CGE model for Canada with water input

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Table of Contents

Detail level SAM

- Total accounts: 857.
- Commodities: 524.
- Industries: 244.
- Gross fixed capital formation: 53.
- Economic agents: HH, NPSH, CORP, GOV.
- Years: 2010-2017.

Detail level SAM

Balancing using linear programming:

```
Entries retrieved:  26000
Entries retrieved:  27000
Entries retrieved:  28000
Entries retrieved:  29000
Runtime Use matrix =  7.179  seconds.


----------  First report on statistical error (SE)  ----------
------------------------------------------------------------
SE is the difference between the sum of income (rows) and the
sum of expenses (columns) on each account of the SAM.

Unbalanced accounts: 12 (out of 857).

HH1 = -939271.0
NPSH1 = -938.0
CORP1 = -4083691.0
GOV1 = -3435405.0
HH3 = 3269333.0
NPSH3 = 6.0
GOV3 = -2134.0
HH_CAP = 23213701.0
NPSH_CAP = -43459.0
CORP_CAP = -23861371.0
GOV_CAP = -54620.0
RoW = 5937849.0


Sum of SE:  0.0
Sum of absolute values of SE:  64841.778  Millions.
Average absolute SE per account:  75.661  Millions.
------------------------------------------------------------
------------------------------------------------------------
```

```
Number of constraints = 103

x[HH1,GOV1] =  5022962.0
x[NPSH1,RoW] =  44391.0
x[CORP1,HH1] =  4083691.0
x[GOV1,RoW] =  8458367.0
x[HH_CAP,HH3] =  3269333.0
x[NPSH_CAP,NPSH1] =  43453.0
x[NPSH_CAP,NPSH3] =  6.0
x[CORP_CAP,GOV_CAP] =  23861371.0
x[GOV_CAP,GOV3] =  -2134.0
x[GOV_CAP,RoW] =  23918125.0
x[RoW,HH_CAP] =  26483034.0


----------  Final report on statistical error (SE)  ----------
------------------------------------------------------------
SE is the difference between the sum of income (rows) and the
sum of expenses (columns) on each account of the SAM.

Unbalanced accounts: 0 (out of 857).



Sum of SE:  0.0
Sum of absolute values of SE:  0.0  Millions.
Average absolute SE per account:  0.0  Millions.
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------------------------------------------------------------
```
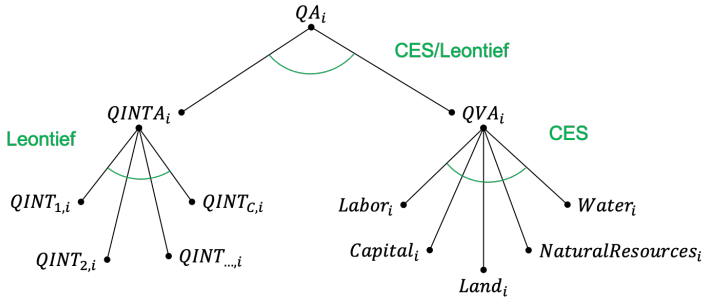

Aggregated SAM

- User-selected industries to single out (e.g. Water & sewage systems).
- User-selected commodities to single out (e.g. Water distribution and irrigation).
- Industries aggregated using medium-level aggregation: 47.
- Commodities aggregated using medium-level aggregation: 66.
- Gross fixed capital formation: 47.
- Total accounts: 180.

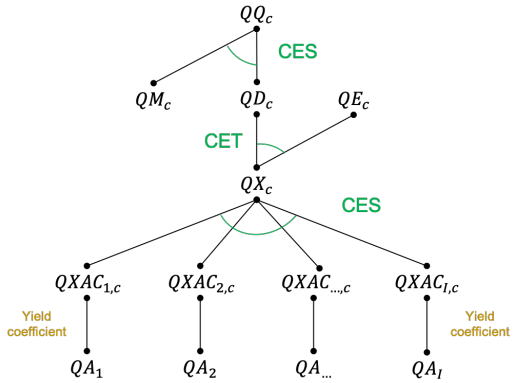
Structure of the CGE model

Production structure:



