Supplementary Information: Computable General Equilibrium Model for Canada

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1 Sets

SS	**Superset**
$C \subset SS$	**Commodities.**
$CD \subset C$	Commodities with domestic sales of domestic output.
$CDN \subset C$	Complement of CD.
$CE \subset C$	Exported commodities.
$CEN \subset C$	Complement of CE.
$CM \subset C$	Imported commodities.
$CMN \subset C$	Complement of CM.
$CX \subset C$	Commodities with domestic output.
$M \subset SS$	**Margins (transaction costs).**
$I \subset SS$	**Industries.**
$ICES \subset I$	Industries with CES function at the top level.
$ILEO \subset I$	Industries with Leontief function at the top level.
$ICW \subset I$	Industries that use capital and water as primary factors.
$ICWN \subset I$	Complement of ICW.
$TAX \subset SS$	**Taxes.**
$F \subset SS$	**Primary factors.**
$FM \subset F$	Mobile factors.
$FNM \subset F$	Non-mobile factors.
$FL1 \subset F$	Factors in level 1 value-added function.
$FL2 \subset F$	Factors in level 2 value-added function.
$FML1 = FM \cap FL1$	Mobile factors in level 1 value-added function.
$FNML1 = FMN \cap FL1$	Non-mobile factors in level 1 value-added function.

$FML2 = FM \cap FL2$	Mobile factors in level 2 value-added function.
$FNML2 = FMN \cap FL2$	Non-mobile factors in level 2 value-added function.

$A \subset SS$	**Economic agent.**
$ANG \subset A$	Non-government economic agent.

$ACAP \subset SS$	**Capital accounts of economic agents.**
$ANGCAP \subset ACAP$	Capital accounts of non-government economic agents.

$GFCF \subset SS$	**Gross fixed capital formation accounts.**

2 Variables

2.1 Endogenous variables

1. DPI Producer's index price for domestically marketed output.
2. EXR Exchange rate.
3. $GOVSHR$ Government consumption share in nominal absorption.
4. $INVSHR$ Investment share in nominal absorption.
5. $TABS$ Total nominal absorption.
6. GDP GDP calculated using the consumption approach.
7. PA_i Unitary industry price, $i \in I$.
8. PDD_c Unitary demand price for comm. produced and sold domestically, $c \in C$.
9. PDS_c Unitary supply price for comm. produced and sold domestically, $c \in C$.
10. PE_c Unitary export price in domestic currency, $c \in C$.
11. $PINTA_i$ Aggregate intermediate input price for industry $i \in I$.
12. PM_c Unitary import price in domestic currency, $c \in C$.
13. PQ_c Unitary composite commodity price, $c \in C$.
14. PVA_i Unitary value-added price, $i \in I$.
15. PX_c Unitary aggregate producer price $c \in C$.
16. $PXAC_{i,c}$ Unitary producer price of commodity $c \in C$ for industry $i \in I$.
17. PCW_i Unitary price of bundle Capital-Water primary input, $i \in I$.
18. QA_i Quantity of industry $i \in I$.
19. QD_c Quantity sold domestically of domestic commodity output $c \in CD$.

20. QE_c Quantity of exported commodities $c \in CE$.
21. $QF_{f,i}$ Quantity demanded of factor $f \in F$ by industry $i \in I$.
22. $QDISP_{c,a}$ Quantity demanded of commodity $c \in C$ by agent $a \in A$.
23. $QINTA_i$ Quantity demanded of intermediate input by industry $i \in I$.
24. $QINT_{c,i}$ Quantity of commodity $c \in C$ as intermediate input to industry $i \in I$.
25. $QGFCF_{c,g}$ Quantity of commodity $c \in C$ demanded by GFCF account $g \in GFCF$.
26. QCW_i Quantity of bundle Capital-Water primary input by industry $i \in I$.
27. $QINV_c$ Quantity of change in inventories, $c \in C$.
28. QM_c Quantity of imported commodities, $c \in CM$.
29. QQ_c Quantity of commodities supplied to domestic market, $c \in C$.
30. QVA_i Quantity of value-added by industry $i \in I$.
31. QX_c Quantity of aggregated domestic output, $c \in CX$.
32. $QXAC_{i,c}$ Quantity of output commodity $c \in C$ produced by industry $i \in I$.
33. $QT_{c,m}$ Quantity of commodities $c \in C$ consumed by margin service $m \in M$.
34. $MAR_{m,c}$ Margins $m \in M$ paid by commodity $c \in C$.
35. $MSUM_m$ Sum of margins $m \in M$ across all commodities.
36. WFM_{fm} Unitary price of mobile factor $fm \in FM$.
37. $WFS_{fnm,i}$ Unitary price of non-mobile factor $fnm \in FNM$, $i \in I$.
38. YF_f Income of factor $f \in F$.
39. YI_a Primary income of agent $a \in A$.
40. $YCAP_{acap}$ Capital income of agent $acap \in ACAP$.
41. $TRPICAP_{a,acap}$ Transfers from primary income $a \in A$ to capital account for agent $acap \in ACAP$.
42. $INVESTGFCF_g$ Total investment across agents on $g \in GFCF$.
43. $LEND_{acap}$ Lending by agent $acap \in ACAP$.
44. $BORROW_{acap}$ Borrowing by agent $acap \in ACAP$.
45. $ROWBRW$ Borrowing of the rest of the world from domestic institutions.
46. $ROWLND$ Lending of the rest of the world to domestic institutions.
47. $TAXVAR$ Sum of all commodity or industry taxes.
48. $ZEROVAR$ Dummy variable that must be equal to zero after shocks.

