0 \to K^+K^-)$	0	$-1/\sqrt{6}$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}_X(D_0 \to \pi^+K^-)$	$1/\sqrt{2}$	0	0
$\mathcal{A}_X(D_0 \to K^+\pi^-)$	$1/\sqrt{2}$	0	0

Combining this with zeroth order CKM-leading portion, we now have

Decay ampl. $\mathcal{A}_1$	$\Sigma\langle 1\| 1\|0\rangle$	$\Delta\langle 0\| 0\|0\rangle$	$\Sigma\langle 1\| 1\|0\rangle\mathcal{O}(\epsilon)$	$\Sigma\langle 0\| 0\|0\rangle\mathcal{O}(\epsilon)$	$\Delta\langle 1\| 1\|0\rangle$
$\mathcal{A}_1(D_0 \to \pi^+\pi^-)$	$1/\sqrt{2}$	$\frac{1}{\sqrt{2}}$	0	$-1/\sqrt{6}$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}_1(D_0 \to K^+K^-)$	$-1/\sqrt{2}$	$\frac{1}{\sqrt{2}}$	0	$-1/\sqrt{6}$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}_1(D_0 \to \pi^+K^-)$	1	0	$1/\sqrt{2}$	0	0
$\mathcal{A}_1(D_0 \to K^+\pi^-)$	-1	0	$1/\sqrt{2}$	0	0

Neglecting the terms proportional to Δ, we are left with 3 reduced matrix elements, hence 1 sum rule,

$$\mathcal{A}_1(D_0 \to \pi^+\pi^-) - \mathcal{A}_1(D_0 \to K^+K^-) = \frac{1}{\sqrt{2}}(\mathcal{A}_1(D_0 \to \pi^+K^-) - \mathcal{A}_1(D_0 \to K^+\pi^-)) \tag{2.21}$$

Compare this to Eq. (A12) in 1203.6659, the difference may come from convention.

2.3 Second Order U-Spin Breaking

For 2nd order U-spin breaking, the Hamiltonian would have terms proportional to

$$\mathcal{H}_{X^2} = \epsilon^2\Bigg(\Sigma_{SCS}\Bigg[\sqrt{\frac{2}{5}}(3,0) - \Bigg(\frac{2}{\sqrt{15}} + \sqrt{\frac{1}{3}}\Bigg)(1,0)\Bigg] \tag{2.22}$$

$$+ \Sigma_{CF}\Bigg[\frac{2}{\sqrt{15}}(3,-1) + \Bigg(\frac{1}{2\sqrt{3}} + \frac{1}{2}\Bigg)(2,-1) + \Bigg(-\frac{\sqrt{3}}{10} + \frac{1}{2}\Bigg)(1,-1)\Bigg] \tag{2.23}$$

$$+ \Sigma_{DCS}\Bigg[\frac{2}{\sqrt{15}}(3,1) - \Bigg(\frac{1}{2\sqrt{3}} + \frac{1}{2}\Bigg)(2,1) + \Bigg(-\frac{\sqrt{3}}{10} + \frac{1}{2}\Bigg)(1,1)\Bigg]\Bigg) \tag{2.24}$$

Acting on the initial state like before, we get the following table

Decay ampl. $\mathcal{A}_{X^2}$	$\Sigma\langle 1\|\,1\,\|0\rangle\,\epsilon^2$
$\mathcal{A}_{X^2}(D_0 \to \pi^+\pi^-)$	$-(\frac{2}{\sqrt{15}} + \frac{1}{\sqrt{3}})$
$\mathcal{A}_{X^2}(D_0 \to K^+K^-)$	$\frac{2}{\sqrt{15}} + \frac{1}{\sqrt{3}}$
$\mathcal{A}_{X^2}(D_0 \to \pi^+K^-)$	$-\frac{\sqrt{3}}{10} + \frac{1}{2}$
$\mathcal{A}_{X^2}(D_0 \to K^+\pi^-)$	$-\frac{\sqrt{3}}{10} + \frac{1}{2}$

Combining with the zeroth order and first order, we will have 4 decay amplitudes and 4 reduced matrix elements, hence we will have 0 sum rules. Now, comparing with 1203.6659, there are two more reduced matrix elements for 2nd order U-spin breaking instead of 1. Delve into this a bit deeper later.

3 $B \to P^+P^-$

Now we look at the case of $B \to P^+P^-$, where B is a U-spin doublet containing $(B^0 B_s)$. Using the convention of hep-ph/0609089,

$$\left|B^0\right\rangle \equiv \left|d\bar{b}\right\rangle = \left|\frac{1}{2}, \frac{1}{2}\right\rangle \tag{3.1}$$

$$\left|B_s\right\rangle \equiv \left|s\bar{b}\right\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \tag{3.2}$$

Compared to before, where the Hamiltonian was a combination of SCS, CF, and DCS decays, in this case the Hamiltonian is based on $\Delta U = \pm 1/2$. From hep-ph/0609089 and 1308.3348, we have 2 components for $\mathcal{H}_{eff}$:

1. $|\Delta S| = 1$ comes from the $\bar{b} \to \bar{s}$ transition and transforms like $\bar{s} \sim \left|\frac{1}{2}, \frac{1}{2}\right\rangle$. This corresponds to $\Delta U = 1/2$.
2. $\Delta S = 0$ comes from the $\bar{b} \to \bar{d}$ transition and transforms like $\bar{d} \sim -\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$. This corresponds to $\Delta U = -1/2$.