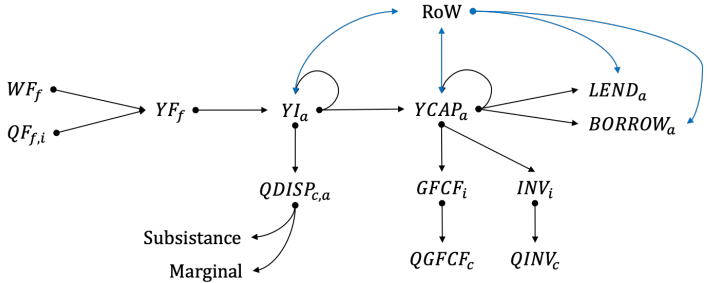
Structure of the CGE model

Domestic output or consumption:



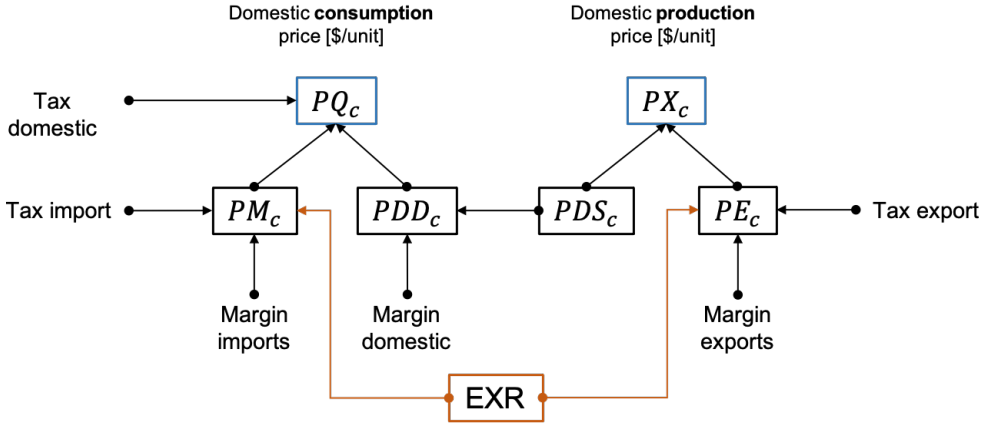
Structure of the CGE model

Economic agents income flows:



Structure of the CGE model

Price structure:



Natural resources & Land

Following the GTAP documentation[1], the share of value-added for a sector-specific primary input is

$$\theta_R = \frac{\theta_{va}\ \sigma_{va}}{\theta_{va}\ \eta_s + \sigma_{va}}$$

- θ_R is the share of the resource (land or natural resource) wrt value-added.
- θ_{va} is the share of value-added.
- σ_{va} is the elasticity of substitution of value-added.
- η_s is the elasticity of supply of the resource.
- Industries with land: 6 (agriculture).
- Industries with natural resources: 16 (forestry, fishing, mining).

[1] Hertel, et al., "Chapter 12.A Primary Factor Shares", GTAP documentation.

Water input

- Water flows [m^3] and payments for water [\$] are obtained from StatsCan.
- These values are used to get the water price [$\$/\mathrm{m}^3$] by sector.
- Missing years are interpolated.
- Water input payments are taken from Capital.
- Total number of industries with water input: 100.
- Aggregated sectors:
  - agriculture
  - mining
  - utilities
  - manufacturing
- SAM adjusted to include new accounts.

Summary of CGE model

- User defines mobile or industry-specific primary input.
- Price block equations: 12
- Production block equations: 18
- Economic agent block equations: 16.
- System block equations: 10.
- Total number of variables: $6\mathrm{A} + 15\mathrm{C} + \mathrm{F} + |\mathrm{FM}| + 7\mathrm{I} + \mathrm{AC} + 3\mathrm{CI} + \mathrm{FI} + |\mathrm{FNM}|\mathrm{I} + 9$.
- Total number of equations: $6\mathrm{A} + 15\mathrm{C} + \mathrm{F} + |\mathrm{FM}| + 7\mathrm{I} + \mathrm{AC} + 3\mathrm{CI} + \mathrm{FI} + |\mathrm{FNM}|\mathrm{I} + 10$.

Example: Primary input supply shock

Assume water supply is increased by 15%.

$\mathrm{WF}_{\mathrm{labor}} = 1.0004$
$\mathrm{WF}_{\mathrm{capital}} = 1.0004$
$\mathrm{WF}_{\mathrm{water}} = 0.8457$
$\mathrm{WF}_{\mathrm{land,agro}} = 0.9993$
$\mathrm{WF}_{\mathrm{NR,agro}} = 0.9993$
$\mathrm{WF}_{\mathrm{NR,mining}} = 1.0005$
$\mathrm{EXR} = 0.9998$
$\Delta\mathrm{GDP} = 0.036\%$

$\mathrm{CHECK} = -2.8 \times 10^{-8}$

Assume capital supply is increased by 15% & natural resources in mining increases by 10%.

