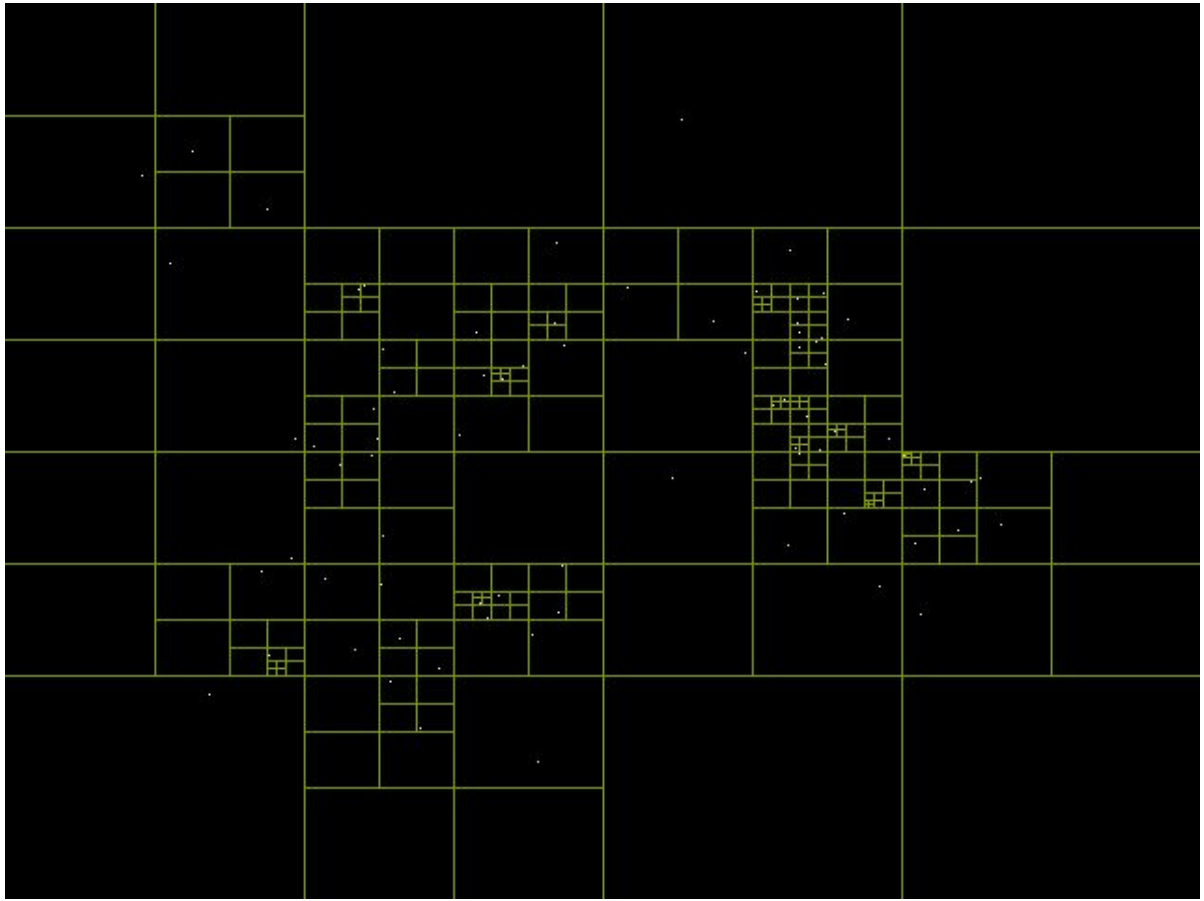
Fast Multipole Method (FMM)

Jorge Arellano



Developed in 1987 by Leslie Greengard & Vladimir Rokhlin Jr., FMM was originally developed as a fast algorithm for approximating the N-body problem

$$O(N^2) \quad \text{to} \quad O(N)$$

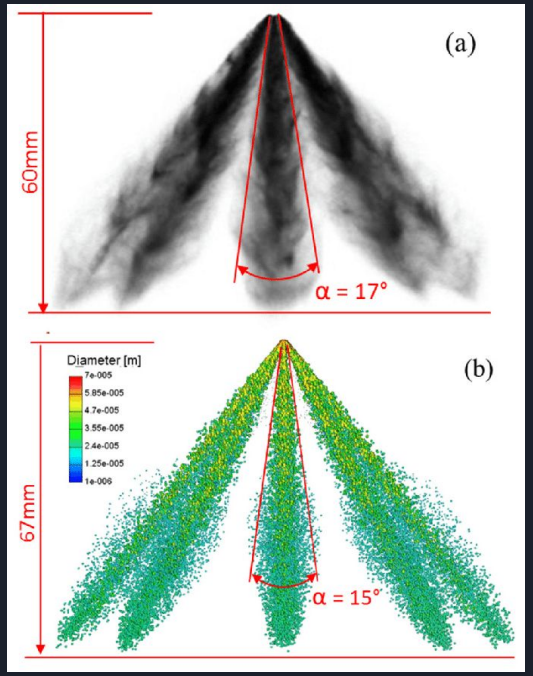
Motivation

“Complexity Trumps Hardware.”