In this way the Hamiltonian is

$$\mathcal{H}_{eff} \sim -\Sigma_d(\frac{1}{2},-\frac{1}{2}) + \Sigma_s(\frac{1}{2},\frac{1}{2}) \tag{3.3}$$

3.1 Zeroth Order

In zeroth order there are 2 reduced matrix elements. Unlike the 4x4 case, we not only have tree amplitudes, but also penguin amplitudes contributing to the process, but because we have modded out the portion of the amplitude corresponding to the weak interaction, these are essentially the same. Thus, we have the following table for the 8 amplitudes.

Decay ampl. $\mathcal{A}_0$	$\langle 0\| \frac{1}{2} \|\frac{1}{2}\rangle$	$\langle 1\| \frac{1}{2} \|\frac{1}{2}\rangle$
$\mathcal{A}_0(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}_0(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}_0(B^0 \to \pi^+K^-)$	0	0
$\mathcal{A}_0(B^0 \to K^+\pi^-)$	0	-1
$\mathcal{A}_0(B_s \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\mathcal{A}_0(B_s \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$\mathcal{A}_0(B_s \to \pi^+K^-)$	0	-1
$\mathcal{A}_0(B_s \to K^+\pi^-)$	0	0

Note that there are two amplitudes that are 0. This is because these processes are heavily suppressed. For the process $B^0 \to \pi^+K^-$, we need a term in the Hamiltonian that transforms like $\bar{d}\bar{d}s$. For the process $B_s \to K^+\pi^-$, we need a term in the Hamiltonian that transforms like $\bar{s}\bar{s}d$. These correspond to a term, $\mathcal{H} \supset \mathcal{O}^{3/2}_{\pm 3/2}$. Because of this, we get one more reduced matrix elements, so the full table for the zeroth order is.

Decay ampl. $\mathcal{A}_0$	$\langle 0 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{3}{2} \vert \frac{1}{2} \rangle$
$\mathcal{A}_0(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0
$\mathcal{A}_0(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$\mathcal{A}_0(B^0 \to \pi^+K^-)$	0	0	1
$\mathcal{A}_0(B^0 \to K^+\pi^-)$	0	-1	0
$\mathcal{A}_0(B_s \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0
$\mathcal{A}_0(B_s \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0
$\mathcal{A}_0(B_s \to \pi^+K^-)$	0	-1	0
$\mathcal{A}_0(B_s \to K^+\pi^-)$	0	0	1

This is of rank 3, so we get 5 sum rules.

$$\mathcal{A}_0(B^0 \to \pi^+\pi^- = \mathcal{A}_0(B_s \to K^+K^-) \tag{3.4}$$
$$\mathcal{A}_0(B^0 \to K^+K^-) = \mathcal{A}_0(B_s \to \pi^+\pi^-) \tag{3.5}$$
$$\mathcal{A}_0(B^0 \to K^+\pi^-) = \mathcal{A}_0(B_s \to \pi^+K^-) \tag{3.6}$$
$$\mathcal{A}_0(B^0 \to \pi^+K^-) = \mathcal{A}_0(B_s \to K^+\pi^-) \tag{3.7}$$
$$\mathcal{A}_0(B^0 \to \pi^+\pi^-) - \mathcal{A}_0(B^0 \to K^+K^-) = \sqrt{2}\mathcal{A}_0(B_s \to \pi^+K^-) \tag{3.8}$$
$$\tag{3.9}$$

If we ignore the subleading term, we ignore the last column and get one more sum rule

$$\mathcal{A}_0(B_0 \to \pi^+K^-) = 0 \tag{3.10}$$

3.2 First Order U-Spin Breaking

Much like before, the spurion introduces a triplet, for the leading term

$$-(1,0) \otimes (\frac{1}{2}, \pm\frac{1}{2}) = -\frac{1}{\sqrt{3}}(\frac{1}{2}, \pm\frac{1}{2}) \pm \sqrt{\frac{2}{3}}(\frac{3}{2}, \pm\frac{1}{2}) \tag{3.11}$$

We get 3 more reduced matrix elements of $\mathcal{O}(\epsilon)$

Decay ampl. $\mathcal{A}_0$	$\langle 0\| \frac{1}{2} \left\| \frac{1}{2} \right\rangle$	$\langle 1\| \frac{1}{2} \left\| \frac{1}{2} \right\rangle$	$\langle 1\| \frac{3}{2} \left\| \frac{1}{2} \right\rangle$
$\mathcal{A}_0(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B^0 \to \pi^+K^-)$	0	0	0
$\mathcal{A}_0(B^0 \to K^+\pi^-)$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to \pi^+\pi^-)$	$-\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to K^+K^-)$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to \pi^+K^-)$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to K^+\pi^-)$	0	0	0