$\mathrm{WF}_{\mathrm{labor}} = 1.0735$
$\mathrm{WF}_{\mathrm{capital}} = 0.9079$
$\mathrm{WF}_{\mathrm{water}} = 1.1167$
$\mathrm{WF}_{\mathrm{land,agro}} = 1.1165$
$\mathrm{WF}_{\mathrm{NR,agro}} = 1.1165$
$\mathrm{WF}_{\mathrm{NR,mining}} = 0.9436$
$\mathrm{EXR} = 0.9722$
$\Delta\mathrm{GDP} = 6.06\%$

$\mathrm{CHECK} = 2.5 \times 10^{-6}$

Price equations block I

Import price:

$$PM_c = pwm_c(1 + tm_c)EXR + icm_c \qquad \forall c \in CM \tag{1}$$

Export price:

$$PE_c = pwe_c(1 - te_c)EXR - ice_c \qquad \forall c \in CE \tag{2}$$

Demand price of domestic non-traded commodities:

$$PDD_c = PDS_c + icm_c \qquad \forall c \in CD \tag{3}$$

Absorption (domestic consumption at market value)

$$PQ_c QQ_c(1 - tq_c) = PDD_c QD_c + PM_c QM_c \qquad \forall c \in (CD \cup CM) \tag{4}$$

Marketed output value

$$PX_c QX_c = PDS_c QD_c + PE_c QE_c \qquad \forall c \in CX \tag{5}$$

Industry price

$$PA_i = \sum_{c \in C} PXAC_{i,c}\theta_{i,c} \qquad \forall i \in I \tag{6}$$

Price equations block II

Aggregate intermediate input price

$$PINTA_i = \sum_{c \in C} PQ_c ica_{c,i} \qquad \forall i \in I \tag{7}$$

Activity revenue

$$PA_i QA_i (1 - ta_i) = PVA_i QVA_i + PINTA_i QINTA_i \qquad \forall i \in I \tag{8}$$

Total margins on the input side

$$MAR_c = QM_c icm_c + QE_c ice_c + QD_c icd_c \qquad \forall c \in C \tag{9}$$

Sum of total margins on the input side

$$MARSUM = \sum_{c \in C} MAR_c \tag{10}$$

Consumer price index (exogenous)

$$\overline{CPI} = \sum_{c \in C} PQ_c cwts_c \tag{11}$$

Producer price index

$$DPI = \sum_{c \in C} PDS_c dwts_c \tag{12}$$

Production equations block I

Top level constant elasticity of substitution (CES) production function

$$QA_i = \alpha_i^A \left[\delta_i^A QVA_i^{-\rho_i^a} + (1-\delta_i^A) QINTA_i^{-\rho_i^a} \right]^{-\frac{1}{\rho_i^A}} \qquad \forall i \in ICES \tag{13}$$

$$QVA_i = \left[\frac{\delta_i^a PINTA_i}{(1-\delta_i^a) PVA_i} \right]^{\frac{1}{1+\rho_i^a}} QINTA_i \qquad \forall i \in ICES \tag{14}$$

Top level Leontief production function

$$QVA_i = iva_i QA_i \qquad \forall i \in ILEO \tag{15}$$

$$QINTA_i = inta_i QA_i \qquad \forall i \in ILEO \tag{16}$$

Value-added CES function

$$QVA_i = \alpha_i^{va} \left[\sum_{f \in F} \delta_i^{va} QF_{f,i}^{-\rho_i^{va}} \right]^{-\frac{1}{\rho_i^{va}}} \qquad \forall i \in I \tag{17}$$

$$WFM_f QF_{f,i}^{(\rho_i^{va}+1)} (\alpha_i^{va})^{\rho_i^{va}} = \delta_i^{va} PVA_i QVA_i^{(\rho_i^{va}+1)} \qquad \forall i \in I, f \in FM \tag{18}$$

$$WFS_{f,i} QF_{f,i}^{(\rho_i^{va}+1)} (\alpha_i^{va})^{\rho_i^{va}} = \delta_i^{va} PVA_i QVA_i^{(\rho_i^{va}+1)} \qquad \forall i \in I, f \in FNM \tag{19}$$