2.2 Exogenous variables

1. $\overline{CPI}$ Consumer's price index.
2. $\overline{QFSM}_f$ Total supply of mobile factors $f \in FM$.
3. $\overline{QFSS}_{f,i}$ Supply of non-mobile factors $f \in FNM$, $i \in I$.

3 Parameters

3.1 Endogenous parameters

1. pwm_c Import unitary price in composite foreign currency, $c \in CM$.
2. pwe_c Export unitary price in composite foreign currency, $c \in CE$.
3. tm_c Import tax rate, $c \in CM$.
4. te_c Export tax rate, $c \in CE$.
5. icm_c Unitary transaction costs of imported commodities, $c \in CM$.
6. ice_c Unitary transaction costs of exported commodities, $c \in CE$.
7. icd_c Unitary transaction costs of commodities produced and sold domestically, $c \in CD$.
8. $im_{c,m}$ Share of total payments to margin $m \in M$ allocated to buy commodities $c \in C$.
9. tq_c Tax rate of commodity $c \in C$.
10. $\theta_{i,c}$ Yield of commodity output $c \in C$ produced by industry $i \in I$.
11. $ica_{c,i}$ Quantity of commodity $c \in C$ consumed by industry $i \in I$.
12. ta_i Net tax rate of industry $i \in I$.
13. α_i^I Efficiency of the CES industry function, $i \in I$.
14. δ_i^I Share parameter in the CES industry function, $i \in I$.
15. iva_i Share of value-added of industry output $i \in I$ in the Leontief industry function.
16. $inta_i$ Share of intermediate of industry output $i \in I$ in the Leontief industry function.
17. α_i^{va1} Efficiency of the CES value-added function 1 of $i \in I$.
18. $\delta_{f,i}^{va1}$ Share for factor $f \in F$ in the CES value-added function 1 of $i \in I$.
19. α_i^{va2} Efficiency of the CES value-added function 2 of $i \in I$.
20. $\delta_{f,i}^{va2}$ Share parameter for factor $f \in F$ in the CES value-added function 2 of $i \in I$.
21. α_c^{ac} Efficiency of the CES domestic aggregation function of $c \in C$.
22. $\delta_{i,c}^{ac}$ Share for industry $i \in I$ in the CES domestic aggregation function of $c \in C$.
23. α_c^T Efficiency of the CET function, $c \in C$.