Including the subleading term, which goes as

$$(1,0) \otimes (\frac{3}{2}, \pm\frac{3}{2}) = \mp\sqrt{\frac{3}{5}}(\frac{3}{2}, \pm\frac{3}{2}) + \sqrt{\frac{2}{5}}(\frac{5}{2}, \pm\frac{5}{2}) \tag{3.12}$$

we get the following table

Decay ampl. $\mathcal{A}_0$	$\langle 0\| \frac{1}{2} \left\| \frac{1}{2} \right\rangle$	$\langle 1\| \frac{1}{2} \left\| \frac{1}{2} \right\rangle$	$\langle 1\| \frac{3}{2} \left\| \frac{1}{2} \right\rangle$	$\langle 1\| \frac{3}{2} \left\| \frac{1}{2} \right\rangle$
$\mathcal{A}_0(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B^0 \to \pi^+K^-)$	0	0	0	$\sqrt{\frac{3}{5}}$
$\mathcal{A}_0(B^0 \to K^+\pi^-)$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B_s \to \pi^+\pi^-)$	$-\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B_s \to K^+K^-)$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B_s \to \pi^+K^-)$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_0(B_s \to K^+\pi^-)$	0	0	0	$-\sqrt{\frac{3}{5}}$

where the last column is the subleading portion. Combining the first order and second order we get

Decay ampl. $\mathcal{A}_1$	$\langle 0\vert \frac{1}{2} \vert \frac{1}{2}\rangle$	$\langle 1\vert \frac{1}{2} \vert \frac{1}{2}\rangle$	$\langle 1\vert \frac{3}{2} \vert \frac{1}{2}\rangle$	$\langle 0\vert \frac{1}{2} \vert \frac{1}{2}\rangle$	$\langle 1\vert \frac{1}{2} \vert \frac{1}{2}\rangle$	$\langle 1\vert \frac{3}{2} \vert \frac{1}{2}\rangle$	$\langle 1\vert \frac{3}{2} \vert \frac{1}{2}\rangle$
$\mathcal{A}_1(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B^0 \to \pi^+K^-)$	0	0	1	0	0	0	$\sqrt{\frac{3}{5}}$
$\mathcal{A}_1(B^0 \to K^+\pi^-)$	0	-1	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B_s \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B_s \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B_s \to \pi^+K^-)$	0	-1	0	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$	0
$\mathcal{A}_1(B_s \to K^+\pi^-)$	0	0	1	0	0	0	$-\sqrt{\frac{3}{5}}$

This is of rank 7, so we get one sum rule

$$\begin{aligned} \mathcal{A}_1(B^0 \to \pi^+\pi^-) - \mathcal{A}_1(B^0 \to K^+K^-) - \sqrt{2}\mathcal{A}_1(B^0 \to K^+\pi^-) \\ -\mathcal{A}_1(B_s \to \pi^+\pi^-) + \mathcal{A}_1(B_s \to K^+K^-) - \sqrt{2}\mathcal{A}_1(B_s \to \pi^+K^-) = 0 \end{aligned} \tag{3.13}$$

3.3 Second Order U-Spin Breaking

From here we get at least one more reduced matrix element, so my intuition is that we get no sum rules at 2nd order, double check this later.

3.4 Subleading is the same order

This portion is from the fact that the subleading vs leading terms come from the CKM factors and the extra loop factors. Since we mod those out, then the reduced matrix elements of the subleading and the leading terms should be of the same order. Hence the table for the first order should be.

Decay ampl. $\mathcal{A}_0$	$\langle 0 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{3}{2} \vert \frac{1}{2} \rangle$
$\mathcal{A}_0(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B^0 \to \pi^+K^-)$	0	0	$\sqrt{\frac{3}{5}}$
$\mathcal{A}_0(B^0 \to K^+\pi^-)$	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to \pi^+\pi^-)$	$-\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to K^+K^-)$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to \pi^+K^-)$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$
$\mathcal{A}_0(B_s \to K^+\pi^-)$	0	0	$-\sqrt{\frac{3}{5}}$

And to first order the full table would be

Par	Decay ampl. $\mathcal{A}_1$	$\langle 0 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{3}{2} \vert \frac{1}{2} \rangle$	$\langle 0 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{1}{2} \vert \frac{1}{2} \rangle$	$\langle 1 \vert \frac{3}{2} \vert \frac{1}{2} \rangle$
a	$\mathcal{A}_1(B^0 \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
b	$\mathcal{A}_1(B^0 \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
c	$\mathcal{A}_1(B^0 \to \pi^+K^-)$	0	0	1	0	0	$\sqrt{\frac{3}{5}}$
d	$\mathcal{A}_1(B^0 \to K^+\pi^-)$	0	-1	0	0	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{6}}$
e	$\mathcal{A}_1(B_s \to \pi^+\pi^-)$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{6}}$
f	$\mathcal{A}_1(B_s \to K^+K^-)$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	0	$-\frac{1}{\sqrt{12}}$	$\frac{1}{\sqrt{12}}$	$-\frac{1}{\sqrt{6}}$
g	$\mathcal{A}_1(B_s \to \pi^+K^-)$	0	-1	0	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{6}}$
h	$\mathcal{A}_1(B_s \to K^+\pi^-)$	0	0	1	0	0	$-\sqrt{\frac{3}{5}}$

