Stat 250: Stat Principles and Practices Central Limit Theorem illustration

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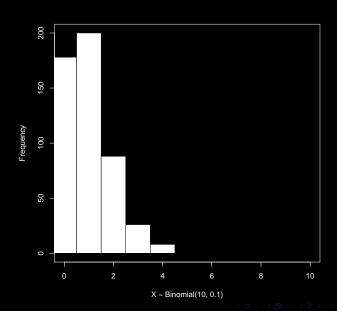
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Set-up: Binomial(10, p)

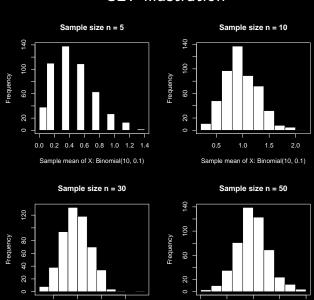
Generate 500 data sets of size n Illustration with n = 5, p = 0.1

| Sample | 1 | 2 | 3 | 4 | 5 | \bar{x} |
|------------|---|---|---|---|---|-----------|
| Data set 1 | 0 | 0 | 1 | 2 | 0 | 0.6 |
| Data set 2 | 0 | 0 | 1 | 3 | 0 | 8.0 |
| Data set 3 | 0 | 1 | 2 | 2 | 2 | 1.4 |
| Data set 4 | 0 | 0 | 0 | 3 | 1 | 8.0 |
| Data set 5 | 1 | 2 | 2 | 0 | 0 | 1.0 |
| : | | | • | | | |

Binomial(10, p) histogram



CLT illustration



Sample mean of X: Binomial(10, 0.1) Sample mean of X: Binomial(10, 0.1)

4.0 4.5

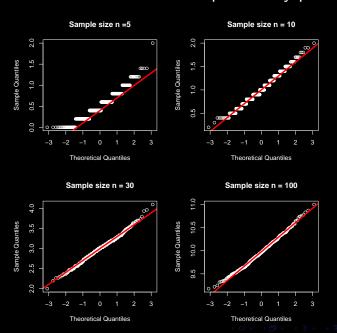
5.0 5.5 6.0

2.5

3.5 4.0

3.0

CLT illustration: normal probability plot



Foreshadowing: Statistical Inference

- Collect binomial(n, p) random sample: X_1, X_2, \dots, X_n .
- Normal approximation to the binomial
- Another CLT: For "large n and reasonable p,"

$$rac{\sum X_i}{n} \sim AN\left(p, \sqrt{rac{p(1-p)}{n}}
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• Use the average to estimate *p* and sampling distribution to assess variability.