24. δ_c^T Share parameter in the CET function, $c \in C$.
25. α_c^Q Efficiency of the Armington function, $c \in C$.
26. δ_c^Q Share parameter in the Armington function, $c \in C$.
27. $shif_c$ Share of income of factor $f \in F$ transferred to agent $a \in A$.
28. $shii_{a,a'}$ Share of income of agent $a' \in A$ transferred to agent $a \in A$.
29. $capshii_{a,a'}$ Share of capital income of agent $a' \in ACAP$ transferred to capital income of agent $a \in ACAP$.
30. $trnsfr_{a,RoW}$ Transfer in foreign currency from RoW to agent $a \in A$.
31. $trnsfr_{RoW,a}$ Transfer in foreign currency from agent $a \in A$ to RoW.
32. $captrnsfr_{a,RoW}$ Capital transfer in foreign currency from RoW to agent $a \in ACAP$.
33. $captrnsfr_{RoW,a}$ Capital transfer in foreign currency from agent $a \in ACAP$ to RoW.
34. $fftnfr$ Borrowing from domestic accounts to RoW.
35. $disp_a$ Share of current income allocated to disposable expenditure by agent $a \in A$.
36. $\gamma_{c,a}$ Subsistance consumption of commodity $c \in C$ by agent $a \in A$.
37. $\beta_{c,a}$ Marginal consumption of commodity $c \in C$ by agent $a \in A$.
38. lnd_a Share of capital income of agent $a \in ACAP$ allocated to lending.
39. $gfcf_{i,a}$ Share of capital income of agent $a \in ACAP$ allocated to GFCF account $g \in GFCF$.
40. $gfcfind_{c,i}$ Share of consumption of commodity $c \in C$ by GFCF account $g \in GFCF$.
41. $invnt_c$ Share of change in inventory of commodity $c \in C$ in terms of sum of total commodity change.
42. $invA_a$ Capital change due to change in commodity stock in agent $a \in A$.
43. $distort_{f,i}$ Price distortion of mobile factor $f \in FM$ for industry $i \in I$.

3.2 Exogenous parameters

1. $cwts_c$ Weight of commodity $c \in C$ in the CPI (consumer's price index).
2. $dwts_c$ Weight of commodity $c \in C$ in the PPI (producer's price index).
3. ρ_i^I Exponent of the CES industry function, $i \in I$.
4. ρ_i^{va1} Exponent of the CES value-added function 1, $i \in I$.
5. ρ_i^{va2} Exponent of the CES value-added function 2, $i \in I$.
6. ρ_c^{ac} Exponent of the CES domestic commodity aggregation function, $c \in C$.
7. ρ_c^T Exponent of the CET function, $c \in C$.

8. ρ_c^Q Exponent of the CES Armington function, $c \in C$.

4 Equations

4.1 Price block

Import unitary price:

$$PM_c = pwm_c(1 + tm_c)EXR + icm_c \qquad \forall c \in CM. \tag{1}$$

Export unitary price:

$$PE_c = pwe_c(1 - te_c)EXR - ice_c \qquad \forall c \in CE. \tag{2}$$

Demand unitary price of domesticly consumed commodities:

$$PDD_c = PDS_c + icd_c \qquad \forall c \in CD. \tag{3}$$

Absorption:

$$PQ_cQQ_c(1 - tq_c) = PDD_cQD_c + PM_cQM_c \qquad \forall c \in (CD \cup CM). \tag{4}$$

Output at market value:

$$PX_cQX_c(1 - tq_c) = PDS_cQD_c + PE_cQE_c \qquad \forall c \in CX. \tag{5}$$

Industry unitary price:

$$PA_i = \sum_{c \in C} PXAC_{i,c} \qquad \forall i \in I. \tag{6}$$

Intermediate input unitary price:

$$PINTA_i = \sum_{c \in C} PQ_c ica_{c,i} \qquad \forall i \in I. \tag{7}$$

Industry revenue:

$$PA_iQA_i(1 - ta_i) = PVA_iQVA_i + PINTA_iQINTA_i \qquad \forall i \in I. \tag{8}$$

Total margins (transaction costs) on the input side:

$$MAR_{'MARDOM',c} = QD_c icd_c \qquad \forall i \in I. \tag{9}$$

$$MAR_{'MAREXP',c} = QE_c ice_c \qquad \forall i \in I. \tag{10}$$

$$MAR_{'MARIMP',c} = QM_c icm_c \qquad \forall i \in I. \tag{11}$$

Sum of total margins on the input side:

$$MSUM_m = \sum_{c \in C} MAR_{m,c} \qquad \forall m \in M. \tag{12}$$

Consumer's price index:

$$\overline{CPI} = \sum_{c \in C} PQ_c cwts_c. \tag{13}$$

Producer's price index:

$$DPI = \sum_{c \in C} PDS_c dwts_c. \tag{14}$$

4.2 Production block

Level 1 production function: CES technology

$$QA_i = \alpha_i^I \left[\delta_i^I QVA_i^{-\rho_i^I} + \left(1-\delta_i^I\right) QINTA_i^{-\rho_i^I} \right]^{-1/\rho_i^I} \quad \forall i \in ICES. \tag{15}$$

$$\delta_i^I PINTA_i QINTA_i^{1+\rho_i^I} = \left(1-\delta_i^I\right) PVA_i QVA_i^{1+\rho_i^I} \quad \forall i \in ICES. \tag{16}$$