In this way, we get 2 sum rules as this matrix is of rank 6, using the above parametrizations

$$a - b - \frac{\sqrt{5}+\sqrt{10}}{4+2\sqrt{2}}c + (\frac{1}{2} - \frac{1}{\sqrt{2}})d - (\frac{1}{2} - \frac{1}{\sqrt{2}})g + \frac{\sqrt{5}+\sqrt{10}}{4+2\sqrt{2}}h = 0 \tag{3.14}$$

$$-\frac{\sqrt{5}+\sqrt{10}}{4+2\sqrt{2}}c + (\frac{1}{2} + \frac{1}{\sqrt{2}})d + e - f + \frac{1}{\sqrt{2}(2+\sqrt{2})}g + \frac{\sqrt{5}+\sqrt{10}}{4+2\sqrt{2}}h = 0 \tag{3.15}$$

Similar to above, if we go to 2nd order, we get no more sum rules. EXPLICITLY SHOW THIS.

4 $B \to BP^+P^-$

This portion extends on the work done in 1811.11188 and is the 16x16 case. The paper focuses on the three body decays of the Λ_c^+ and Ξ_c^+ baryons. We extend by considering the full U-spin picture of these decays.

Overall, the decays related to $\Lambda_c^+ \to pK^-K^+$ via U-spin are 16.

6 of which are related through SCS decays

1. $\Lambda_c^+ \to pK^-K^+$
2. $\Lambda_c^+ \to p\pi^-\pi^+$
3. $\Lambda_c^+ \to \Sigma^+\pi^-K^+$
4. $\Xi_c^+ \to \Sigma^+K^-K^+$
5. $\Xi_c^+ \to \Sigma^+\pi^-\pi^+$
6. $\Xi_c^+ \to pK^-\pi^+$

4 of which are related via Cabibbo-favored (CF) decays

7. $\Lambda_c^+ \to pK^-\pi^+$
8. $\Lambda_c^+ \to \Sigma^+K^-K^+$
9. $\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$
10. $\Xi_c^+ \to \Sigma^+K^-\pi^+$

4 of which are related via Doubly-Cabibbo-Suppressed (DCS) decays

11. $\Lambda_c^+ \to p\pi^-K^+$
12. $\Xi_c^+ \to pK^-K^+$
13. $\Xi_c^+ \to p\pi^-\pi^+$
14. $\Xi_c^+ \to \Sigma^+\pi^-K^+$

2 of which are not related through any of the three possible decay modes

15. $\Lambda_c^+ \to \Sigma^+K^-\pi^+$
16. $\Xi_c^+ \to p\pi^-K^+$

We use the group-theoreitc parametrization of U-spin as done on the paper:

$$\left|\Lambda_c^+\right\rangle \equiv |udc\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle \tag{4.1}$$

$$|p\rangle \equiv |uud\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle \tag{4.2}$$

$$\left|K^+\right\rangle \equiv |u\bar{s}\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle \tag{4.3}$$

$$-\left|\pi^-\right\rangle \equiv |d\bar{u}\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle \tag{4.4}$$

$$\left|\Xi_c^+\right\rangle \equiv |usc\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{4.5}$$

$$\left|\Sigma^+\right\rangle \equiv |uus\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{4.6}$$

$$\left|\pi^+\right\rangle \equiv \left|u\bar{d}\right\rangle = -\left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{4.7}$$

$$-\left|K^-\right\rangle \equiv |s\bar{u}\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{4.8}$$

The group theoretical decomposition for the final states is done through the order $(B\otimes P^-)\otimes P^+$. Similar to as done above, the final states, using the group-theoretic parametrization are,

$$pK^-K^+ = -\sqrt{\frac{1}{3}}\left|\frac{3}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_1 - \frac{1}{\sqrt{2}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_0 \tag{4.9}$$

$$\Sigma^+\pi^-\pi^+ = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_1 - \frac{1}{\sqrt{2}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_0 \tag{4.10}$$

$$\Sigma^+\pi^-K^+ = -\sqrt{\frac{1}{3}}\left|\frac{3}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_1 + \frac{1}{\sqrt{2}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_0 \tag{4.11}$$

$$pK^-\pi^+ = \sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{6}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_1 + \frac{1}{\sqrt{2}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_0 \tag{4.12}$$

$$p\pi^-\pi^+ = \sqrt{\frac{1}{3}}\left|\frac{3}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle_1 \tag{4.13}$$

$$\Sigma^+K^-K^+ = -\sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle_1 \tag{4.14}$$

$$p\pi^-K^+ = -\left|\frac{3}{2},\frac{3}{2}\right\rangle \tag{4.15}$$

$$\Sigma^+K^-\pi^+ = \left|\frac{3}{2},-\frac{3}{2}\right\rangle \tag{4.16}$$

4.1 Zeroth Order

From this point, we consider just 2 generations.