Production equations block II

Intermediate input demand

$$QINT_{c,i} = ica_{c,i} QINTA_i \qquad \forall c \in C, i \in I \tag{20}$$

Commodity output

$$QXAC_{i,c} = \theta_{i,c} QA_i \qquad \forall i \in I, c \in CX \tag{21}$$

Output aggregation CES function

$$QX_c = \alpha_c^{ac} \left[\sum_{i \in I} \delta_{i,c}^{ac} QXAC_{i,c}^{-\rho_c^{ac}} \right]^{-\frac{1}{\rho_c^{ac}}} \qquad \forall c \in CX \tag{22}$$

$$PXAC_{i,c} QXAC_{i,c}^{(\rho_c^{ac}+1)} (\alpha_c^{ac})^{\rho_c^{ac}} = \delta_c^{ac} PX_c QX_c^{(\rho_c^{ac}+1)} \qquad \forall i \in I, c \in CX \tag{23}$$

Output constant elasticity of transformation (CET) function

$$QX_c = \alpha_c^t \left[\delta_c^t QE_c^{\rho_c^t} + (1 - \delta_c^t) QD_c^{\rho_c^t} \right]^{\frac{1}{\rho_c^t}} \qquad \forall c \in (CD \cap CE) \tag{24}$$

$$QE_c = \left[\frac{(1 - \delta_c^t) PE_c}{\delta_c^t PDS_c} \right]^{\frac{1}{\rho_i^t - 1}} QD_c \qquad \forall c \in (CD \cap CE) \tag{25}$$

Production equations block III

$$QX_c = QD_c + QE_c \qquad \forall c \in (CD \cap CEN) \cup (CE \cap CDN) \tag{26}$$

Composite CES (Armington) supply function

$$QQ_c = \alpha_c^q \left[\delta_c^q QM_c^{-\rho_c^q} + (1 - \delta_c^q) QD_c^{-\rho_c^q} \right]^{-\frac{1}{\rho_c^q}} \qquad \forall c \in (CD \cap CM) \tag{27}$$

$$QM_c = \left[\frac{\delta_c^q PDD_c}{(1 - \delta_c^q) PM_c} \right]^{\frac{1}{\rho_i^q + 1}} QD_c \qquad \forall c \in (CD \cap CM) \tag{28}$$

$$QQ_c = QD_c + QM_c \qquad \forall c \in (CD \cap CMN) \cup (CM \cap CDN) \tag{29}$$

Margin commodities

$$PQ_c QT_c = im_c MSUM \qquad \forall c \in C \tag{30}$$

Economic agents block I

Factor income

$$YF_f = \sum_{i \in I} WFW_f \, QF_{f,i} \qquad \forall f \in FM \tag{31}$$

$$YF_f = \sum_{i \in I} WFS_{f,i} \, QF_{f,i} \qquad \forall f \in FNM \tag{32}$$

Income of domestic nongovernment agents

$$YI_a = \sum_{f \in F} shif_{a,f} \, YF_f + \sum_{a' \in ANG} shii_{a,a'} \, YI_{a'} + shii_{a,GOV} \, \overline{CPI} + trnsfr_{a,RoW} \, EXR \qquad \forall a \in ANG \tag{33}$$

Government income

$$\begin{aligned} YI_{GOV} = & \sum_{f \in F} shif_{GOV,f} \, YF_f + \sum_{a' \in ANG} shii_{GOV,a'} \, YI_{a'} + trnsfr_{GOV,RoW} \, EXR + \sum_{i \in I} ta_i \, PA_i \, QA_i \\ & + \sum_{c \in C} tq_c \, PQ_c \, QQ_c + \sum_{c \in CM} tm_c \, pwm_c \, PM_c \, QM_c \, EXR + \sum_{c \in CE} te_c \, pwe_c \, PE_c \, QE_c \, EXR \end{aligned} \tag{34}$$

Expenditure of domestic nongovernment agents

$$\left(1 - \sum_{a' \in A} shii_{a',a}\right) YI_a = trnfr_{RoW,a} \, EXR + disp_a \, YI_a + TRPICAP_a \qquad \forall a \in ANG \tag{35}$$

Economic agents block II

Expenditure of government

$$YI_{GOV} = \left(\sum_{a' \in A} shii_{a',GOV} \right) \overline{CPI} + trnfr_{RoW,GOV} EXR + disp_{GOV} YI_{GOV} + TRPICAP_{GOV} \tag{36}$$