Level 1 production function: Leontief technology

$$QVA_i = iva_i QA_i \qquad \forall i \in ILEO. \tag{17}$$

$$QINTA_i = inta_i QA_i \qquad \forall i \in ILEO. \tag{18}$$

Value-added production function 1:

$$QVA_i = \alpha_i^{va1} \left(\sum_{f \in FL1} \delta_{f,i}^{va1} QF_{f,i}^{-\rho_i^{va1}} + \delta_{WatCap,i}^{va1} QCW_i^{-\rho_i^{va1}} \right)^{-1/\rho_i^{va1}} \qquad \forall i \in I. \tag{19}$$

$$\left(\alpha_i^{va1}\right)^{\rho_i^{va1}} distort_{f,i} WFM_f QF_{f,i}^{(1+\rho_i^{va1})} = \delta_{f,i}^{va1} PVA_i QVA_i^{(1+\rho_i^{va1})} \quad \forall i \in I, f \in FML1. \tag{20}$$

$$\left(\alpha_i^{va1}\right)^{\rho_i^{va1}} WFS_{f,i} QF_{f,i}^{(1+\rho_i^{va1})} = \delta_{f,i}^{va1} PVA_i QVA_i^{(1+\rho_i^{va1})} \quad \forall i \in I, f \in FNML1. \tag{21}$$

$$\left(\alpha_i^{va1}\right)^{\rho_i^{va1}} PCW_i QCW_i^{(1+\rho_i^{va1})} = \delta_{WatCap,i}^{va1} PVA_i QVA_i^{(1+\rho_i^{va1})} \quad \forall i \in I. \tag{22}$$

Value-added production function 2:

$$QCW_i = \alpha_i^{va2} \left(\sum_{f \in FL2} \delta_{f,i}^{va2} QF_{f,i}^{-\rho_i^{va2}} \right)^{-1/\rho_i^{va2}} \qquad \forall i \in ICW. \tag{23}$$

$$\left(\alpha_i^{va2}\right)^{\rho_i^{va2}} distort_{f,i} WFM_f QF_{f,i}^{(1+\rho_i^{va2})} = \delta_{f,i}^{va2} PCW_i QCW_i^{(1+\rho_i^{va2})} \, \forall i \in ICW, f \in FML2. \tag{24}$$

$$\left(\alpha_i^{va2}\right)^{\rho_i^{va2}} WFS_{f,i} QF_{f,i}^{(1+\rho_i^{va2})} = \delta_{f,i}^{va2} PCW_i QCW_i^{(1+\rho_i^{va2})} \quad \forall i \in ICW, f \in FNML2. \tag{25}$$

$$QCW_i = \sum_{f \in FL2} QF_{f,i} \quad \forall i \in ICWN. \tag{26}$$

$$PCW_i QCW_i = \sum_{f \in FML2} distort_{f,i} WFM_i QF_{f,i} + \sum_{f \in FNML2} WFS_{f,i} QF_{f,i} \quad \forall i \in ICWN. \tag{27}$$

Intermediate input demand:

$$QINT_{c,i} = ica_{c,i} QINTA_i \quad \forall c \in C, i \in I. \tag{28}$$

Commodity output:

$$QXAC_{i,c} = \theta_{i,c} QA_i \quad \forall i \in I, c \in C. \tag{29}$$

Output aggregation function:

$$QX_c = \alpha_c^{ac}\left(\sum_{i\in I}\delta_{i,c}^{ac}QXAC_{i,c}^{-\rho_c^{ac}}\right)^{-1/\rho_c^{ac}} \qquad \forall c \in CX. \tag{30}$$

$$(\alpha_c^{ac})^{\rho_c^{ac}}\, PXAC_{i,c}QXAC_{i,c}^{(1+\rho_c^{ac})} = \delta_{i,c}^{ac}PX_cQX_c^{(1+\rho_c^{ac})} \quad \forall i \in I, c \in CX. \tag{31}$$