$$\mathcal{H}_{eff} = \Sigma_{SCS}(1,0) + \Sigma_{CF}(1,-1) + \Sigma_{DCS}(1,1) \tag{4.17}$$

With this, $\langle I|\, \mathcal{H}_{eff}$ become

$$\mathcal{H}_{eff}\left|\Lambda_c^+\right\rangle = \Sigma\left(\sqrt{\frac{2}{3}}\left|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle\right) + \Sigma_{\text{CF}}\left(\sqrt{\frac{1}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle\right) + \Sigma_{\text{DCS}}\left|\frac{3}{2},\frac{3}{2}\right\rangle \tag{4.18}$$

$$\mathcal{H}_{eff}\left|\Xi_c^+\right\rangle = \Sigma\left(\sqrt{\frac{2}{3}}\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2},-\frac{1}{2}\right\rangle\right) + \Sigma_{\text{CF}}\left|\frac{3}{2},-\frac{3}{2}\right\rangle + \Sigma_{\text{DCS}}\left(\sqrt{\frac{1}{3}}\left|\frac{3}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2},\frac{1}{2}\right\rangle\right) \tag{4.19}$$

We then get the following table:

Par	Decay ampl. $\mathcal{A}$	$\left\langle\frac{1}{2}\right\|_0 1 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{1}{2}\right\|_1 1 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{3}{2}\right\| 1 \left\|\frac{1}{2}\right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	$-\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$
f	$\Xi_c^+ \to pK^-\pi^-$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$
g	$\Lambda_c^+ \to pK^-\pi^+$	$-\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$\frac{1}{3}$
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$-\frac{2}{3}$	$-\frac{1}{3}$
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$\frac{1}{3}$
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	1
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	-1
l	$\Xi_c^+ \to pK^-K^+$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{3}$
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$\frac{2}{3}$	$\frac{1}{3}$
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{3}$
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0

From this table, which is rank 3, we get the following 11 U-spin sum rules as there are only 14 nonzero amplitudes.

3 from SCS decays

1. $\mathcal{A}(\Lambda_c^+ \to pK^-K^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+)$
2. $\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = -\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)$
3. $\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+)$

5 from CF and DCS decays

4. $\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+)$
5. $\mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) = -\mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+)$
6. $\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to pK^-k^+)$
7. $\mathcal{A}(\Lambda_c^+ p\pi^-K^+) = -\mathcal{A} \to \Sigma^+K^-\pi^+)$
8. $\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+) - \mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) = \mathcal{A}(\Xi_c^+ \to \Sigma^+K^-\pi^+)$

3 from the combination of all 3

9. $\mathcal{A}(\Lambda_c^+ \to pK^-K^+) = \frac{1}{4\sqrt{2}}\mathcal{A}(\Xi_c^+ \to pK^-K^+) - \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+) + \frac{3}{4\sqrt{2}}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+)$
10. $\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) = -\frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to pK^-K^+) - \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+)$
11. $\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) = \frac{3}{4\sqrt{2}}\mathcal{A}(\Xi_c^+ \to pK^-K^+) - \frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+) + \frac{1}{4\sqrt{2}}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+)$

CHECK THE SIGNS on this one. After this, consider three generations and look at the subleading terms with the last 2 decays as well.

4.2 First Order U-Spin Breaking

The U-spin breaking gives rise to a triplet spurion operator. Performing the tensor product on the original Hamiltonian,

$$(1,0) \otimes (1,0) = \sqrt{\frac{2}{3}}(2,0) - \sqrt{\frac{1}{3}}(0,0) \tag{4.20}$$

$$(1,0) \otimes (1,-1) = \sqrt{\frac{1}{2}}(2,-1) + \sqrt{\frac{1}{2}}(1,-1) \tag{4.21}$$

$$(1,0) \otimes (1,1) = \sqrt{\frac{1}{2}}(2,1) - \sqrt{\frac{1}{2}}(1,1) \tag{4.22}$$