Disposable income

$$PQ_c QDISP_c = PQ_c \gamma_{c,a} + \beta_{c,a} \left(disp_a YI_a - \sum_{c' \in C} PQ_{c'} \gamma_{c',a} \right) \quad \forall c \in C, a \in A \tag{37}$$

Capital income of domestic non-goverment agents

$$YCAP_a = TRPICAP_a + \sum_{a' \in ANGCAP} capshii_{a,a'} YCAP_{a'} + capshii_{a,GOV} \overline{CPI} + captrnsfr_{a,RoW} EXR + BORROW_a \qquad \forall a \in ANGCAP \tag{38}$$

Capital income of the government agent

$$YCAP_{GOV} = TRPICAP_{GOV} + \sum_{a' \in ANGCAP} capshii_{GOV,a'} YCAP_{a'} + captrnsfr_{GOV,RoW} EXR + BORROW_{GOV} \tag{39}$$

Capital expenses of non-government capital accounts

$$\left(1 - \sum_{a' \in A} capshii_{a',a} - \sum_{g \in GFCF} gfcfcoef_{g,a} \right) YCAP_a = captrnsfr_{RoW,a} EXR + LEND_a + INV_a \quad \forall a \in ANGCAP \tag{40}$$

Economic agents block III

Government capital expenses

$$\left(1 - \sum_{g \in GFCF} gfcfcoef_{g,a}\right) YCAP_{GOV} = \left(\sum_{a' \in A} capshii_{a',GOV}\right) \overline{CPI} + captrnsfr_{RoW,GOV}\, EXR + LEND_{GOV} + INV_{GOV} \tag{41}$$

Lending

$$LEND_a = lnd_a YI_a \quad \forall a \in ACAP \tag{42}$$

Gross fixed capital formation (industry)

$$QGFCFind_g = \sum_{a \in ACAP} gfcfcoef_{g,a} YCAP_a \qquad \forall g \in GFCF \tag{43}$$

Gross fixed capital formation (commodity)

$$PQ_c QGFCF_c = \sum_{g \in GFCF} gfcfind_g QGFCFind_g \qquad \forall c \in C \tag{44}$$

Inventories

$$PQ_c QINV_c = invnt_c \sum_{a \in ACAP} INV_a \qquad \forall c \in C \tag{45}$$

Change in capital due to a change in inventory

$$INV_a = invA_a YCAP_a \qquad \forall a \in ACAP \tag{46}$$

System equations block I

RoW borrowing from domestic agents

$$RoWBRW = fftrnfrEXR \tag{47}$$

Financial flows

$$RoWLND + \sum_{a \in ACAP} LEND_a = RoWBRW + \sum_{a \in ACAP} BORROW_a \tag{48}$$

RoW balance (foreign currency)

$$\begin{aligned} &\sum_{c \in CM} pwm_c QM_c + \sum_{a \in A} trnsfr_{RoW,a} + \sum_{a \in ACAP} captrnsfr_{RoW,a} + \frac{RoWBRW}{EXR} \\ &= \sum_{c \in CE} pwe_c QE_c + \sum_{a \in A} trnsfr_{a,RoW} + \sum_{a \in ACAP} captrnsfr_{a,RoW} + \frac{RoWLND}{EXR} \end{aligned} \tag{49}$$

Commodity balance

$$QQ_c = \sum_{i \in I} QINT_{c,i} + \sum_{a \in A} QDISP_{c,a} + QGFCF_c + QINV_c + QT_c \qquad \forall c \in C \tag{50}$$

System equations block II

Factor supply (mobile)

$$\overline{QFSM}_f = \sum_{i \in I} QF_{f,i} \qquad \forall f \in FM \tag{51}$$

Factor supply (sector-specific)

$$\overline{QFSS}_{f,i} = QF_{f,i} \qquad \forall f \in FNM, i \in I \tag{52}$$

Total absorption

$$TABS = \sum_{c \in C} PQ_c \left(\sum_{a \in A} QDISP_{c,a} + QGFCF_c + QINV_c + QT_c \right) \tag{53}$$

Ratio investment - absorption

$$INVSHR\ TABS = \sum_{c \in C} PQ_c \left(QGFCF_c + QINV_c \right) \tag{54}$$

Ratio government consumption - absorption

$$GOVSHR\ TABS = \sum_{c \in C} PQ_c QDISP_{c,GOV} \tag{55}$$

GDP

$$GDP = TABS + \sum_{c \in CE} PE_c QE_c - \sum_{c \in CM} PM_c QM_c \tag{56}$$

Inclusion of emissions on the CGE model