Output CET function:

$$QX_c = \alpha_c^T\left[\delta_c^T QE_c^{\rho_c^T} + (1-\delta_c^T)QD_c^{\rho_c^T}\right]^{1/\rho_c^T} \quad \forall c \in (CD \cap CE). \tag{32}$$

$$\delta_c^T PDS_cQD_c^{(1-\rho_c^T)} = \left(1-\delta_c^T\right)PE_cQE_c^{(1-\rho_c^T)} \quad \forall c \in (CD \cap CE). \tag{33}$$

$$QX_c = QD_c + QE_c \quad \forall c \in (CD \cup CE) - (CD \cap CE). \tag{34}$$

Composite Armington supply function:

$$QQ_c = \alpha_c^q\left[\delta_c^q QM_c^{\rho_c^q} + (1-\delta_c^q)QD_c^{\rho_c^q}\right]^{1/\rho_c^q} \quad \forall c \in (CD \cap CM). \tag{35}$$

$$\delta_c^q PDD_cQD_c^{(1-\rho_c^q)} = (1-\delta_c^q)\,PM_cQM_c^{(1-\rho_c^q)} \quad \forall c \in (CD \cap CM). \tag{36}$$

$$QQ_c = QD_c + QM_c \quad \forall c \in (CD \cup CM) - (CD \cap CM). \tag{37}$$

Consumption of commodities due to margins (transaction costs):

$$PQ_cQT_c = im_{c,m}MSUM_m \quad \forall c \in C, m \in M. \tag{38}$$

4.3 Agents block

Factor income:

$$YF_f = \sum_{i\in I} distort_{f,i}WFM_fQF_{f,i} \qquad \forall f \in FM. \tag{39}$$

$$YF_f = \sum_{i\in I} WFS_{f,i}QF_{f,i} \qquad \forall f \in FNM. \tag{40}$$

Income of domestic nongovernment agents:

$$YI_a = \sum_{f\in F} shif_{a,f}YF_f + \sum_{a'\in ANG} shii_{a,a'}YI_{a'} + shii_{a,'GOV'}\overline{CPI} + trnsfr_{a,RoW}EXR \quad \forall a \in ANG \tag{41}$$

Government income:

$$TAXVAR = \sum_{i\in I} ta_iPA_iQA_i + \sum_{c\in C} tq_cPQ_cQQ_c + \sum_{c\in CM} tm_cpwm_cPM_iQM_i + \sum_{i\in I} te_cpwe_cPE_iQE_i. \tag{42}$$

$$YI_{'GOV'} = \sum_{f \in F} shif_{'GOV',f} YF_f + \sum_{a \in ANG} shii_{'GOV',a} YI_a + trnsfr_{'GOV',RoW} EXR + TAXVAR \quad (43)$$

Expenditure of domestic nongovernment agents:

$$YI_a = \sum_{a' \in A} shii_{a',a} YI_a + trnsfr_{RoW,a} EXR + disp_a YI_a + TRPICAP_a \quad \forall a \in ANG \quad (44)$$

Expenditure of government:

$$YI_{'GOV'} = \sum_{a \in A} shii_{a,'GOV'} \overline{CPI} + trnsfr_{RoW,'GOV'} EXR + disp_{'GOV'} YI_{'GOV'} + TRPICAP_{'GOV'} \quad (45)$$

Disposable income:

$$PQ_c QDISP_{c,a} = PQ_c \gamma_{c,a} + \beta_{c,a} \left(disp_a YI_a - \sum_{c' \in C} PQ_{c'} \gamma_{c',a} \right) \quad \forall c \in C, a \in A \quad (46)$$

Capital income of non-government agents:

$$YCAP_a = TRPICAP_a + \sum_{a' \in ANG} capshii_{a,a'} YCAP_{a'} + capshii_{a,'GOV'} \overline{CPI} + captrnsfr_{a,RoW} EXR + BORROW_a \quad \forall a \in ANGCAP \quad (47)$$

Capital income of government:

$$YCAP_{'GOV'} = TRPICAP_{'GOV'} + \sum_{a \in ANG} capshii_{'GOV',a} YCAP_a + captrnsfr_{'GOV',RoW} EXR + BORROW_{'GOV'} \quad (48)$$

Capital expenses of non-government accounts:

$$\left(1 - \sum_{a' \in ACAP} capshii_{a',a} - \sum_{g \in GFCF} gfcf\_coef_{g,a} \right) YCAP_a = captrnsfr_{RoW,a} EXR + LEND_a + INV_a \quad \forall a \in ANGCAP \quad (49)$$