Hence $\mathcal{H}_X \left|I\right\rangle$ is

$$\mathcal{H}_X \left|\Lambda_c^+\right\rangle = \Sigma\Big[-\sqrt{\frac{2}{5}}(2)\left|\frac{5}{2},\frac{1}{2}\right\rangle + \frac{2}{\sqrt{15}}(2)\left|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}(0)\left|\frac{1}{2},\frac{1}{2}\right\rangle\Big]$$
$$+ \Sigma_{\text{CF}}\Big[\sqrt{\frac{1}{5}}(2)\left|\frac{5}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{3}{10}}(2)\left|\frac{3}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{1}{6}}(1)\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}(1)\left|\frac{1}{2},-\frac{1}{2}\right\rangle\Big]$$
$$+ \Sigma_{\text{DCS}}\Big[\sqrt{\frac{2}{5}}(2)\left|\frac{5}{2},\frac{3}{2}\right\rangle - \sqrt{\frac{1}{10}}(2)\left|\frac{3}{2},\frac{3}{2}\right\rangle - \sqrt{\frac{1}{2}}(1)\left|\frac{3}{2},\frac{3}{2}\right\rangle\Big] \tag{4.23}$$
$$\mathcal{H}_X \left|\Xi_c^+\right\rangle = \Sigma\Big[\sqrt{\frac{2}{5}}(2)\left|\frac{5}{2},-\frac{1}{2}\right\rangle + \frac{2}{\sqrt{15}}(2)\left|\frac{3}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}(0)\left|\frac{1}{2},-\frac{1}{2}\right\rangle\Big]$$
$$+ \Sigma_{\text{CF}}\Big[\sqrt{\frac{2}{5}}(2)\left|\frac{5}{2},-\frac{3}{2}\right\rangle + \sqrt{\frac{1}{10}}(2)\left|\frac{3}{2},-\frac{3}{2}\right\rangle + \sqrt{\frac{1}{2}}(1)\left|\frac{3}{2},-\frac{3}{2}\right\rangle\Big]$$
$$+ \Sigma_{\text{DCS}}\Big[\sqrt{\frac{1}{5}}(2)\left|\frac{5}{2},\frac{1}{2}\right\rangle + \sqrt{\frac{3}{10}}(2)\left|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{6}}(1)\left|\frac{3}{2},\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}(1)\left|\frac{1}{2},\frac{1}{2}\right\rangle\Big] \tag{4.24}$$

This leads to the following table, up to some CKM matrix elements

Par	Decay ampl. $\mathcal{A}_X$	$\left\langle \frac{1}{2} \right\|_0 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_0 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 2 \left\| \frac{1}{2} \right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
f	$\Xi_c^+ \to pK^-\pi^+$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
g	$\Lambda_c^+ \to pK^-\pi^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
l	$\Xi_c^+ \to pK^-K^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0	0	0	0
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0	0	0	0

Combining this with the zeroth order terms, we get the following table

Par	Decay ampl. $\mathcal{A}$	$\left\langle \frac{1}{2} \middle\|_0 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \middle\|_1 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \middle\| 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \middle\|_0 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \middle\|_1 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \middle\| 1 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \middle\|_0 0 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \middle\|_1 0 \middle\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \middle\| 2 \middle\| \frac{1}{2} \right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	$-\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
f	$\Xi_c^+ \to pK^-\pi^-$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
g	$\Lambda_c^+ \to pK^-\pi^+$	$\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{3}$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	1	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	-1	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
l	$\Xi_c^+ \to pK^-K^+$	$\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$-\frac{2}{3}$	$\frac{1}{3}$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$-\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0	0	0	0	0	0	0
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0	0	0	0	0	0	0

There are 14 decays, 9 possible amplitudes, thus there are 5 sum rules: 1 from the CF and DCS decays and 4 from all three.

1. $\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to p\pi^-K^+) + \mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+)$
$= \mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) + \mathcal{A}(\Xi_c^+ \to \Sigma^+K^-\pi^+) + \mathcal{A}(\Xi_c^+ \to pK^-K^+) + \mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+)$

2. $\mathcal{A}(\Lambda_c^+ \to p\pi^-K^+) = \sqrt{2}\mathcal{A}(\Lambda_c^+ \to pK^-K^+) - \sqrt{2}\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) + \sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$
$+\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) - \mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+)$

3. $\mathcal{A}(\Xi_c^+ \to pK^-K^+) = -\sqrt{2}\mathcal{A}(\Lambda_c^+ \to pK^-K^+) - \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) + \sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$
$+\frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) + \sqrt{2}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) - \sqrt{2}\mathcal{A}(\Xi_c^+ \to pK^-\pi^+)$
$+\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+)$

4. $\mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+) = \frac{\sqrt{2}}{2}\mathcal{A}(\Lambda_c^+ \to pK^-K^+) - \frac{\sqrt{2}}{2}\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) - \frac{\sqrt{2}}{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$
$+\frac{\sqrt{2}}{2}\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) + \frac{\sqrt{2}}{2}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) + \frac{\sqrt{2}}{2}\mathcal{A}(\Xi_c^+ \to pK^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+)$

5. $\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+) = \sqrt{2}\mathcal{A}(\Lambda_c^+ \to pK^-K^+) - \frac{1}{\sqrt{2}}\mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) - \sqrt{2}\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+)$
$+\frac{1}{\sqrt{2}}\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) - \sqrt{2}\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) + \sqrt{2}\mathcal{A}(\Xi_c^+ \to pK^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to pK^-\pi^+)$