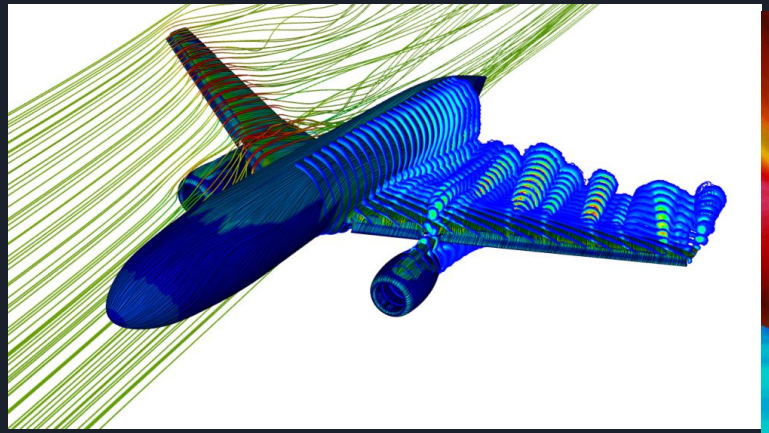
The current trend in computer architecture is moving towards multi-core processors and many-core processors where the Byte/flop ratio is decreasing with every generation.

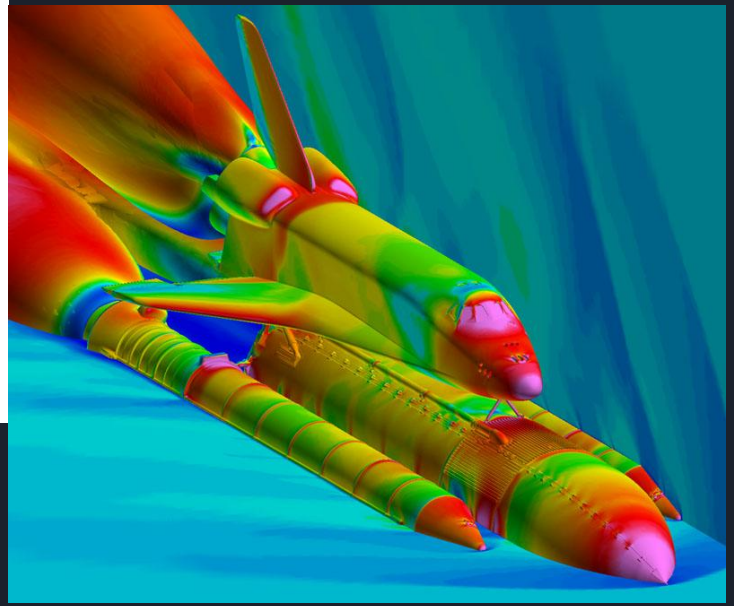
Applications:

Spray Simulations:

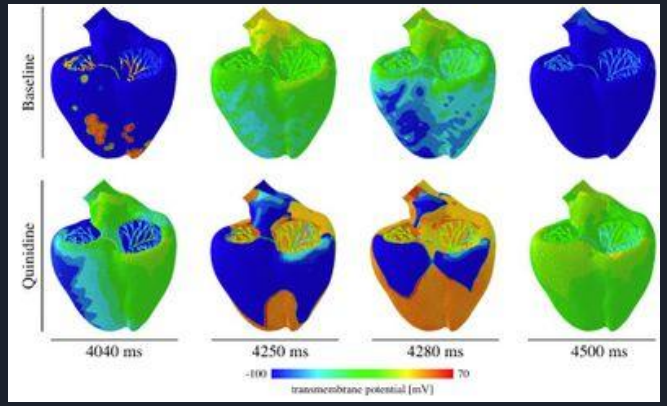


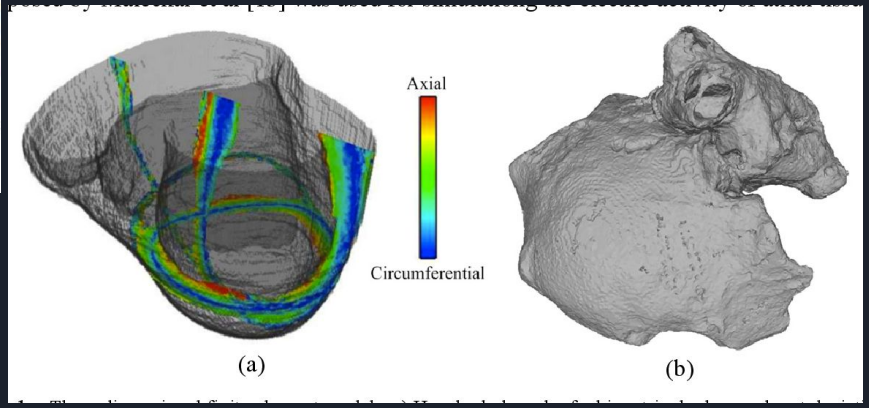
Applications: Aerodynamics





Applications: Healthcare

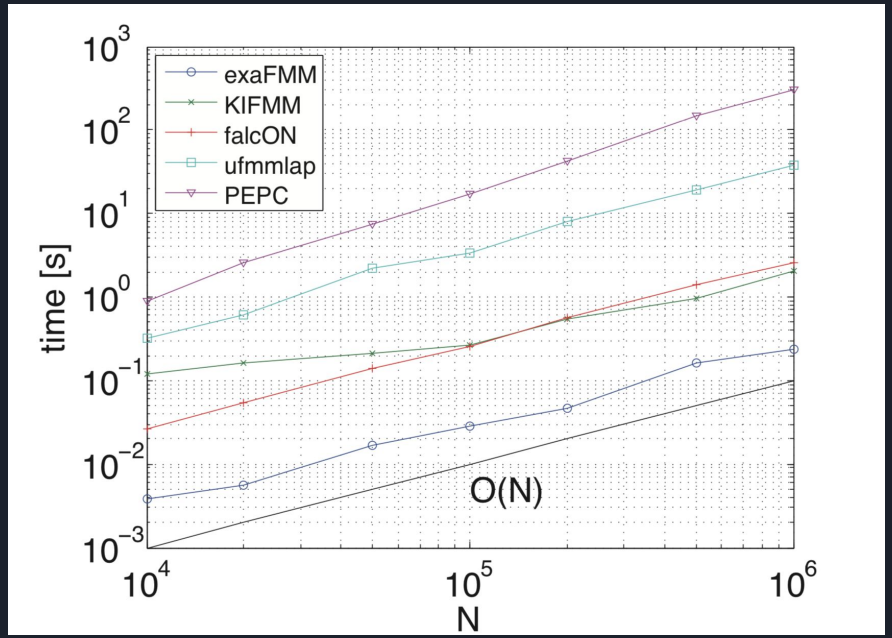




Simulation of Celestial Bodies

https://youtu.be/W04TzMMpp9A

Benchmarks



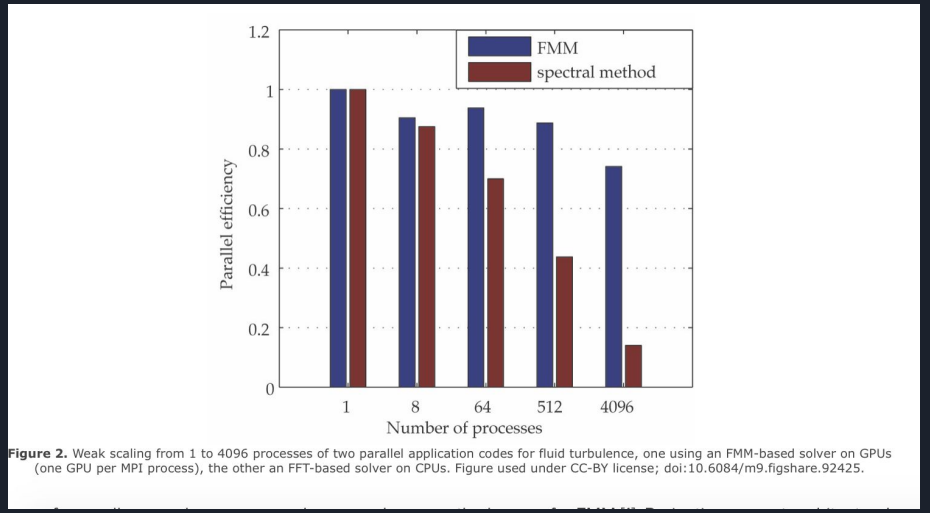


Figure 2. Weak scaling from 1 to 4096 processes of two parallel application codes for fluid turbulence, one using an FMM-based solver on GPUs (one GPU per MPI process), the other an FFT-based solver on CPUs. Figure used under CC-BY license; doi:10.6084/m9.figshare.92425.

Conclusion

FMM works!

But only useful when the problem can be parallelized

Low byte/flop (dense lin-alg) tend to have high complexity O(N) and algorithms with low complexity (FFT, Sparse Lin-alg) have high Byte/flop. FMM has an impressive combination of O(N) complexity and a Byte/flop that is even lower than matrix-matrix multiplication.

A possible alternative more PDE solvers.

references

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