Capital expenses of government:

$$\left(1 - \sum_{g \in GFCF} gfcf\_coef_{g,'GOV'} \right) YCAP_{'GOV'} = \left(\sum_{a' \in ACAP} capshii_{a','GOV'} \right) \overline{CPI} + captrnsfr_{RoW,'GOV'} EXR + LEND_{'GOV'} + INV_{'GOV'} \quad (50)$$

Lending:

$$LEND_{acap} = lnd_{acap} YI_a \quad \forall acap \in ACAP, a \in A \tag{51}$$

Gross fixed capital formation (investment):

$$INVESTGFCF_g = \sum_{a \in ACAP} gfcf\_coef_{g,a} YCAP_{acap} \quad \forall g \in GFCF \tag{52}$$

Gross fixed capital formation (consumption):

$$PQ_c QGFCF_c = gfcfind_{c,g} INVESTGFCF_g \quad \forall c \in C, g \in GFCF \tag{53}$$

Inventories:

$$PQ_c QINV_c = invnt_c \sum_{acap \in ACAP} invA_{acap} YCAP_{acap} \quad \forall c \in C \tag{54}$$

4.4 System block

Rest of the world borrowing from domestic institutions:

$$ROWBRW = fftrnfrEXR \tag{55}$$

Financial flows (restriction replaced below to include ZEROVAR):

$$ROWLND + \sum_{acap \in ACAP} LEND_{acap} = ROWBRW + \sum_{acap \in ACAP} BORROW_{acap} \tag{56}$$

RoW balance in composite foreign currency:

$$\sum_{c \in CM} pwm_c QM_c + \sum_{a \in A} trnsfr_{RoW,a} + \sum_{acap \in ACAP} captrnsfr_{RoW,acap} + \frac{ROWBRW}{EXR} = \sum_{c \in CE} pwe_c QE_c + \sum_{a \in A} trnsfr_{a,RoW} + \sum_{acap \in ACAP} captrnsfr_{acap,RoW} + \frac{ROWLND}{EXR} \tag{57}$$

Commodity balance:

$$QQ_c = \sum_{i \in I} QINT_{c,i} + \sum_{a \in A} QDISP_{c,a} + \sum_{g \in GFCF} QGFCF_{c,g} + \sum_{m \in M} QT_{c,m} + QINV_c \quad \forall c \in C \tag{58}$$

Factor supply (mobile):

$$\overline{QFSM}_f = \sum_{i \in I} QF_{f,i} \qquad \forall f \in FM \tag{59}$$

Factor supply (non-mobile):

$$\overline{QFSS}_{f,i} = QF_{f,i} \qquad \forall f \in FNM, i \in I \tag{60}$$

Total absorption:

$$TABS = \sum_{c \in C} PQ_c \left(\sum_{a \in A} QDISP_{c,a} + \sum_{g \in GFCF} QGFCF_{c,g} + QINV_c \right) \tag{61}$$

Ratio investment to absorption:

$$INVSHR * TABS = \sum_{c \in C} PQ_c \left(\sum_{g \in GFCF} QGFCF_{c,g} + QINV_c \right) \tag{62}$$

Government consumption:

$$GOVSHR * TABS = \sum_{c \in C} PQ_c QDISP_{c,'GOV'} \tag{63}$$

GDP using the consumption approach:

$$GDP = TABS + \sum_{c \in CE} PE_c QE_c - \sum_{c \in CM} PM_c QM_c \tag{64}$$

4.5 Other constraints

Non-negative prices and quantities:

$$PM_c, PE_c, PDS_c, PDD_c, PX_c, PQ_c \geq 0 \quad \forall c \in C \tag{65}$$

$$QM_c, QE_c, QD_c, QX_c, QQ_c \geq 0 \quad \forall c \in C \tag{66}$$

$$PVA_i, PINTA_i, PCW_i \geq 0 \quad \forall i \in I \tag{67}$$

$$QVA_i, QINTA_i, QCW_i \geq 0 \quad \forall i \in I \tag{68}$$

Constraints to avoid having zero values on CES functions (quantities ought to be at least one part in ten thousand of baseline):

$$QD_c \geq 10^{-4} QD_c^{(baseline)} \quad \forall c \in C \tag{69}$$

$$QE_c \geq 10^{-4} QE_c^{(baseline)} \quad \forall c \in C \tag{70}$$

$$QM_c \geq 10^{-4} QM_c^{(baseline)} \quad \forall c \in C \tag{71}$$

$$QXAC_{i,c} \geq 10^{-4} QXAC_{i,c}^{(baseline)} \quad \forall i \in I, c \in C \tag{72}$$