4.3 Second Order U-spin Breaking

From 2nd order U-spin breaking, the Hamiltonian would have terms proportional to

$$(1,0)\otimes(1,0)\otimes(1,0) = (1,0)\otimes\left[\sqrt{\frac{2}{3}}(2,0) - \sqrt{\frac{1}{3}}(0,0)\right] \tag{4.25}$$

$$= \sqrt{\frac{2}{5}}(3,0) - \frac{4}{\sqrt{15}}(1,0)_2 - \sqrt{\frac{1}{3}}(1,0)_1 \tag{4.26}$$

$$(1,0)\otimes(1,0)\otimes(1,-1) = (1,0)\otimes\left[\frac{1}{\sqrt{2}}(2,-1) + \frac{1}{\sqrt{2}}(1,-1)\right] \tag{4.27}$$

$$= \frac{2}{\sqrt{15}}(3,-1) + \frac{1}{2\sqrt{3}}(2,-1)_2 - \frac{\sqrt{2}}{10}(1,-1)_2 + \frac{1}{2}(2,-1)_1 + \frac{1}{2}(1,-1)_1 \tag{4.28}$$

$$(1,0)\otimes(1,0)\otimes(1,1) = (1,0)\otimes\left[\frac{1}{\sqrt{2}}(2,1) - \frac{1}{\sqrt{2}}(1,1)\right] \tag{4.29}$$

$$= \frac{2}{\sqrt{15}}(3,1) - \frac{1}{2\sqrt{3}}(2,1)_2 - \frac{\sqrt{3}}{10}(1,1)_2 - \frac{1}{2}(2,1)_1 + \frac{1}{2}(1,1)_1 \tag{4.30}$$

From here, there are at least six more reduced matrix elements, making this a rank 16 matrix, so no sum rules from tree level.

4.4 Subleading Terms

The last two decays $\Lambda_c^+ \to \Sigma^+K^-\pi^+$ and $\Xi_c^+ \to p\pi^-K^+$ necessitate terms in the Hamiltonian corresponding to $\mathcal{O}^2_{\pm 2}$. So, we get one more reduced matrix element, $\langle 3/2|\,2\,|1/2\rangle$ corresponding to this subleading term. Furthermore, we have the subleading portion of the Hamiltonian for the SCS decays that go as $\Delta(0,0)$, and get 2 more reduced matrix elements from it.

So at zeroth order we have the following table

Par	Decay ampl. $\mathcal{A}$	$\left\langle\frac{1}{2}\right\|_0 1 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{1}{2}\right\|_1 1 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{3}{2}\right\| 1 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{1}{2}\right\|_0 0 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{1}{2}\right\|_1 0 \left\|\frac{1}{2}\right\rangle$	$\left\langle\frac{3}{2}\right\| 2 \left\|\frac{1}{2}\right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	$-\frac{\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	0	$\sqrt{\frac{2}{3}}$	0
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{\sqrt{2}}{3}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	$-\frac{\sqrt{2}}{3}$	0	$\sqrt{\frac{2}{3}}$	0
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0
f	$\Xi_c^+ \to pK^-\pi^-$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	$\frac{\sqrt{2}}{3}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{6}}$	0
g	$\Lambda_c^+ \to pK^-\pi^+$	$-\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$-\frac{2}{3}$	$-\frac{1}{3}$	0	0	0
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$\frac{2}{\sqrt{3}}$	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	1	0	0	0
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	-1	0	0	0
l	$\Xi_c^+ \to pK^-K^+$	$-\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	0
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$\frac{2}{3}$	$\frac{1}{3}$	0	0	0
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$\frac{2}{\sqrt{3}}$	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	0
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0	0	0	$-\frac{2}{\sqrt{5}}$
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0	0	0	$\frac{2}{\sqrt{5}}$

As this is rank 6, we should get 10 sum rules:
1 from SCS decays

$$\begin{aligned}\mathcal{A}(\Lambda_c^+ \to pK^-K^+) + \mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-K^+) \\ +\mathcal{A}(\Xi_c^+ \to pK^-\pi^+) - \mathcal{A}(\Lambda_c^+ \to p\pi^-\pi^+) - \mathcal{A}(\Xi_c^+ \to \Sigma^+K^-K^+) = 0\end{aligned} \tag{4.31}$$

5 from CF and DCS decays

$$\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+\pi^-K^+) \tag{4.32}$$
$$\mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) = -\mathcal{A}(\Xi_c^+ \to p\pi^-\pi^+) \tag{4.33}$$
$$\mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to pK^-k^+) \tag{4.34}$$
$$\mathcal{A}(\Lambda_c^+ \to p\pi^-K^+) = -\mathcal{A}(\Xi_c^+ \to \Sigma^+K^-\pi^+) \tag{4.35}$$
$$\mathcal{A}(\Lambda_c^+ \to pK^-\pi^+) + \mathcal{A}(\Lambda_c^+ \to \Sigma^+\pi^-\pi^+) - \mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-K^+) = \mathcal{A}(\Xi_c^+ \to \Sigma^+K^-\pi^+) \tag{4.36}$$

1 from the last 2 processes

$$\mathcal{A}(\Lambda_c^+ \to \Sigma^+K^-\pi^+) = -\mathcal{A}(\Xi_c^+ \to p\pi^-K^+) \tag{4.37}$$

4 from the combination of all of them, using the parametrization above

$$a + \frac{1}{4}(-4+\sqrt{2})b + (1-\sqrt{2})c + (-1+\frac{3}{2\sqrt{2}})d + (1-\sqrt{2})e + f + g = 0 \tag{4.38}$$
$$\frac{1}{2}(2-\sqrt{2})a - b + \frac{1}{2}(2-\sqrt{2})c + \frac{1}{2}(-2+\sqrt{2})d + e + f + h = 0 \tag{4.39}$$
$$a + \frac{1}{4}(-4-3\sqrt{2})b + (1+\sqrt{2})c + \frac{1}{4}(-4-\sqrt{2})d + (1+\sqrt{2})e + f + i = 0 \tag{4.40}$$
$$\frac{1}{2}(2-\sqrt{2})a + (-1+\frac{1}{\sqrt{2}})b + \frac{1}{2}(2-\sqrt{2})c - d + e + f + i = 0 \tag{4.41}$$

Now in first order, with the three generations and the subleading term, we get 3 more reduced matrix elements compared to before. Combining with the zeroth order, we get will have 15 reduced matrix elements, giving us 1 sum rule. Work out the actual CG coefficients and the actual sum rule later.

Performing the tensor product on the subleading Hamiltonian, we get terms that go as

$$(1,0) \otimes (0,0) = (1,0) \tag{4.42}$$
$$(1,0) \otimes (2,\pm 2) = \mp\sqrt{\frac{2}{3}}(2,\pm 2) + \frac{1}{\sqrt{3}}(3,\pm 2) \tag{4.43}$$

so we get 3 more reduced matrix elements to first order U-spin breaking, $\langle 1/2|_0\, 1\, |1/2\rangle$, $\langle 1/2|_1\, 1\, |1/2\rangle$, and $\langle 3/2|\, 2\, |1/2\rangle$. Note that these are also subleading. We then get the following table

From this we get 1 sum rule.

Second order, we have no sum rules.

4.5 Subleading is the same order

Much like above, since the weak portion of the amplitude determines whether an element is subleading or not and we modded that out, the subleading matrix elements should be the same order as the leading matrix elements. Therefore, the above table for the first order U-spin breaking becomes:

Par	Decay ampl. $\mathcal{A}_X$	$\left\langle \frac{1}{2} \right\|_0 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_0 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 2 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_0 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 2 \left\| \frac{1}{2} \right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	0
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$	0	$-\frac{\sqrt{2}}{3}$	0
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	0
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	0	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$	0	$\frac{\sqrt{2}}{3}$	0
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	0	0	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	0
f	$\Xi_c^+ \to pK^-\pi^+$	0	0	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	0
g	$\Lambda_c^+ \to pK^-\pi^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$	0	0	0
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$	0	0	0
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$	0	0	0
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$	0	0	0
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$	0	0	0
l	$\Xi_c^+ \to pK^-K^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$	0	0	0
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$	0	0	0
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$	0	0	0
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0	0	0	0	0	0	$\frac{2}{\sqrt{15}}$
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0	0	0	0	0	0	$\frac{2}{\sqrt{15}}$

Par	Decay ampl. $\mathcal{A}_X$	$\left\langle \frac{1}{2} \right\|_0 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 1 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_0 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{1}{2} \right\|_1 0 \left\| \frac{1}{2} \right\rangle$	$\left\langle \frac{3}{2} \right\| 2 \left\| \frac{1}{2} \right\rangle$
a	$\Lambda_c^+ \to pK^-K^+$	$\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
b	$\Lambda_c^+ \to p\pi^-\pi^+$	0	$-\frac{\sqrt{2}}{3}$	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
c	$\Lambda_c^+ \to \Sigma^+\pi^-K^+$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{3\sqrt{2}}$	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
d	$\Xi_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	0	0	$-\frac{\sqrt{2}}{3}$	$-\frac{2}{3\sqrt{5}}$
e	$\Xi_c^+ \to \Sigma^+\pi^-\pi^+$	$-\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	0	$\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
f	$\Xi_c^+ \to pK^-\pi^+$	$\frac{1}{\sqrt{6}}$	$\frac{1}{3\sqrt{2}}$	0	$-\sqrt{\frac{1}{6}}$	$-\frac{1}{3\sqrt{2}}$	$\frac{2}{3\sqrt{5}}$
g	$\Lambda_c^+ \to pK^-\pi^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
h	$\Lambda_c^+ \to \Sigma^+K^-K^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
i	$\Lambda_c^+ \to \Sigma^+\pi^-\pi^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
j	$\Xi_c^+ \to \Sigma^+K^-\pi^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
k	$\Lambda_c^+ \to p\pi^-K^+$	0	0	$-\frac{1}{\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
l	$\Xi_c^+ \to pK^-K^+$	$-\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
m	$\Xi_c^+ \to p\pi^-\pi^+$	0	$\frac{\sqrt{2}}{3}$	$\frac{1}{3\sqrt{2}}$	0	0	$\sqrt{\frac{1}{10}}$
n	$\Xi_c^+ \to \Sigma^+\pi^-K^+$	$\sqrt{\frac{1}{6}}$	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	0	0	$-\sqrt{\frac{1}{10}}$
o	$\Lambda_c^+ \to \Sigma^+K^-\pi^+$	0	0	0	0	0	$\frac{2}{\sqrt{15}}$
p	$\Xi_c^+ \to p\pi^-K^+$	0	0	0	0	0	$\frac{2}{\sqrt{15}}$