5 Summary

Total number of variables ($|\cdot|$ refers to cardinality):
$5 * |A| + 12 * |C| + |F| + |FM| + 8 * |I| + |GFCF| + |A| * |C| + |C| * |GFCF| +$
$3 * |C| * |I| + |F| * |I| + |FNM| * |I| + |M| + 2 * |M| * |C| + 9$

Total number of equality equations:
$5 * |A| + 12 * |C| + |F| + |FM| + 8 * |I| + |GFCF| + |A| * |C| + |C| * |GFCF| +$
$3 * |C| * |I| + |F| * |I| + |FNM| * |I| + |M| + 2 * |M| * |C| + 10$

Difference: 1 additional equation.
Balance: 1 variable is added (ZEROVAR) and equation 56 is changed to

$ZEROVAR = ROWBRW - ROWLND + \sum_{acap \in ACAP} (BORROW_{acap} - LEND_{acap})$

6 CES parameters

Table 1: CES exponent values for industries.

Account	Industry	ρ_I	ρ_{va1}	ρ_{va2}
I033	Water, sewage and other systems	0.6	0.7	0.2
111	Crop production	0.6	0.7	0.2
112	Animal production and aquaculture	0.6	0.7	0.2
113	Forestry and logging	0.6	0.7	0.2
114	Fishing, hunting and trapping	0.6	0.7	0.2
115	Support activities for agriculture and forestry	0.6	0.7	0.2
211	Oil and gas extraction	0.6	0.7	0.2
212	Mining and quarrying (except oil and gas)	0.6	0.7	0.2
213	Support activities for mining, and oil and gas extraction	0.6	0.7	0.2
221	Utilities	0.6	0.7	0.2
230	Construction	0.6	0.7	0.2
311	Food manufacturing	0.6	0.7	0.2
312	Beverage and tobacco product manufacturing	0.6	0.7	0.2
313-314	Textile and textile product mills	0.6	0.7	0.2
315-316	Clothing and leather and allied product manufacturing	0.6	0.7	0.2
321	Wood product manufacturing	0.6	0.7	0.2
322	Paper manufacturing	0.6	0.7	0.2
323	Printing and related support activities	0.6	0.7	0.2
324	Petroleum and coal product manufacturing	0.6	0.7	0.2
325	Chemical manufacturing	0.6	0.7	0.2
326	Plastics and rubber products manufacturing	0.6	0.7	0.2
327	Non-metallic mineral product manufacturing	0.6	0.7	0.2
331	Primary metal manufacturing	0.6	0.7	0.2
332	Fabricated metal product manufacturing	0.6	0.7	0.2
333	Machinery manufacturing	0.6	0.7	0.2
334	Computer and electronic product manufacturing	0.6	0.7	0.2
335	Electrical equipment, appliance and component manufacturing	0.6	0.7	0.2
336	Transportation equipment manufacturing	0.6	0.7	0.2
337	Furniture and related product manufacturing	0.6	0.7	0.2
339	Miscellaneous manufacturing	0.6	0.7	0.2
410	Wholesale trade	0.6	0.7	0.2
440-450	Retail trade	0.6	0.7	0.2
480-490	Transportation and warehousing	0.6	0.7	0.2
510	Information and cultural industries	0.6	0.7	0.2
520	Finance and insurance	0.6	0.7	0.2
530	Real estate and rental and leasing	0.6	0.7	0.2
540	Professional, scientific and technical services	0.6	0.7	0.2
550	Holding companies	0.6	0.7	0.2
560	Administrative and support, waste management and remediation	0.6	0.7	0.2
610	Educational services	0.6	0.7	0.2
620	Health care and social assistance	0.6	0.7	0.2

710	Arts, entertainment and recreation	0.6	0.7	0.2
720	Accommodation and food services	0.6	0.7	0.2
810	Other services by businesses and non-profit institutions	0.6	0.7	0.2
911	Governments, other federal government services	0.6	0.7	0.2
912	Governments, other provincial and territorial government services	0.6	0.7	0.2
913	Governments, other municipal government services	0.6	0.7	0.2
914	Governments, other aboriginal government services	0.6	0.7	0.2

Table 2: CES exponent values for commodities.

Account	Industry	ρ_{ac}	ρ_t	ρ_q
C048	Water delivered by water works and irrigation systems	0.643	0.643	0.821
C049	Sewage and dirty water disposal and cleaning services	0.643	0.643	0.821
M111B	Grains and other crop products	0.231	0.231	0.615
M112A	Live animals	0.5	0.5	0.75
M11D0	Other farm products	0.231	0.231	0.615
M11E0	Forestry products and services	0.6	0.6	0.8
M1140	Fish, crustaceans, shellfish and other fishery products	0.2	0.2	0.6
M1150	Support services related to farming and forestry	0.6	0.6	0.8
M21B0	Mineral fuels	0.808	0.808	0.904
M2122	Metal ores and concentrates	0.661	0.661	0.831
M2123	Non-metallic minerals	0.655	0.655	0.828
M2130	Mineral support services	0.2	0.2	0.6
M21A0	Mineral and oil and gas exploration	0.655	0.655	0.828
M2200	Utilities	0.643	0.643	0.821
M23A0	Residential construction	0.474	0.474	0.737
M23B0	Non-residential buildings	0.474	0.474	0.737
M23C0	Engineering construction	0.474	0.474	0.737
M23D0	Repair construction services	0.474	0.474	0.737
M31C0	Food and non-alcoholic beverages	0.13	0.13	0.565
M312A	Alcoholic beverages and tobacco products	0.13	0.13	0.565
M31D0	Textile products, clothing, and products of leather and similar materials	0.733	0.733	0.867
M3210	Wood products	0.706	0.706	0.853
M3220	Wood pulp, paper and paper products and paper stock	0.706	0.706	0.853
M3230	Printed products and services	0.661	0.661	0.831
M3240	Refined petroleum products (except petrochemicals)	0.524	0.524	0.762
M3250	Chemical products	0.697	0.697	0.841
M3260	Plastic and rubber products	0.697	0.697	0.848
M3270	Non-metallic mineral products	0.655	0.655	0.828
M3310	Primary metallic products	0.733	0.733	0.867
M3320	Fabricated metallic products	0.733	0.733	0.867
M3330	Industrial machinery	0.753	0.753	0.877
M3350	Electrical equipment, appliances and components	0.773	0.773	0.792
M336A	Transportation equipment	0.767	0.767	0.884
M3363	Motor vehicle parts	0.643	0.643	0.821
M3370	Furniture and related products	0.2	0.2	0.6

M3B00	Computer electronics and other manufactured products	0.773	0.773	0.886
M4100	Wholesale margins and commissions	0.474	0.474	0.737
M4A00	Retail margins, sales of used goods and commissions	0.474	0.474	0.737
M4B00	Transportation and related services	0.474	0.474	0.737
M51D0	Information and cultural services	0.2	0.2	0.6
M51E0	Published and recorded media products	0.2	0.2	0.6
M5170	Telecommunications	0.474	0.474	0.737
M52C0	Depository credit intermediation	0.474	0.474	0.737
M5F00	Other finance and insurance	0.474	0.474	0.737
M53D0	Real estate, rental and leasing and rights to non-finan. intang. assets	0.474	0.474	0.737
M53C0	Imputed rental of owner-occupied dwellings	0.474	0.474	0.737
M541E	Professional services (except software and research and development)	0.2	0.2	0.6
M5E00	Software	0.2	0.2	0.6
M5417	Research and development	0.2	0.2	0.6
M5G00	Administrative and support, head office, waste management and rem.	0.474	0.474	0.737
M6100	Education services	0.474	0.474	0.737
M6200	Health and social assistance services	0.474	0.474	0.737
M7100	Arts, entertainment and recreation services	0.474	0.474	0.737
M7200	Accommodation and food services	0.474	0.474	0.737
M8100	Other services	0.474	0.474	0.737
M9A00	Sales of other services by Non-Profit Institutions Serving Households	0.474	0.474	0.737
M9B00	Sales of other government services	0.474	0.474	0.737
N0000	Services provided by Non-Profit Institutions Serving Households	0.474	0.474	0.737
G6100	Education services provided by government sector	0.474	0.474	0.737
G6200	Health services provided by government sector	0.474	0.474	0.737
G9110	Other federal government services	0.474	0.474	0.737
G9120	Other provincial and territorial government services	0.474	0.474	0.737
G9130	Other municipal government services	0.474	0.474	0.737